C H A P T E R - V

THEORETICAL CONSIDERATIONS

5.1.0. GENERAL

The current research in the area of jointed rocks is centered around to develop a physical and mathematical model amenable to numerical and computational techniques for the analysis and design of structures in and on jointed rocks. The approach has been to understand the mechanism of sliding between the bodies and to delineate the principal factors that differ from sliding between the two bodies assumed in classical mechanics. In classical approach sliding between the two bodies is considered as the energy spent only in friction assuming that the sliding bodies are rigid. To consider the sliding of jointed rocks equivalent to sliding of two rigid bodies is a gross over simplification and entails exclusion of the fundamental factors governing the mechanism of sliding. Besides the energy spent in the basic friction, the mechanism involves the additional components of energy spent in the process of deformations associated with dilatancy. However, the proper phenomenological parameter giving the quantification of the energy components for dilatancy has not been properly established. By incorporating a dilatancy parameter it should be possible to modify the classical Mohr-Coulomb criterion and consequently generate an equation which can be used to develop constitutive relationship for accomplishing numerical solutions.

5.2.0. MECHANISTIC CONCEPT OF SLIDING FOR JOINTED ROCKS

It has been established that the ratio of energy input to the energy out put on an element consisting of two sliding rigid bodies is minimum if these two bodies slide at a critical orientation consistant with the classical laws of friction. The critical angle of sliding depends on the frictional properties of the interface. If the sliding bodies are made to slide at an orientation deviating from the critical orientation there will be change in the ratio of energy input to energy output which is reflected in the deformation of element subjected to stress. In case of interface material following Coulomb's law the effect would be the shear deformation associated with volume change reflective of dilatancy. The approach therefore should be to consider the equilibrium conditions for sliding between two bodies to determine the critical angle of sliding so as to slide under minimum energy ratio and to evalute the effect of dilation resultant due to the sliding of the bodies at an orientation deviating from the critical angle of sliding.

5.3.0. FAILURE CRITERION FOR JOINTED ROCK

Figure 5.1 represents a basic equilibrium diagram for two elemental blocks in contact on a sliding plane having physical properties of cohesion and friction.

Refer Fig. 5.1

Resolving the cohesion forces in P and Q directions and balancing by friction, the resultant forces are:

$$P - C_f \times \operatorname{Sec}\beta \operatorname{Cos}\beta = R \qquad \text{....5.3.1}$$

$$Q + C_f \times \operatorname{Sec}\beta \operatorname{Sin}\beta = T \qquad \text{....5.3.2}$$

Now considering the frictional forces, we get,

$$\frac{R}{T} = \tan \phi = \tan (\phi_f + \beta) \qquad \text{....5.3.3}$$

Substituting for R and T from equations 5.3.1 and 5.3.2

We get,

$$\frac{P - C_f x}{Q + C_f x \tan\beta} = \tan(\phi + \beta) \qquad \text{....5.3.4}$$

From which

$$P - Q\tan(\phi_f + \beta) + \frac{C_f x \operatorname{Sec}^2\beta}{1 - \tan\phi_f \tan\beta} \qquad \text{....5.3.5}$$

For axisymmetrical stress conditions as in case of conventional triaxial compression test where $\sigma_1 > \sigma_2 = \sigma_3$ the equilibrium equations can be obtained by integrating the principal stresses along a sliding plane oriented at an angle α to the major principal plane.

Refer Fig. 5.2 (a) and 5.2 (b)

$P = \sigma_1 b$, $Q = \sigma_3 b \tan\alpha$ and $x = b \tan\alpha$

Substituting these values into equation 5.3.5 We get,

$$\sigma_1 = \sigma_3 \tan\alpha \tan(\phi_f + \beta) + \frac{C_f \tan\alpha \operatorname{Sec}^2\beta}{1 - \tan{-}\beta \tan\phi_f} \qquad \text{....5.3.6}$$

$$\frac{\sigma_1}{\sigma_3} = \tan\alpha \tan(\phi_f + \beta) + \frac{C_f}{\sigma_3} \frac{\tan\alpha \operatorname{Sec}^2\beta}{1 - \tan\beta \tan\phi_f} \qquad \text{..5.3.7}$$

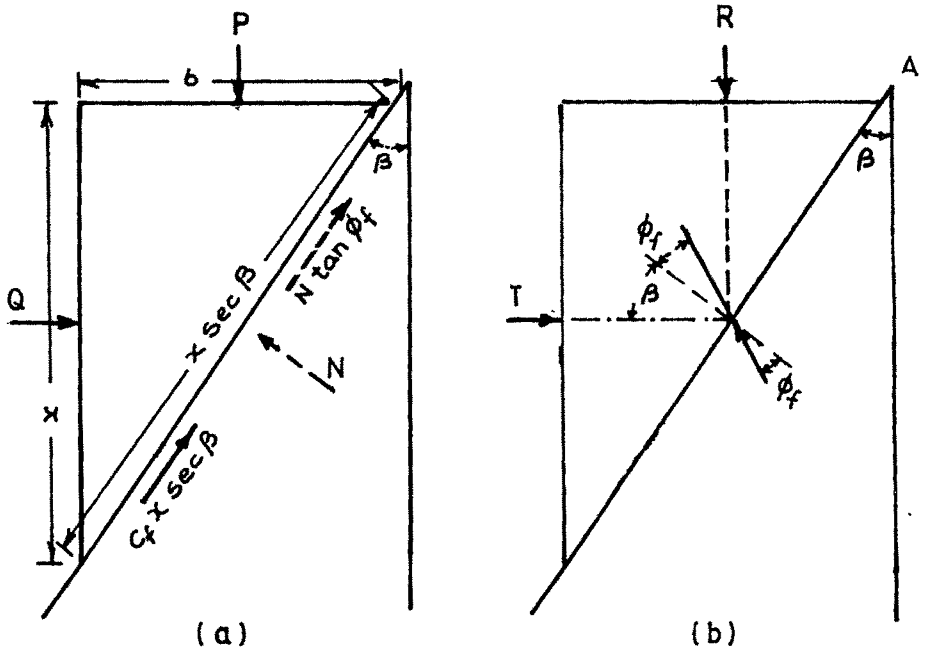

FIG. 5·1 BASIC EQUILIBRIUM DIAGRAM

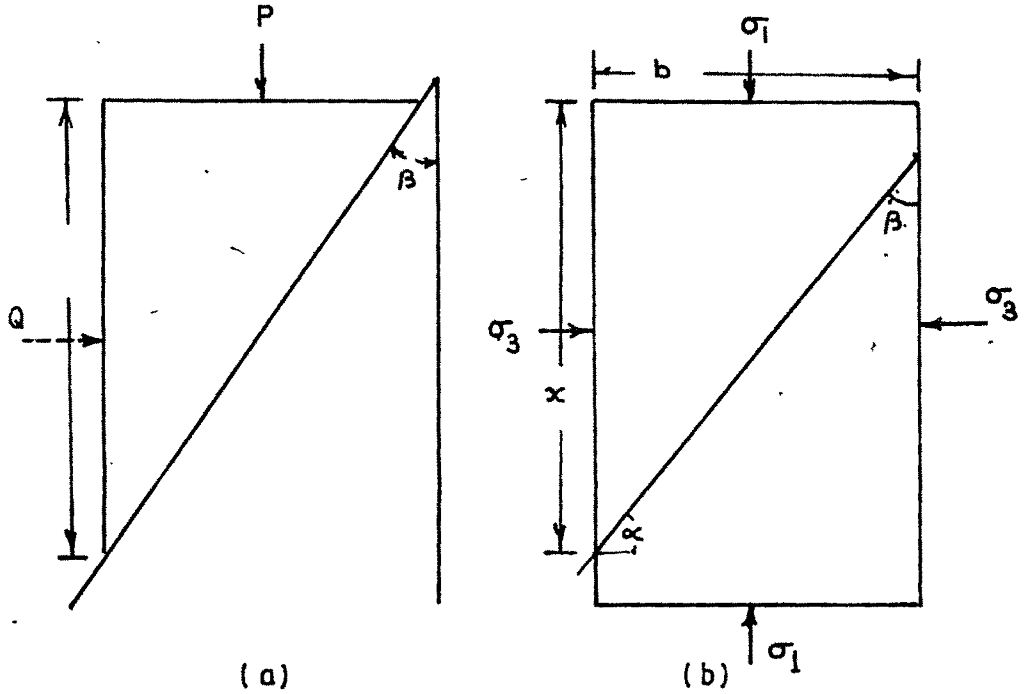

FIG. 5·2 EQUILIBRIUM DIAGRAM FOR AXISYMMETRICAL STRESS CONDITIONS ($\sigma_1 > \sigma_2 = \sigma_3$)

Refer Fig. 5.3

When the cylindrical specimen is subjected to stress conditions $\sigma_1 > \sigma_2 = \sigma_3$ there will be changes in the major and minor principal strains denoted by $\delta\epsilon_1$ and $\delta\epsilon_3$ respectively. It can be shown by the geometry that the ratio of principal strains will be given by;

$$\frac{2\,\delta\epsilon_3}{\delta\epsilon_1} = \tan\alpha \tan\beta \qquad \text{.... 5.3.8}$$

As a consequence of strain increaments due to stress increments there will be increment in internal work which can be given by

$$\Delta W = \sigma_1\,\delta\epsilon_1 - 2\sigma_3\,\delta\epsilon_3 \qquad \text{....5.3.9}$$

Or

$$\Delta W = \sigma_1\,\delta\epsilon_1 \left[1 - \frac{2\sigma_3\,\delta\epsilon_3}{\sigma_1\,\delta\epsilon_1}\right] \qquad \text{....5.3.10}$$

The ratio of ENERGY INPUT to ENERGY OUTPUT at any instant will be given by

$$\Delta E = \frac{\sigma_1\,\delta\epsilon_1}{2\sigma_3\,\delta\epsilon_3} = \frac{\sigma_1}{\sigma_3}\tan\alpha\tan\beta \qquad \text{....5.3.11}$$

Substituting in equation 5.3.10 We get,

$$\Delta W = \sigma_1\,\delta\epsilon_1 \left[1 - \frac{1}{\Delta E}\right] \qquad \text{....5.3.12}$$

For minimum ΔW, the ΔE must be minimum.

For ΔE to be minimum,

$$\frac{d\Delta E}{d\beta} = 0 \qquad \text{....5.3.13}$$

This gives,

$$\beta_c = (45 - \phi_f/2) \qquad \text{....5.3.14}$$

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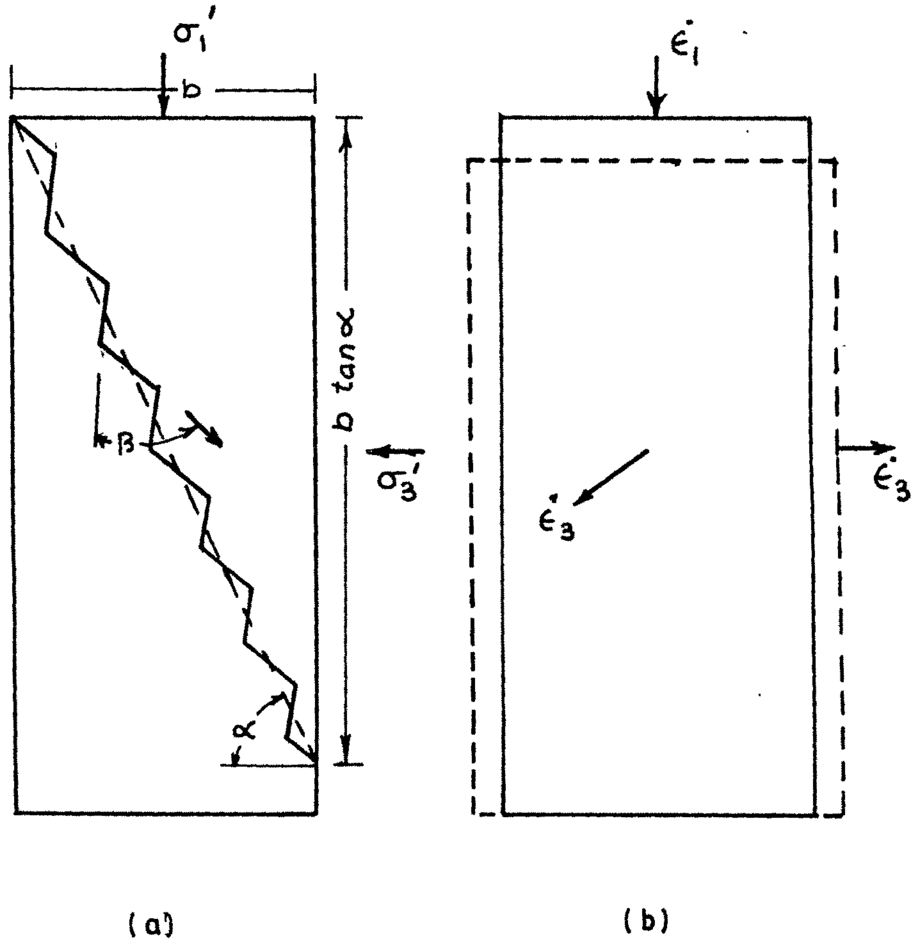

FIG. 5·3 STRESS AND STRAIN RATIO RELATIONSHIPS

It is the critical direction of sliding so as to absorb minimum work.

Substituting $\beta = (45 - \phi_f/2)$ in equation 5.3.7 We get,

$$\frac{\sigma_1}{\sigma_3} = \tan\alpha \tan\left(45+\frac{\phi_f}{2}\right) + \frac{C_f}{\sigma_3}\,\frac{\tan\alpha \sec^2(45-\phi_f/2)}{1-\tan(45+\phi_f/2)\tan\phi} \quad ..5.3.15$$

Which will reduce to

$$\sigma_1 = \tan\alpha\left[\sigma_3 \tan(45+\phi_f/2) + 2\,C_f\right] \quad ....5.3.16$$

And, substituting $\beta = (45-\phi_f/2)$ in equation 5.3.8 We get,

$$\frac{2\,\delta\epsilon_3}{\delta\epsilon_1} = \tan\alpha \tan(45-\phi_f/2) \quad ....5.3.16\ (a)$$

$$= \frac{\tan\alpha}{\tan(45+\phi_f/2)} \quad ....5.3.16\ (b)$$

The volumetric change, in a specimen is:

$$\delta v = \delta\epsilon_1 + (-2\,\delta\epsilon_3) \quad ....5.3.17$$

Therefore,

$$\frac{2\,\delta\epsilon_3}{\delta\epsilon_1} = \left(1 - \frac{\delta v}{\delta\epsilon_1}\right) \quad ....5.3.18$$

Thus, $\dfrac{\tan\alpha}{\tan(45+\phi_f/2)}$ represents the measure of dilation at sliding denoted by D

If there is 'no dilatancy' $\delta v = 0$ whence

$\tan\alpha = \tan(45+\phi_f/2)$

i.e. $\alpha = 90° - \beta_c = 45° + \dfrac{\phi_f}{2}$ 5.3.19

Mohr-Coulumb failure criterion is strictly applicable if $\delta\vartheta = 0$. If $\delta\vartheta \neq 0$, Mohr-Coulumb failure criterion needs to be modified for dilatancy effect.

The classical Coulumb law is:

$$\tau_f = C_f + \sigma_f \tan \phi_f \qquad \ldots.5.3.20$$

This law is required to be modified for the effect of dilatancy, which can be achieved by dividing major principal stress σ_1 with a dilatancy parameter, $D = \tan\alpha / \tan(45+\phi_f/2)$

Figure 5.4 is the represantation of Coulumb's equation modified for dilatancy.

$$\tau_f = \chi + \sigma_f \tan \Psi \qquad \ldots.5.3.21$$

where

$$\tau_f = \left(\frac{\sigma_1}{D} - \sigma_3\right)_f \; ; \; \sigma_f = \left(\frac{\sigma_1}{D} + \sigma_3\right)_f$$

$$C_f = \frac{\chi}{2} \mathrm{Sec}^2 \phi_f \; ; \; \mathrm{Sin}\, \phi_f = \tan \Psi$$

In context of a unified numerical solution on the basis of plasticity the failure equation needs to be expressed in convenient form of stress invariants, J_1, J_2 and J_3. For mathematical amenability Nayak and Zienkiewick (1963) proposed an alternative form for third stress invarient J_3 as:

$$\mathrm{Sin}\, 3\phi_o = \left[\frac{-3\sqrt{3}}{2} \cdot \frac{J_3}{J_2^{3/2}}\right] \qquad \ldots.5.3.22$$

and $\quad -\pi/6 \leq \phi \leq \pi/6$

Thus, the three principal stresses of σ_{ij} can be given as for $\sigma_1 > \sigma_2 > \sigma_3$

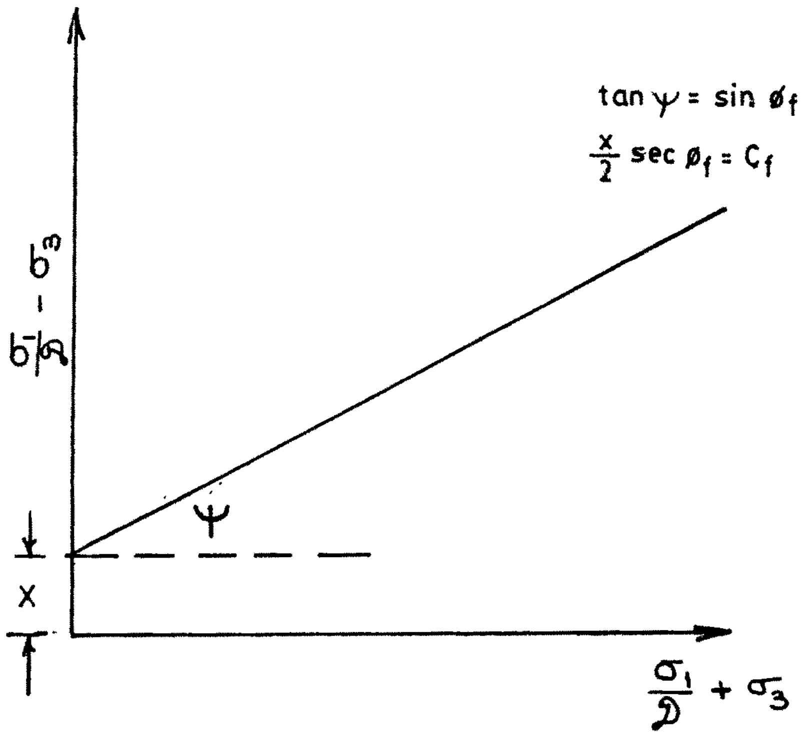

FIG. 5·4 MOHR COULOMB'S EQUATION MODIFIED FOR DILATANCY

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \frac{2}{\sqrt{3}} \sqrt{J_2} \begin{Bmatrix} \sin(\phi_o + 2/3\pi) \\ \sin \phi_o \\ \sin(\phi_o + \frac{4}{3}\pi) \end{Bmatrix} + \begin{Bmatrix} J_1/3 \\ J_1/3 \\ J_1/3 \end{Bmatrix} \quad \ldots .5.3.23$$

From above equation 5.3.23, Mohr-Coulomb failure criterion can be expressed as

$$F = J_1/3 \sin \phi_f + \sqrt{J_2} \cos \phi_o - \frac{\sqrt{J_2}}{\sqrt{3}} \sin \phi_o \sin \phi_f - C \cos \phi_f \quad ..5.3.24$$

Figure 5.5 shows yield surface in the principal stress space for the equation 5.3.24 which is a right hexagonal pyramid, the axis of which is inclined with the stress axis. The intersection of the pyramid with π plane $\sigma_1+\sigma_2+\sigma_3=0$ is shown by the dotted line.

Drucker and Prager (1952) proposed an approximation to Mohr-Coulomb law as a Heuber-Mises yield criterion in the form of circular cone with the intersection shown in figure 5.6. The generalized criterion can be written as below:-

$$F = \sqrt{J_2} - \lambda J_1 - K = 0 \quad \ldots .5.3.25$$

Where λ and K are positive material parameters.
Refer Fig. 5.6

The values of λ and K can be expressed in terms of angle of internal friction ϕ_f and cohesion C_f. The values of λ and K for conventional triaxial compression test where the circular cone with the intersection shown in Fig. 5.6. the constants are

$$\lambda = \frac{2 \sin \phi_f}{\sqrt{3}(3-\sin \phi_f)} \quad \ldots .5.3.26\ (a)$$

and

$$K = \frac{6\ C \cos \phi_f}{\sqrt{3}(3-\sin \phi_f)} \quad \ldots .5.3.26\ (b)$$

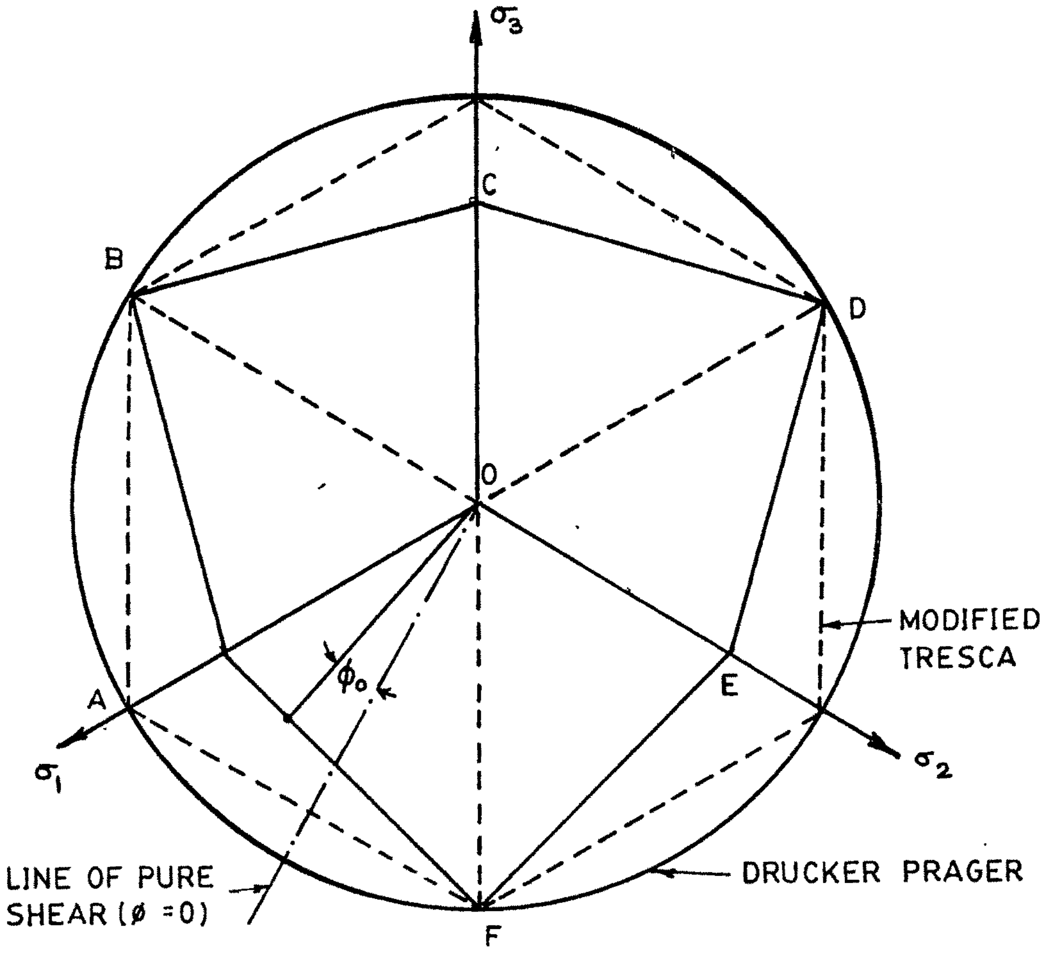

FIG. 5·5 INTERSECTION OF PYRAMID WITH π PLANE

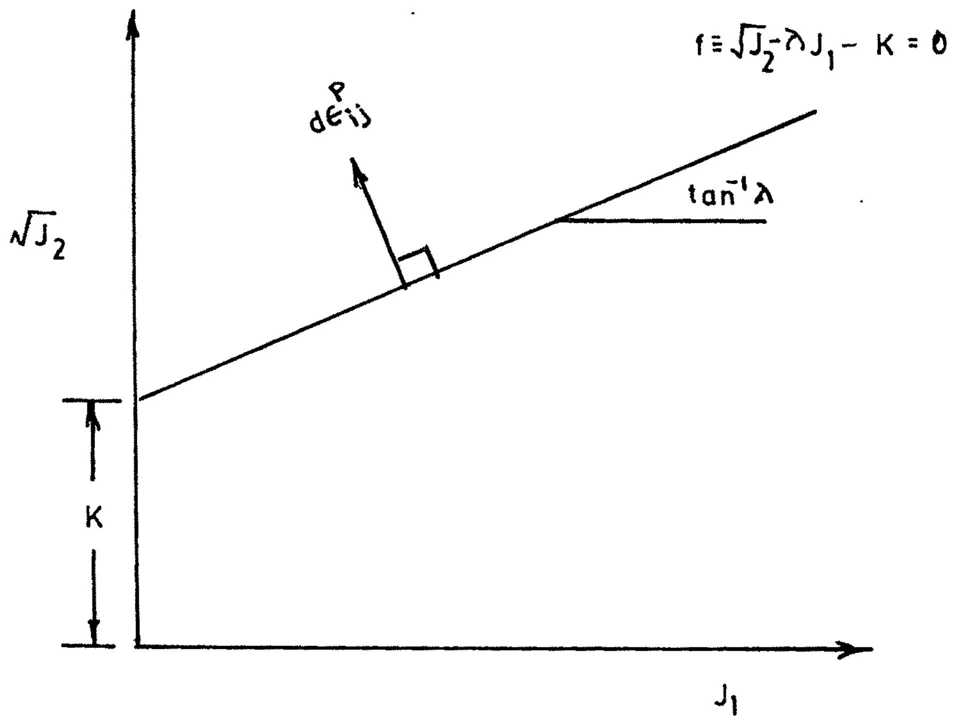

FIG. 5·6 MODIFICATION OF MOHR COULOMB LAW BY DRUCKER PRAGER and another cone with an intersection with the inner circle in figure 5.7 the constants will be

$$\lambda = \frac{2 \sin \phi_f}{\sqrt{3}\,(3+\sin \phi_f)} \qquad \text{....5.3.27 (a)}$$

and

$$K = \frac{6\,C \cos \phi_f}{\sqrt{3}\,(3+\sin \phi_f)} \qquad \text{....5.3.27 (b)}$$

5.4.0. CONSTITUTIVE RELATIONS FOR JOINTED ROCKS

It has been the contention of many research workers to treat jointed rock as blocks separated by joints with special properties as against treating the jointed rock as continuum and proposed joint elements for conducting finite element analysis.

The approach is similar as solid elements that is the development of stiffness matrix $[K]$

$$[K]_i = \int_v [B]^T [C^{ep}]_i \, [B]\, dv \qquad \text{....5.4.1}$$

where $[B]$ = transformation matrix

v = volume

$[C^{ep}]_i$ = constitutive matrix

Then the element equations are written as

$$[K]_i \{g\} = \{Q\} \qquad \text{....5.4.2}$$

where $\{g\}$ = vector of nodal displacement

$\{Q\}$ = vector of nodal forces.

The constitutive matrix for the jointed element can be written as

$$[C]_i^{ep} = \left[C^e \{KS,\ K_n\} - \left[C^p(K_s, K_n, \{du_r^p\}\right]\right] \qquad \text{....5.4.3}$$

where $\{du_i^p\}$ = vector of incremental relative displacement.

The first part of constitutive matrix can be obtained by considering the behaviour as linear elastic or non linear elastic such as hyperbolic simulation through theory of elasticity.

The second part of consitutive matrix may be obtained on the basis of yield and flow criterion of theory of plasticity.

In article 5.3.0 an yield function as suggested by Drucker-Prager is a modification to Mohr-Coulomb criterion for facilitating the use of frame work of theory of plasticity. The yield function is expressed as euqtion 5.3.25

$$F = \sqrt{J_2} - \lambda J_1 - K = 0$$

To evalute the normal vector the derivation of yield function with stresses is determined for its use in general numerical solution progress.

$$\frac{\partial F}{\partial \{\sigma\}} = C_1 \frac{\partial (J_1/3)}{\partial \{\sigma\}} + C_2 \frac{\partial \sqrt{J_2}}{\partial \{\sigma\}} + C_3 \frac{\partial J_3}{\partial \{\sigma\}} \quad \ldots.5.4.5$$

For Drucker-Prager case, $C_1 = 3\lambda$, $C_2 = 1$, $C_3 = 0$.

5.5.0. CONCLUDING REREMARKS

A mechanistic model for the shearing behaviour of jointed rock is proposed from first principles. An approximate but in convenient form a yield function in terms of stress invarients is presented and a possible approach for developing the constitutive relationship has been indicated. With the theoretical development presented it should be possible to analyse the experimental investigations proposed for the present project and should be able to contribute a step further in understanding a shearing behaviour of jointed rock.