CHAPTER 1

GENERAL INTRODUCTION

BEARINGS

When two dry metallic surfaces which are parts of a machine move relatively in contact with each other, friction and wear are caused. While friction consumes and wastes energy, wear causes loss of material. Thus the life of the machine is considerably reduced. To minimize this we insert substances such as viscous fluid, liquid metal etc., called lubricants, to keep the moving surfaces apart so as to allow them to slide on each other with the least effort.

The lubricants are selected according to the types of the bearing used. Emulsions of oil and water are used as lubricants for cutting tools while water itself is used in some marine bearings. Milk and air are used as lubricants in the bearings of cream separators and in some high speed spindles respectively. In addition to their function of reducing friction, the lubricants usually perform the additional function of carrying away the major portion of the heat generated by friction. Thus, the theory of lubrication deals not only with the ways and means of minimizing friction and wear but with the viscous dissipation of heat also.

Owing to rapid industrialization there has been a spurt in machines and bearings are the most prominent constituents of todays' machines. Bearing problems arise whenever the mechanical engineer has to transmit mechanical power from one point to another. These are similar to those faced by the electrical engineer in the transmission of electric power. The electrical engineer has to choose the insulators with the same care as the mechanical engineer chooses the bearings. Cameron [1] says that the functions of the insulators and the bearings are similar and the outcome of their failure in service is no less serious.

The lubrication of two surfaces moving relative to each other depends upon a number of factors like load, relative velocity of the surfaces, geometry of the surfaces, physical and chemical properties of lubricants and the metals out of which the surfaces are made.

The efforts of the lubrication engineers are all centered around improving upon bearing performance by making use of recent technological advancements. The development of powder metallurgy gave rise to considerable use of porous bearings. Owing to their low cost, ready availability, design simplicity and self-lubricating properties, the porous bearings have become increasingly popular and practically indispensable for many applications in automotive machines, domestic appliances and industry.

1.1 Description of porous bearings

A porous bearing consists of a bearing bush of porous sintered metal. Porosity is defined as the ratio of pore volume to the total volume of a given sample of material and permeability as a measure of the ease with which fluids pass through a porous material [2]. While the former is dimensionless, the latter has the dimension of the square of length and is independent of density and the viscosity of the fluid. Porous bearings were suggested in 1920's, perhaps to improve upon the heat conductivity limitations of oil-soaked wooden bearings.

The commonly used porous bearing material [3] contains 90 % copper and 10 % tin. After blending powdered copper and tin into a mixture, the mixture is pressed under high pressure to half its original volume. Then it is sintered into a coherent solid. The bore and faces are then sized in a burnishing operation. The porosity arises out of the incomplete solidification of compressed particles of mixture so that the pores are intercommunicating. After their fabrication the pores of the porous bearings are filled with lubricant by vacuum impregnation. If any lubricant is lost from the film during operation, it is replaced by the lubricant, stored in the pores, which serves as an additional supply and which remains effective throughout the bearing life.

1.1.1 Advantages and disadvantages

As the continuity of lubricant supply is maintained, porous bearings can run hydrodynamically longer without maintenance and are more stable than the equivalent conventional bearings. The self-lubricating natures of the porous bearings overcomes the need for pipes, pumps, etc. and simplifies the problems of machine design. Moreover, the possibility of soiling manufactured articles handled in textile, printing, baking and other industries by lubricant drip is avoided. The porous bearings can be easily fitted even in positions which are not readily accessible and the need for frequent maintenance is overcome. The friction in porous bearings is less than that in the corresponding non-porous ones. As a result of mass production, the porous bearings are cheaper than the equivalent non-porous ones.

Moreover, in a porous journal bearing the effect of cavitation is reduced [4] because it operates at lower pressures than the corresponding non-porous one for a given speed. The design of externally pressurized porous thrust bearings is compact and there is an even distribution of pressure at the thrust face compared to the conventional ones. Gas porous bearings are excellent for feasibility demonstrations. They are advantageously used where large bearing gaps are required and less design work is possible.

In spite of the above advantages, the porosity introduces some disadvantages also. The presence of porosity results in a loss of mechanical strength and a reduction in film pressure and hence in the load capacity in comparison to the identical non-porous bearings. In squeeze film bearings the time taken to attain a specified film thickness is also reduced. Efforts to overcome some of the above disadvantages have been made by the introduction of electromagnetic effects and making the bearing bush to have multi-layered non-homogeneous porous housing [5,6].

1.1.2 Applications

The applications of porous bearings are wide and varied. The porous bearings are used in many devices such as vacuum cleaners, extractor fans, motor car starters, hair dryers etc. They are also used in business machines, farm and construction equipments, aircraft automotive accessories etc. Heller et.al [7] mention the use of hydrostatic gas bearings in miniature turbomachinery.

Sneck [8] has given various applications of gas lubricated porous bearings. Gas bearing is used in the tape-support section of a magnetic recording device. The moving tape never contacts the bearing, and there is no bearing inertia with rapid starting and stopping. Gas porous bearings are also used in precision machine tools, fixtures, electrostatic printing machines, guidance devices for magnetic film and photographic film transports, high temperature environments as in textile industry, dirty environments as in production line machine tools, and in optical systems where low friction and precision positioning are important. These variety of applications show how indispensable the porous bearings are to the designers.

It is usually assumed that the type of lubrication found in porous bearings, lubricated only by the oil initially within their structure, is mixed or boundary. Though this might be true for low porosity bearings supporting high loads and running at low speeds, for lightly loaded high porosity bearings running at moderate to high speeds fluid film or hydrodynamic conditions are achieved. In the sequel we will be mainly concerned with fluid film lubrication in which the conditions of operations of the bearing are such that the film of lubricant is so thick that metal to metal contact between the moving surfaces is prevented and the only friction is that which occurs in the fluid film.

1.2 MODIFIED REYNOLDS EQUATIONS FOR POROUS BEARINGS

The porous bearings operate under complex conditions. Their analysis shows that the flow of lubricant occurs in the film region, in the porous region and across the interface of these regions. So, there is a coupling between the flows in the two regions.

In the following sections we present the mathematical formulation of the problems and give the derivations of modified Reynolds equations for hydrodynamic and hydromagnetic lubrication and hydromagnetic lubrication of a porous bearing with anisotropic permeability and slip velocity, which serve as the basic equations for the analysis of the problems of porous metal lubrication.

1.2.1 Reynolds equation for hydrodynamic lubrication

In deriving this equation we make the following assumptions:

1. The lubricant is Newtonian with constant density and viscosity and the flow is laminar.
2. There are no external forces acting on the lubricant.
3. The lubricant film thickness is very small in comparison with the dimensions of the bearing, as

a consequence of which curvature of the film may be ignored and rotational velocities may be replaced by translational velocities.

4. There is no variation of pressure across the film.
5. Fluid inertia is small compared to viscous forces.
6. Velocity gradients across the flim predominate.
7. The porous region is homogeneous and isotropic.
8. The flow in the porous region is governed by Darcy's law.
9. Pressures and normal velocity components are continuous at the interfaces.
10. Bearing is press-fitted in a solid housing.

Let us consider a fluid film of thickness $h = h(x, z)$ between two surfaces with porous facings of permeability k (Fig. 1). The upper and lower surfaces move with velocities (U_2, V_2, W_2) and (U_1, V_1, W_1) respectively.

Making the above assumptions of hydrodynamic lubrication, the equations governing the steady flow of the lubricant in different regions are

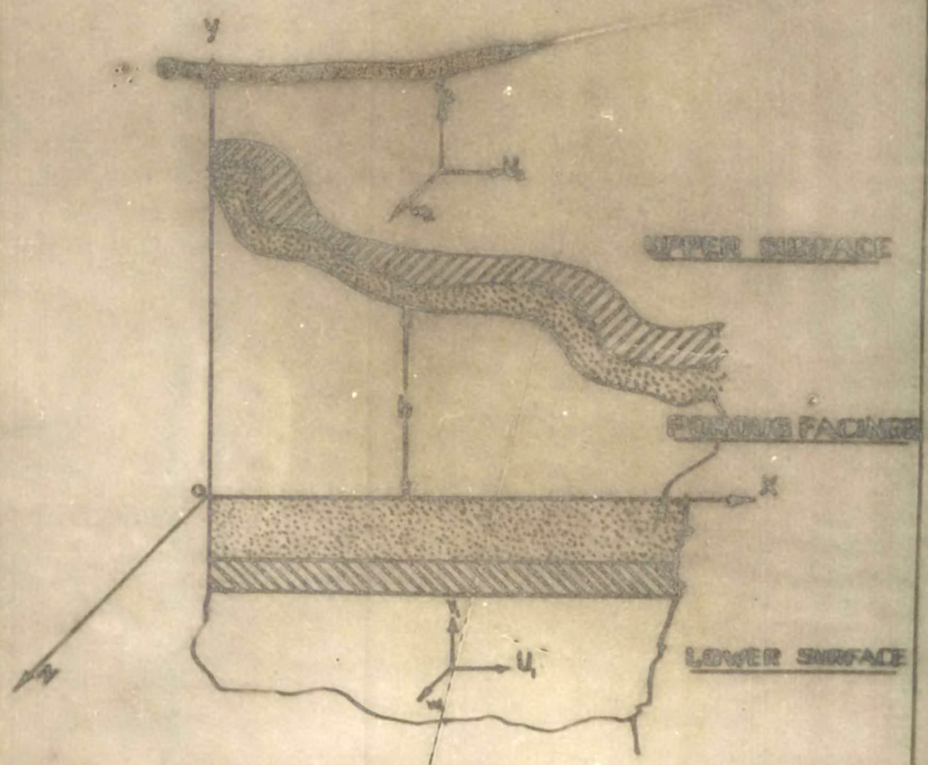

FIG. 1. POROUS BEARING.

Film region :

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \qquad (1)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial z} \qquad (2)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (3)$$

$$\frac{\partial}{\partial x}\int_0^h u \, dy + v_h - v_0 + \frac{\partial}{\partial z}\int_0^h w \, dy = 0 \qquad (4)$$

Porous region of the lower surface :

$$\vec{V} = -\frac{k}{\mu} \nabla P_1 \qquad (5)$$

$$\nabla . \vec{V} = 0 \qquad (6)$$

Porous region of the upper surface :

$$\vec{V}^* = -\frac{k}{\mu} \nabla P_2 \qquad (7)$$

$$\nabla . \vec{V}^* = 0 \qquad (8)$$

where $\vec{q}$, $\vec{V}$, $\vec{V}^*$ are the velocities and p, P_1, P_2 the pressures of the fluid in the flim region, and the lower and the upper porous regions respectively.

Solving equations (1) and (2) for u and w under the no-slip boundary conditions

$$u = U_2 \quad \text{when} \quad y = h, \quad u = U_1 \quad \text{when} \quad y = 0$$

$$w = W_2 \quad \text{when} \quad y = h, \quad w = W_1 \quad \text{when} \quad y = 0$$

and substituting in equation (4) we obtain

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z} \right)$$

$$= 6\mu \left[\frac{\partial}{\partial x}\left\{ (U_1+U_2)h \right\} + \frac{\partial}{\partial z}\left\{ (W_1+W_2)h \right\} + 2(v_h - v_0) \right] \qquad (9)$$

Equations (5) - (8) show that P_1 and P_2 satisfy the Laplace equation

$$\nabla^2 P = 0 \qquad (10)$$

Owing to the continuity of velocities at the interfaces, we have

$$v_0 = V_1 - \frac{k}{\mu} \left(\frac{\partial P_1}{\partial y} \right)_{y=0} \qquad (11)$$

and

$$v_h = V_2 - \frac{k}{\mu} \left(\frac{\partial P_2}{\partial y} \right)_{y=h} \qquad (12)$$

Substituting equations (11) - (12) into (9) we obtain the modified Reynolds equation for porous bearings as

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z} \right)$$

$$= 6\mu \left[\frac{\partial}{\partial x}\left\{(U_1 + U_2)h\right\} + \frac{\partial}{\partial z}\left\{(W_1 + W_2)h\right\} + 2(V_2 - V_1) \right] - 12k \left[\left(\frac{\partial P_2}{\partial y} \right)_{y=h} - \left(\frac{\partial P_1}{\partial y} \right)_{y=0} \right] \qquad (13)$$

Corollary 1.(1)

In most applications we consider the upper surface as non-porous and moving with a uniform velocity U in the x-direction together with a normal velocity V_h and that the lower surface is stationary and has a porous facing of thickness H. So, taking $U_2 = U$, $U_1 = 0$, $W_1 = W_2 = 0$, $V_1 = 0$, $P_1 = P$, $V_2 = V_h$ and $\left(\frac{\partial P_2}{\partial y} \right)_{y=h} = 0$, equation (13) reduces to

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z} \right)$$

$$= 6\mu U \frac{\partial h}{\partial x} + 12\mu V_h + 12k \left(\frac{\partial P}{\partial y} \right)_{y=0} \qquad (14)$$

where the pressure P in the porous region satisfies the Laplace equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0 \qquad (15)$$

Equation (14) which gives the film pressure is a coupled equation consisting of the pressure in the porous region. Using the Morgan-Cameron approximation [9] that, when H is small, the pressure in the porous region can be replaced by the average pressure with respect to the bearing-wall thickness and which was extensively used by Prakash and Vij [10] and others, it is uncoupled by substituting

$$\left(\frac{\partial P}{\partial y} \right)_{y=0} = - H \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) \qquad (16)$$

in it. Thus we obtain the modified equation

$$\frac{\partial}{\partial x}\left[(h^3 + 12kH) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z}\left[(h^3 + 12kH) \frac{\partial p}{\partial z} \right] = 6 \mu U \frac{\partial h}{\partial x} + 12 \mu V_h \qquad (17)$$

Hence the problem of finding the film pressure is reduced to the solution of equation (17) with appropriate boundary conditions.

1.2.2 Reynolds equation for hydromagnetic lubrication

The problem of using liquid metals as lubricants has recently become of interest. The study of the lubrication properties of liquid metals shows that liquid metals can be advantageously used in preference to ordinary oils in space vehicles where extremely high temperatures and speed occur, because of their high operating temperatures and thermal conductivity. The application of an external magnetic field results in electromagnetic pressurization in liquid metals owing to their large electrical conductivity. The study of hydrodynamic lubrication with an external magnetic field is called hydromagnetic or magnetohydrodynamic (MHD) lubrication.

The general MHD equations are complicated owing to the coupling of Maxwell's equations with hydrodynamic equations. So, we make the following simplifying assumptions :

1. The Lorentz force $\vec{J} \times \vec{B}$ is the only external force acting on the lubricant.

2. The induced magnetic fields are negligible in comparison to the applied magnetic field.

3. The flow in the porous medium satisfies the modified Darcy's law due to Ene [11].

We consider a lubricant film of thickness $h = h(x,z,t)$ within a slider bearing. The slider is non-porous and moving with a relative velocity U in the x-direction together with a normal velocity V_h. The lower surface has a porous facing of thickness H which is backed by a solid wall. The surfaces are supposed to be non-conducting while the fluid is considered to be conducting electrically. A uniform magnetic field is applied in the y-direction as in Fig. 2. The general equations governing the flow in the film region [12] are

$$\rho(\vec{q}\cdot\nabla)\vec{q} = -\nabla p + \mu\nabla^2\vec{q} + \vec{J}\times\vec{B}$$

$$\nabla\times\vec{B} = \mu_e\vec{J}$$

$$\nabla\times\vec{E} = 0$$

$$\nabla\cdot\vec{B} = 0$$

$$\nabla\cdot\vec{J} = 0$$

$$\vec{J} = \sigma(\vec{E} + \vec{q}\times\vec{B})$$

$$\nabla\cdot\vec{q} = 0$$

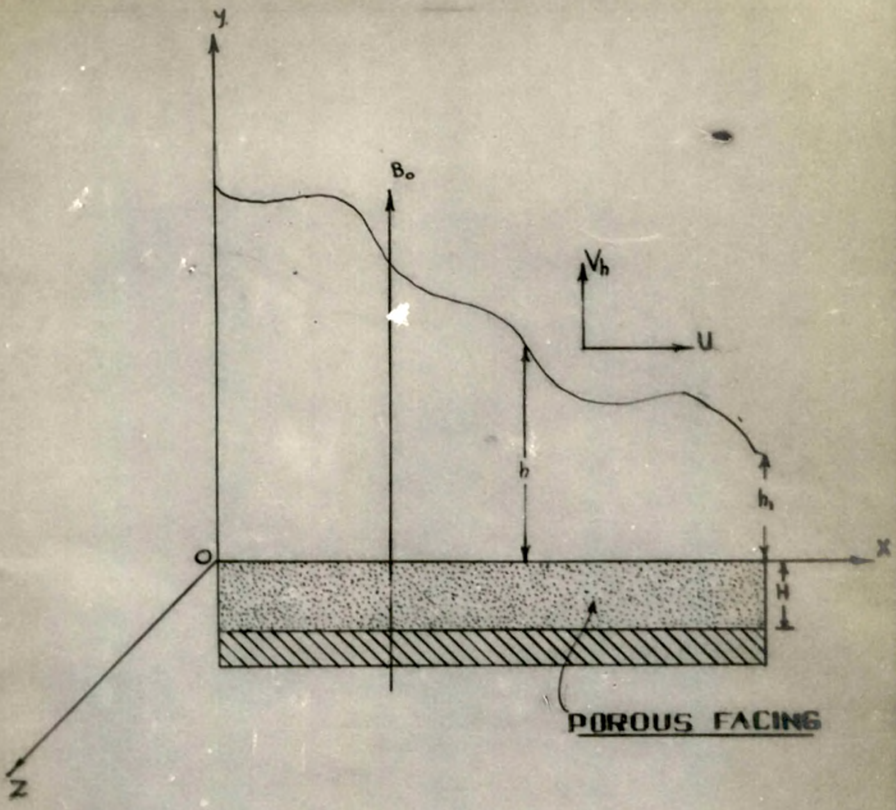

FIG. 2. POROUS BEARING WITH A TRANSVERSE MAGNETIC FIELD.

Following the usual assumptions these reduce to

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - J_z B_0 = 0 \tag{18}$$

$$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} + J_x B_0 = 0 \tag{19}$$

$$J_z = \sigma (E_z + u B_0) \tag{20}$$

$$J_x = \sigma (E_x - w B_0) \tag{21}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{22}$$

From equations (18), (20) and (19), (21) we have

$$\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\mu} B_0^2 u = \frac{1}{\mu} \left(\frac{\partial p}{\partial x} + \sigma E_z B_0 \right) \tag{23}$$

and

$$\frac{\partial^2 w}{\partial y^2} - \frac{\sigma}{\mu} B_0^2 w = \frac{1}{\mu} \left(\frac{\partial p}{\partial z} - \sigma E_x B_0 \right) \tag{24}$$

The velocity $\vec{V}$ of the fluid in the isotropic porous region satisfies the modified Darcy's law [11].

$$\vec{V} = -\frac{k}{\mu} \left[\left(\frac{\partial P}{\partial x} + J_z B_0 \right) \vec{i} + \frac{\partial P}{\partial y} \vec{j} + \left(\frac{\partial P}{\partial z} - J_x B_0 \right) \vec{k} \right], \tag{25}$$

the Ohm's law

$$\vec{J} = \sigma \left(\vec{E} + \frac{1}{m^*} \vec{V} \times \vec{B} \right) \tag{26}$$

and the continuity equation

$$\nabla . \vec{V} = 0 \tag{27}$$

Equations (25) - (26) take the form

$$V_x = - \frac{k}{\mu} \left(\frac{\partial P}{\partial x} + \sigma E_z B_0 \right) \frac{1}{c^2} \tag{28}$$

$$V_y = - \frac{k}{\mu} \frac{\partial P}{\partial y} \tag{29}$$

$$V_z = - \frac{k}{\mu} \left(\frac{\partial P}{\partial z} - \sigma E_x B_0 \right) \frac{1}{c^2} \tag{30}$$

where

$$c = \left(1 + \frac{k}{m^*} \frac{M^2}{h_1^2}\right)^{1/2} \text{ and } M = B_0 h_1 \sqrt{\sigma / \mu} \tag{31}$$

Equations (28) - (30) combined with (27) yield

$$\frac{\partial}{\partial x} \left(\frac{\partial P}{\partial x} + \sigma E_z B_0 \right) + c^2 \frac{\partial^2 P}{\partial y^2} + \frac{\partial}{\partial z} \left(\frac{\partial P}{\partial z} - \sigma E_x B_0 \right) = 0 \tag{32}$$

Solving equations (23) - (24) under the no-slip boundary conditions

$$\left.\begin{array}{l} u = 0 \text{ at } y = 0, \quad u = U \quad \text{at } y = h \\ w = 0 \text{ at } y = 0, \quad w = 0 \quad \text{at } y = h \end{array}\right\} \tag{33}$$

we have

$$u = \frac{h_1^2}{\mu M^2} \left(\frac{\partial p}{\partial x} + \sigma E_z B_0 \right) \left[\cosh \frac{My}{h_1} - 1 - \left(\cosh \frac{Mh}{h_1} - 1 \right) \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \right] + U \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \tag{34}$$

and

$$w = \frac{h_1^2}{\mu M^2} \left(\frac{\partial p}{\partial z} - \sigma E_x B_0 \right) \left[\cosh \frac{My}{h_1} - 1 - \left(\cosh \frac{Mh}{h_1} - 1 \right) \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \right] \tag{35}$$

Equation (22) is integrated across the film thickness to yield

$$\frac{\partial}{\partial x} \int_0^h u \, dy + V_h - V_0 + \frac{\partial}{\partial z} \int_0^h w \, dy = 0 \tag{36}$$

Owing to the continuity of normal velocity component at the interface of film and porous matrix we have

$$v_0 = V_{0y} = -\frac{k}{\mu}\left(\frac{\partial P}{\partial y}\right)_{y=0} \tag{37}$$

Integrating equation (32) with respect to y from − H to 0 and using the Morgan-Cameron approximation and the condition that $\left(\frac{\partial P}{\partial y}\right)_{y=-H} = 0$, we have

$$\left(\frac{\partial P}{\partial y}\right)_{y=0} = -\frac{H}{c^2}\left[\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x} + \sigma E_z B_0\right) + \frac{\partial}{\partial z}\left(\frac{\partial p}{\partial z} - \sigma E_x B_0\right)\right] \tag{38}$$

Substituting equations (34), (35) into (36) and making use of (37) − (38), we have

$$\frac{\partial}{\partial x}\left[\left(\frac{\partial p}{\partial x} + \sigma E_z B_0\right)\left\{\frac{kH}{c^2} + \frac{h_1^3}{M^3}\left(\frac{Mh}{h_1} - 2\tanh\frac{Mh}{2h_1}\right)\right\}\right]$$

$$+ \frac{\partial}{\partial z}\left[\left(\frac{\partial p}{\partial z} - \sigma E_x B_0\right)\left\{\frac{kH}{c^2} + \frac{h_1^3}{M^3}\left(\frac{Mh}{h_1} - 2\tanh\frac{Mh}{2h_1}\right)\right\}\right]$$

$$= \frac{\mu}{M} U h_1 \frac{\partial}{\partial x}\left(\tanh\frac{Mh}{2h_1}\right) + \mu V_h \tag{39}$$

This is the modified Reynolds equation for the hydromagnetic lubrication of a porous bearing.

Corollary 1.(2)

By making $B_0 \to 0$, equation (39) reduces to the Reynolds equation (17) for hydrodynamic lubrication as in [10].

1.2.3 Hydromagnetic lubrication equation for an anisotropic porous bearing considering slip velocity

So far we considered the porous region to have isotropic permeability. But some foamy materials and materials made of soft metals [13] are found to have anisotropic permeability. We consider the porosities to be m_x, m_y, m_z and the permeabilities to be k_x, k_y, k_z in the directions of the coordinate axes. Another assumption under which we derived the equations in the previous sections was the no-slip condition. But recent experiments [14,15] demonstrated the existence of slip velocities, comparable to the mean velocities in the porous region, at the interface of fluid film and the porous matrix. We suppose that u_0 and w_0 are the slip velocities in the x and z directions. We assume the fluid to be compressible with the density variation across the film to be neglected.

We consider a film of conducting fluid of thickness $h = h(x, z, t)$ between two electrically non-conducting surfaces. The upper surface is non-porous and moves with a relative velocity U in the x-direction as in Fig. 3. The lower surface has a porous matrix with anisotropic permeability. A uniform transverse magnetic field B_0 is applied. Making the usual assumptions of hydromagnetic lubrication as modified by the above assumptions, the equations governing the flow of the lubricant in the film region are

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{h_1^2} u = \frac{1}{\mu} \left(\frac{\partial p}{\partial x} + \sigma E_z B_0 \right) \qquad (40)$$

$$\frac{\partial^2 w}{\partial y^2} - \frac{M^2}{h_1^2} w = \frac{1}{\mu} \left(\frac{\partial p}{\partial z} - \sigma E_x B_0 \right) \qquad (41)$$

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) + \frac{\partial \rho}{\partial t} = 0 \qquad (42)$$

where

$$M = B_0 h_1 \sqrt{\sigma / \mu} \qquad (43)$$

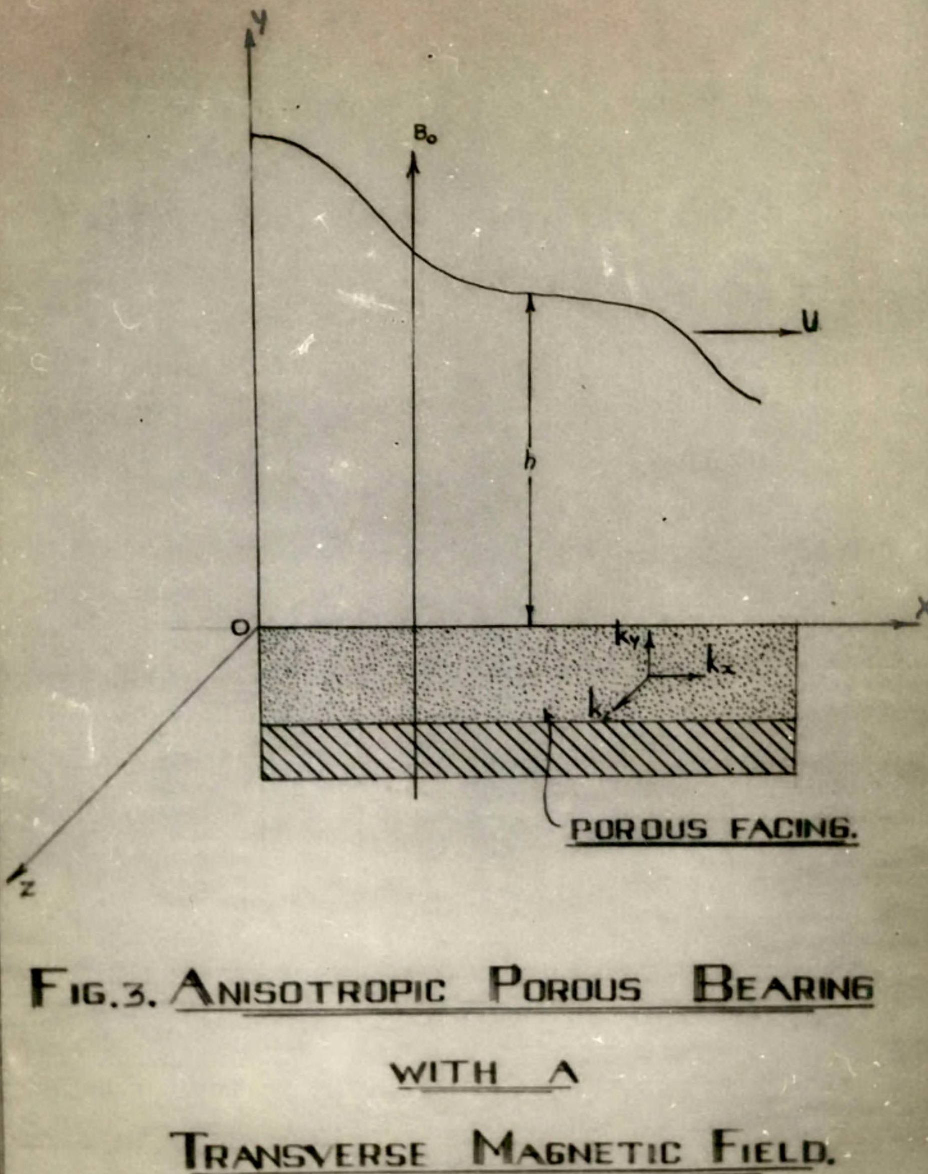

FIG. 3. ANISOTROPIC POROUS BEARING
WITH A
TRANSVERSE MAGNETIC FIELD.

The velocity $\vec{V}$ of the lubricant in the anisotropic porous matrix satisfies the modified Darcy's law generalized as

$$\vec{V} = -\frac{1}{\mu}\left[k_x\left(\frac{\partial P}{\partial x} + J_z B_0\right)\vec{i} + k_y \frac{\partial P}{\partial y}\vec{j} + k_z\left(\frac{\partial P}{\partial z} - J_x B_0\right)\vec{k}\right] \quad (44)$$

and the Ohm's law

$$\vec{J} = \sigma\left[E_x \vec{i} + E_y \vec{j} + E_z \vec{k} - \frac{V_z}{m_x} B_0 \vec{i} + \frac{V_x}{m_z} B_0 \vec{k}\right] \quad (45)$$

Combining equations (44) and (45)

$$\vec{V} = -\frac{1}{\mu}\left[\frac{k_x}{c_x^2}\left(\frac{\partial P}{\partial x} + \sigma E_z B_0\right)\vec{i} + k_y \frac{\partial P}{\partial y}\vec{j} + \frac{k_z}{c_z^2}\left(\frac{\partial P}{\partial z} - \sigma E_x B_0\right)\vec{k}\right] \quad (46)$$

$$= V_x \vec{i} + V_y \vec{j} + V_z \vec{k} \quad (47)$$

where

$$c_x = (1 + \frac{k_x}{m_x} \frac{M^2}{h_1^2})^{1/2} \text{ and } c_z = (1 + \frac{k_z}{m_z} \frac{M^2}{h_1^2})^{1/2} \quad (48)$$

Owing to the continuity of pressure at the interface of film and porous matrix, we have

$$P(x, y, z) \Big|_{y=0} = p(x, z) \quad (49)$$

Rewriting equation (46) at $y = 0$ and using (49) we have

$$\vec{V}_0 = -\frac{1}{\mu} \Big[\frac{k_x}{c_x^2} (\frac{\partial p}{\partial x} + \sigma E_z B_0) \vec{i} + k_y (\frac{\partial P}{\partial y})_{y=0} \vec{j} + \frac{k_z}{c_z^2} (\frac{\partial p}{\partial z} - \sigma E_x B_0) \vec{k} \Big]$$

$$= V_{0x} \vec{i} + V_{0y} \vec{j} + V_{0z} \vec{k} \quad (50)$$

Equation (42) is expressed as

$$\frac{\partial}{\partial x} (\rho \int_0^h u \, dy) + \rho (v_h - v_0) + \frac{\partial}{\partial z} (\rho \int_0^h w \, dy) + \int_0^h \frac{\partial \rho}{\partial t} dy = 0, \quad (51)$$

because ρ is independent of y.

The y-component of the fluid velocity at y = 0 is

$$v_0 = V_{0y} = -\frac{k_y}{\mu}\left(\frac{\partial P}{\partial y}\right)_{y=0} \tag{52}$$

The boundary conditions as in Kulkarni and Vinay Kumar [13] are

$$u(h) = U \tag{53}$$

$$u(0) = u_0 \tag{54}$$

where

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\alpha_x}{\sqrt{k_x}}\left(u_0 - \delta_{\ell 1} V_{0x}\right) \tag{55}$$

and

$$w(h) = 0 \tag{56}$$

$$w(0) = w_0 \tag{57}$$

where

$$\left(\frac{\partial w}{\partial y}\right)_{y=0} = \frac{\alpha_z}{\sqrt{k_z}}\left(w_o - \delta_{\ell 1} V_{0z}\right) \tag{58}$$

and

$$\alpha_x = \frac{5}{\sqrt{m_x}} \quad , \quad \alpha_z = \frac{5}{\sqrt{m_z}} \tag{59}$$

Equations (55) and (58) yield the exact or the approximate velocity slip model according as $\ell = 1$ or 2, and $\delta_{\ell 1}$ is the Kronecker delta.

Solving equations (40) and (41) under the boundary conditions (53) - (54) and (56) - (57) respectively, we have

$$u = u_0 \cosh \frac{My}{h_1} + \frac{h_1^2}{\mu M^2} \left(\frac{\partial p}{\partial x} + \sigma E_z B_0 \right)$$

$$\left[\cosh \frac{My}{h_1} - 1 - \left(\cosh \frac{Mh}{h_1} - 1\right) \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \right]$$

$$+ \left(U - u_0 \cosh \frac{Mh}{h_1}\right) \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \tag{60}$$

and

$$w = w_0 \cosh \frac{My}{h_1} + \frac{h_1^2}{\mu M^2} \left(\frac{\partial p}{\partial z} - \sigma E_x B_0 \right)$$

$$\left[\cosh \frac{My}{h_1} - 1 - \left(\cosh \frac{Mh}{h_1} - 1\right) \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \right]$$

$$- w_0 \cosh \frac{Mh}{h_1} \frac{\sinh \frac{My}{h_1}}{\sinh \frac{Mh}{h_1}} \tag{61}$$

Substituting equations (60) and (61) in (55) and (58) respectively and using (50) , we have

$$u_0 = \left[\mu \frac{M}{h_1} \left(5 + \sigma_x \frac{M}{h_1} \coth \frac{Mh}{h_1} \right)\right]^{-1} \left[\sigma_x \mu \frac{M^2}{h_1^2} \frac{U}{\sinh \frac{Mh}{h_1}}\right.$$

$$\left. - \left(\frac{\partial p}{\partial x} + \sigma E_z B_0\right)\left(5 \frac{M}{h_1} \delta_{\ell 1} \frac{k_x}{c_x^2} + \sigma_x \tanh \frac{Mh}{2h_1} \right) \right] \tag{62}$$

and

$$w_0 = - \frac{\left(\frac{\partial p}{\partial z} - \sigma E_x B_0\right)\left(5 \frac{M}{h_1} \delta_{\ell 1} \frac{k_z}{c_z^2} + \sigma_z \tanh \frac{Mh}{2h_1}\right)}{\mu \frac{M}{h_1} \left(5 + \sigma_z \frac{M}{h_1} \coth \frac{Mh}{h_1} \right)} \tag{63}$$

where

$$\sigma_x = \sqrt{k_x\, m_x} \quad \text{and} \quad \sigma_z = \sqrt{k_z\, m_z} \tag{64}$$

Substituting equations (60) - (63) and (52) into (51) we have the required equation as

$$\frac{\partial}{\partial x}\left[\frac{\rho}{\mu}\left(\frac{\partial p}{\partial x} + \sigma E_z B_0\right)\left\{\frac{5\delta_{\ell 1}\dfrac{k_x}{c_x^2} + \sigma_x \dfrac{\tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)}}{5 + \sigma_x \dfrac{M}{h_1} \coth \dfrac{Mh}{h_1}} \cdot \frac{\tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)} + \frac{\frac{Mh}{h_1} - 2 \tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)^3}\right\}\right]$$

$$+ \frac{\partial}{\partial z}\left[\frac{\rho}{\mu}\left(\frac{\partial p}{\partial z} - \sigma E_x B_0\right)\left\{\frac{5\delta_{\ell 1}\dfrac{k_z}{c_z^2} + \sigma_z \dfrac{\tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)}}{5 + \sigma_z \dfrac{M}{h_1} \coth \dfrac{Mh}{h_1}} \cdot \frac{\tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)} + \frac{\frac{Mh}{h_1} - 2 \tanh \frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)^3}\right\}\right]$$

$$= \frac{\partial}{\partial x}\left[\rho U\left\{1 + \frac{\sigma_x \frac{M}{h_1} \operatorname{cosech}\frac{Mh}{h_1}}{5 + \sigma_x \frac{M}{h_1}\coth\frac{Mh}{h_1}}\right\}\frac{\tanh\frac{Mh}{2h_1}}{\left(\frac{M}{h_1}\right)}\right]$$

$$+ \rho v_h + \rho \frac{k_y}{\mu}\left(\frac{\partial P}{\partial y}\right)_{y=0} + \int_0^h \frac{\partial \rho}{\partial t}\, dy \qquad (65)$$

This is the differential equation for the hydromagnetic lubrication of a porous bearing considering anisotropic permeability and slip velocity.

By making $B_0 \to 0$, it agrees with that obtained by Kulkarni and Vinay Kumar [13] for the non-magnetic case.

1.3 REVIEW OF RELATED WORKS

In spite of the wide use of porous bearings, their analysis and design is of recent origin. This is because of the complex conditions under which they operate. It was only in 1957, the problem of their analysis was done by Morgan and Cameron [9] who formulated the Reynolds equation for porous bearings considering the flow of the lubricant in the porous medium. This modified Reynolds equation was solved for the case of a narrow porous journal bearing on the assumption that a parabolic pressure distribution existed in the porous matrix in the axial direction. Rouleau [16] gave an exact solution of the above problem dropping the above assumption. Later, many investigators tried to bridge the gap between theoretical and experimental results of the performance of porous bearings by introducing many assumptions and simplifications.

We present below a brief review of related topics discussed in this thesis.

<u>Squeeze films between parallel surfaces</u>

A squeeze film is a very thin fluid layer between two surfaces which are parts of a machine, having normal relative motion and thus approaching each other. The relative normal motion of the surfaces causes the fluid to flow towards less constrained boundaries, resulting in the development of high pressures which in turn support the load and hence keep off the approaching surfaces from potential contact. This classical theory of squeeze film between non-porous surfaces has been known for some time. However, when one of the approaching surfaces is porous, only a part of the fluid is squeezed out and the rest flows out through the porous medium. This results in the decrease of the load capacity as well as the time taken to attain a specified film thickness.

Wu [17] analysed the squeeze film behaviour between two annular disks when one of them had a porous facing. He solved the problem analytically and presented the results for pressure distribution, load capacity and film thickness as functions of time in the form of infinite series involving Bessel's functions. He found that both the permeability parameter and the film thickness determined the extent of porous effects which increased in significance when the film thickness decreased. When the permeability parameter increased, the time required for the film thickness to reach any prescribed value decreased.

Wu [18] considered the squeeze film between two parallel rectangular plates with the upper one having a porous facing and approaching the lower one with uniform velocity. He solved the modified Reynolds equation and obtained the pressure distribution, load capacity and film thickness as functions of time in the form of infinite series involving trigonometric functions. He showed that the pressure and response time dropped off rapidly when

$\psi_0/\bar{h}^3$ and ψ_0 increased respectively. Moreover, for the same area, the square had the largest load capacity and, for the same permeability, the increase of porous facing thickness reduced the load capacity.

Prakash and Vij [10] simplified the analysis of Wu [17] by incorporating the Morgan-Cameron approximation. They showed that the results obtained by them were very close to those of Wu [17] for small porous facing thicknesses. They obtained the pressure distribution, load capacity and response time for configurations consisting of circular, annular, elliptic, rectangular and conical plates. If the area of the plate was kept fixed, the circular plate had the highest load capacity and the time taken in reducing a prescribed film thickness was greater for a circular plate than for other geometries.

It was customary to assume the no-slip velocity conditions at the interface of fluid and the porous region. But experiments [14,15] demonstrated the existence of slip velocities at the interface of fluid film and the porous matrix. Sparrow et al [19] extended the analysis [17] using slip velocity assumption. Results for the load capacity and response time were presented. The results indicated that slip velocity further reduced the load capacity and the response time of the porous squeeze film.

Wu [20] and Prakash and Vij [21] extended their earlier analyses [18,10] to include the effect of velocity slip at the porous boundary. They showed that the existence of slip velocity would further reduce the load capacity and the response time, thus supporting the conclusions [19].

The squeeze film behaviour between two circular disks, when one disk had a porous facing and approached the other with uniform velocity, was studied by Murti [22] who obtained expressions for pressure distribution, load capacity and time of approach in terms of Fourier-Bessel series. In addition to the reduction in load-capacity and response time he found that the entire fluid could be squeezed out in a finite time resulting in actual contact of the disks.

Wu [23] analysed the squeeze film between two rotating annular disks, when one had a porous facing. The inertia due to centrifugal force on the fluid was taken into account. The pressure distribution and load capacity were presented in series form while the time-height relation was in integral form. The effect of rotation was to reduce pressure, load capacity and response time. The criteria under which the inertia effects could be neglected were also given. Ting [24] presented an analogue method and simplified the analysis [23] by taking only one disk rotating. His results favourably compared with those of Wu [23]. In addition to obtaining expressions for pressure distribution, load capacity and response time, he obtained a relationship between squeeze time and film thickness for a given load. The squeeze time reduction due to the inertia effect would become small if the porous facing had high permeability and thickness.

Cowling [25] showed that the pressure gradients increased considerably when an electrically conducting fluid flowed under the influence of electromagnetic fields. Elco and Hughes [26] initiated the study of an axial current induced pinch effect on the load capacity of solid bearings with conducting lubricants. Gupta and Sinha [27] considered such effects on two parallel annular disks when the upper one which had a porous facing approached the stationary lower disk with a uniform normal velocity. Expressions were obtained for pressure distribution, load capacity and film thickness as functions of time in the form of Bessel's functions. It was shown that all the above quantities increased due to the pinch effect. Owing to the pinch effect the bearing could sustain an amount of load even when there was no flow. Gupta and Patel [28] extended the analysis [27] by including the velocity slip at the interface of the film and the porous region.

Hingu [29] considered the effect of axial current induced pinch on the squeeze film behaviour between two circular disks when the upper disk had a porous facing and moved normal to itself to approach the lower disk. Expressions for pressure distribution, load capacity and film thickness were obtained in the form of infinite series involving Bessel's functions. He showed that the film pressure, the load capacity and the time of approach increased due to the pinch. Moreover, an amount of load could be sustained by the bearing even when there was no flow. Patel [30] considerably simplified the analyses

[27,29] by incorporating the Morgan-Cameron approximation and showed that his results were very close to those obtained earlier.

Likewise the application of magnetic fields on the squeeze film bearings was found to be considerably advantageous. A number of theoretical and experimental studies [31 - 33] were made of squeeze films between impermeable surfaces. However, Sinha and Gupta [5] initiated the study of the squeeze film behaviour between porous surfaces. They made a theoretical study of the squeeze film behaviour between two parallel rectangular plates, the upper one having a porous facing and approaching normally the lower one, in the presence of a uniformly applied transverse magnetic field. Results were presented for pressure distribution, load capacity and film thickness as functions of time in the form of infinite series. It was shown that hydromagnetic effects could be used to increase the bearing characteristics considerably without altering the size of the bearing. Sinha and Gupta [34] obtained similar results for hydromagnetic squeeze film between porous annular disks.

Patel [35] extended the analysis [34] by including the slip velocity at the porous boundary. Chandrasekhara [36] analysed the squeeze film between two infinite parallel strips, when one had a porous facing, in the presence of a transverse magnetic field and the slip velocity at the porous boundary. It was shown that increases in the load capacity and response time could be attained by the application of the magnetic field. These increases were marked for small values of the permeability and large values of the Hartmann number.

<u>Other geometries</u>

Prakash and Vij [37] considered an infinite slider bearing when the slider moved tangentially with a uniform velocity and when the stator had a porous facing. Using the Morgan-Cameron approximation they uncoupled the equation for film pressure. Then they obtained expressions for pressure, load capacity, friction, coefficient of friction and centre of pressure in closed form. The effect of porosity was to decrease the load capacity and friction, but to increase the coefficient of friction.

Kulkarni and Vinay Kumar [13] obtained a lubrication equation for a general bearing with one surface moving with a relative velocity and the other having a porous matrix with anisotropic permeability. They took into account the slip velocity at the porous boundary.