

CHAPTER 6

STOCHASTIC INVENTORY MODEL UNDER  
PERMISSIBLE DELAY IN PAYMENT ALLOWING  
PARTIAL PAYMENT FOR TWO SUPPLIERS

## CHAPTER 6

### 6.1. INTRODUCTION:

In this chapter, we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods.

### 6.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under permissible delay in payment allowing partial payment is developed on the basis of the following assumptions.

(a)  $T_{li}$  is the time allowed by  $i^{\text{th}}$  supplier where  $i=1, 2$  at which  $\alpha_i$  ( $0 < \alpha_i < 1$ ) fraction of total amount has to be paid to the  $i^{\text{th}}$  supplier where  $i=1, 2$ .

(b)  $T_i$  ( $T_i > T_{li}$ ) is the time at which remaining amount has to be cleared.

(c)  $T_{00}$  is the expected cycle time.  $T_{li}$  and  $T_i$  are known constants and  $T_{00}$  is a decision variable.

(d)  $Ie_i$  = Interest rate earned when purchase made from  $i^{\text{th}}$  supplier where  $i=1, 2$

$Ic_i$  = Interest rate charged by  $i^{\text{th}}$  supplier where  $i=1, 2$ .

(e)  $U_i$  and  $V_i$  are indicator variables for  $i^{\text{th}}$  supplier where  $i=1, 2$

$U_1=0$  if part payment is done at  $T_{11}$  to the first supplier by the businessmen  
= 1 otherwise

$U_2=0$  if part payment is done at  $T_{12}$  to the second supplier by the businessmen  
= 1 otherwise

$V_1=0$  if the balanced amount is cleared at  $T_1$  for the 1<sup>st</sup> supplier by the businessmen  
= 1 otherwise

$V_2=0$  if the balanced amount is cleared at  $T_2$  for the 2<sup>nd</sup> supplier by the businessmen  
= 1 otherwise

In this chapter, we assume that supplier allows a fixed period  $T_{li}$  during which  $\alpha_i$  fraction of total amount has to be paid and remaining amount i.e.  $(1 - \alpha_i)$  fraction has to be cleared up to time  $T_i$ . Hence up to time period  $T_{li}$  no interest is charged for  $\alpha_i$  fraction, but beyond that period, interest will be charged upon not doing promised payment of  $\alpha_i$  fraction. Similarly for  $(1 - \alpha_i)$  fraction no interest will be charged up to time period  $T_i$  but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

Interest earned and interest charged is as follows.

(f) Interest earned on the entire amount up to time period  $T_{li}$  is  $dcT_{li}T_{00}Ie_i$

(g) Interest earned on  $(1 - \alpha_i)$  fraction during the period  $(T_i - T_{li})$  is

$$(1 - \alpha_i)dc(T_i - T_{li})T_{00}Ie_i$$

(h) If part payment is not done at  $T_{li}$  then interest will be earned over  $\alpha_i$  fraction for period  $(T_i - T_{li})$  but interest will also be charged for  $\alpha_i$  fraction for  $(T_i - T_{li})$  period.

$$\text{Interest earned} = dc\alpha_iT_{00}(T_i - T_{li})Ie_i$$

$$\text{Interest charged} = dc\alpha_iT_{00}(T_i - T_{li})Ic_i$$

To discourage not doing promised payment, we assume that  $Ic_i$  is quite larger than  $Ie_i$ .

(i) Interest earned over the amount  $dcT_{00}T_{li}Ie_i$  over the period  $(T_i - T_{li})$  is

$$dcT_{00}T_{li}Ie_i(T_i - T_{li})Ie_i$$

(j) If the remaining amount is not cleared at  $T_i$  then interest will be earned for the period  $(T_{00} - T_i)$  for  $(1 - \alpha_i)$  fraction simultaneously interest will be charged on the same amount for the same period.

$$\text{Interest earned} = dc(1 - \alpha_i)T_{00}(T_{00} - T_i)Ie_i$$

$$\text{Interest charged} = dc(1 - \alpha_i)T_{00}(T_{00} - T_i)Ic_i$$

$$\begin{aligned}
\text{Total interest earned} &= dcT_{1i}T_{00}Ie_i + (1-\alpha_i)dc(T_i-T_{1i})T_{00}Ie_i + dc\alpha_iT_{00}(T_i-T_{1i})Ie_i + \\
&dcT_{00}T_{1i}Ie_i(T_i-T_{1i})Ie_i + V_i \\
&[dc(1-\alpha_i)T_{00}(T_{00}-T_i)Ie_i + dcT_{1i}T_{00}Ie_i(T_i-T_{1i})Ie_i(T_{00}-T_i)Ie_i + dcT_{1i}T_{00}Ie_i(T_{00}-T_i)Ie_i \\
&+ dc(1-\alpha_i)T_{00}(T_i-T_{1i})Ie_i(T_{00}-T_i)Ie_i + \{dc\alpha_iT_{00}Ie_i(T_i-T_{1i})Ie_i - dc\alpha_iT_{00}Ic_i(T_{00}-T_i)\}] \\
\text{Total Interest charged} &= dc\alpha_iT_{00}(T_i-T_{1i})Ic_i + V_i[dc(1-\alpha_i)T_{00}(T_{00}-T_i)Ic_i]
\end{aligned}$$

Total interest earned and charged is as follows.

$$\begin{aligned}
&dcT_{1i}T_{00}Ie_i + (1-\alpha_i)dc(T_i-T_{1i})T_{00}Ie_i + \{dc\alpha_iT_{00}(T_i-T_{1i})Ie_i - \\
&dc\alpha_iT_{00}(T_i-T_{1i})Ic_i\} + dcT_{00}T_{1i}Ie_i(T_i-T_{1i})Ie_i + V_i \\
&[dc(1-\alpha_i)T_{00}(T_{00}-T_i)Ie_i + dcT_{1i}T_{00}Ie_i(T_i-T_{1i})Ie_i(T_{00}-T_i)Ie_i + dcT_{1i}T_{00}Ie_i(T_{00}-T_i)Ie_i \\
&+ dc(1-\alpha_i)T_{00}(T_i-T_{1i})Ie_i(T_{00}-T_i)Ie_i + \{dc\alpha_iT_{00}Ie_i(T_i-T_{1i})Ie_i \\
&- dc\alpha_iT_{00}Ic_i(T_{00}-T_i)\} - dc(1-\alpha_i)T_{00}(T_{00}-T_i)Ic_i]
\end{aligned}$$

### 6.3. OPTIMAL POLICY DECISION FOR THE MODEL:

Analysis of the average cost function requires the exact determination of the transition probabilities  $P_{ij}(t)$ ,  $i, j=0, 1, 2, 3$  for the four state CTMC. The lemma which is used to obtain the transition probabilities is same as discussed in chapter 4, (lemma (4.3.1)) also lemma 4.3.2 to 4.3.5 are also same hence we omit it here.

**Proposition 6.3.1:** The Average cost objective function for two suppliers when delay in payment allowing partial payment is given by  $AC = \frac{C_{00}}{T_{00}}$

$C_{00}$  is given by

$$\begin{aligned}
C_{00} = & A(q_0, r) + P_{01} \{ C_{10} - dcT_{00}T_{11}Ie_1 - (1-\alpha_1)dcT_{00}(T_1 - T_{11})Ie_1 - U_1dc\alpha_1T_{00}(T_1 - T_{11})Ie_1 \\
& + U_1dc\alpha_1T_{00}(T_1 - T_{11})Ic_1 - dcT_{00}T_{11}Ie_1(T_1 - T_{11}')Ie_1 \\
& - V_1 \left[ (1-\alpha_1)dcT_{00}(T_{00} - T_1)Ie_1 + dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right. \\
& \left. + dcT_{00}T_{11}Ie_1(T_{00} - T_1)Ie_1 + (1-\alpha_1)dcT_{00}(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right] \\
& - V_1 \left[ U_1 \{ dc\alpha_1T_{00}Ie_1(T_1 - T_{11})(T_{00} - T_1)Ie_1 \} \right] \\
& + V_1 \left[ U_1 \{ dc\alpha_1T_{00}Ic_1(T_{00} - T_1) + (1-\alpha_1)dcT_{00}Ic_1(T_{00} - T_1) \} \right] \} \\
& + P_{02} \{ C_{20} - dcT_{00}T_{12}Ie_2 - (1-\alpha_2)dcT_{00}(T_2 - T_{12})Ie_2 - U_2dc\alpha_2T_{00}(T_2 - T_{12})Ie_2 \\
& + U_2dc\alpha_2T_{00}(T_2 - T_{12})Ic_2 - dcT_{00}T_{12}Ie_1(T_2 - T_{12})Ie_2 \\
& - V_2 \left[ (1-\alpha_2)dcT_{00}(T_{00} - T_2)Ie_2 + dcT_{00}T_{12}Ie_2(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right. \\
& \left. + dcT_{00}T_{12}Ie_2(T_{00} - T_2)Ie_2 + (1-\alpha_2)dcT_{00}(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right] \\
& - V_2 \left[ U_2 \{ dc\alpha_2T_{00}Ie_2(T_2 - T_{12})(T_{00} - T_2)Ie_2 \} \right] \\
& + V_2 \left[ U_2 \{ dc\alpha_2T_{00}Ic_2(T_{00} - T_2) + (1-\alpha_2)dcT_{00}Ic_2(T_{00} - T_2) \} \right] \} \\
& + P_{03} \left\{ \bar{C} + \rho_1 \left[ \begin{aligned} & C_{10} - dcT_{00}T_{11}Ie_1 - (1-\alpha_1)dcT_{00}(T_1 - T_{11})Ie_1 - U_1dc\alpha_1T_{00}(T_1 - T_{11})Ie_1 \\ & + U_1dc\alpha_1T_{00}(T_1 - T_{11})Ic_1 - dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1 \\ & - V_1 \left[ (1-\alpha_1)dcT_{00}(T_{00} - T_1)Ie_1 + dcT_{00}T_{11}Ie_1(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right. \\ & \left. + dcT_{00}T_{11}Ie_1(T_{00} - T_1)Ie_1 + (1-\alpha_1)dcT_{00}(T_1 - T_{11})Ie_1(T_{00} - T_1)Ie_1 \right] \\ & - V_1 \left[ U_1 \{ dc\alpha_1T_{00}Ie_1(T_1 - T_{11})(T_{00} - T_1)Ie_1 \} \right] \\ & + V_1 \left[ U_1 \{ dc\alpha_1T_{00}Ic_1(T_{00} - T_1) + (1-\alpha_1)dcT_{00}Ic_1(T_{00} - T_1) \} \right] \end{aligned} \right] \\
& + \rho_2 \left[ \begin{aligned} & C_{20} - dcT_{00}T_{12}Ie_2 - (1-\alpha_2)dcT_{00}(T_2 - T_{12})Ie_2 - U_2dc\alpha_2T_{00}(T_2 - T_{12})Ie_2 \\ & + U_2dc\alpha_2T_{00}(T_2 - T_{12})Ic_2 - dcT_{00}T_{12}Ie_1(T_2 - T_{12})Ie_2 \\ & - V_2 \left[ (1-\alpha_2)dcT_{00}(T_{00} - T_2)Ie_2 + dcT_{00}T_{12}Ie_2(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right. \\ & \left. + dcT_{00}T_{12}Ie_2(T_{00} - T_2)Ie_2 + (1-\alpha_2)dcT_{00}(T_2 - T_{12})Ie_2(T_{00} - T_2)Ie_2 \right] \\ & - V_2 \left[ U_2 \{ dc\alpha_2T_{00}Ie_2(T_2 - T_{12})(T_{00} - T_2)Ie_2 \} \right] \\ & + V_2 \left[ U_2 \{ dc\alpha_2T_{00}Ic_2(T_{00} - T_2) + (1-\alpha_2)dcT_{00}Ic_2(T_{00} - T_2) \} \right] \end{aligned} \right] \right\} \\
& \text{and } T_{00} = \frac{q_0}{d + \theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\bar{T} + \rho_1T_{10} + \rho_2T_{20})
\end{aligned}$$

**Proof:** Proof follows using Renewal reward theorem (RRT). The optimal solution for  $q_0$ ,  $q_1$ ,  $q_2$  and  $r$  is obtained by using Newton Rapson method in R programming.

#### 6.4. NUMERICAL EXAMPLE:

There are sixteen different patterns of payments, some of them we consider here.

1.  $U_i=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  and clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , both are satisfied.
2.  $U_i=0$  and  $V_i=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ .
3.  $U_i=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is not satisfied for both the suppliers but all the amount are cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ .
4.  $U_i=0$ ,  $V_1=0$  and  $V_2=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is satisfied for both suppliers and clearing the remaining amount at time  $T_I$  for 1<sup>st</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier.
5.  $U_i=0$ ,  $V_1=1$  and  $V_2=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is satisfied for both suppliers and promise of clearing the remaining amount at time  $T_2$  for 2<sup>nd</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_I$  for 1<sup>st</sup> supplier.
6.  $U_1=0$ ,  $U_2=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is kept for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{I2}$  is not satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , are satisfied for both the suppliers.
7.  $U_1=1$ ,  $U_2=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is not satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{I2}$  is satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$  are satisfied for both the suppliers.

In this section we verify the results by a numerical example. We assume that  $k=\text{Rs. } 5/\text{order}$ ,  $c=\text{Rs. } 1/\text{unit}$ ,  $d=20/\text{units}$ ,  $\theta=4$ ,  $h=\text{Rs. } 5/\text{unit/time}$ ,  $\pi=\text{Rs. } 350/\text{unit}$ ,  $T_{11}=0.6$ ,  $\hat{\pi}=\text{Rs. } 25/\text{unit/time}$ ,  $\alpha_1=0.5$ ,  $\alpha_2=0.6$ ,  $I_{c1}=0.11$ ,  $I_{e1}=0.02$ ,  $I_{c2}=0.13$ ,  $I_{e2}=0.04$ ,  $T_{12}=0.8$ ,  $T_1=0.9$ ,  $T_2=1.1$ ,  $\lambda_1=0.58$ ,  $\lambda_2=0.45$ ,  $\mu_1=3.4$ ,  $\mu_2=2.5$ .

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are  $1/\lambda_1=1.72413794$ ,  $1/\lambda_2=2.2222$ ,  $1/\mu_1=.2941176$  and  $1/\mu_2=.4$  respectively. The long run probabilities are obtained as  $p_0=0.7239588$ ,  $p_1=0.1303126$ ,  $p_2=0.1234989$  and  $p_3=0.02222979$ . The optimal solution for the above numerical example based on the seven patterns of payment is obtained as

$(U_1, U_2, V_1, V_2)$	$q_0$	$q_1$	$q_2$	$r$	AC
(0,0,0,0)	3.2899	30.17858	29.58059	0.745935	6.406068
(0,0,1,1)	2.9496	29.82422	29.14462	0.664672	6.50769
(1,1,0,0)	3.34668	30.15484	29.56186	0.766788	6.37324
(0,0,0,1)	3.04876	29.91408	29.25791	0.690931	6.475395
(0,0,1,0)	3.15503	30.04058	29.41159	0.714835	6.443119
(0,1,0,0)	3.32203	30.16482	29.56969	0.757816	6.386726
(1,0,0,0)	3.31408	30.16817	29.5723	0.75489	6.392686

### Conclusion:

From this we conclude that the cost is minimum if part payment is not done at  $T_{1i}$  but account is cleared at  $T_i$  and the cost is maximum if part payment is done at  $T_{1i}$  but account is not cleared at  $T_i$ , this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

### 6.5. SENSITIVITY ANALYSIS:

To observe the effects of varying parameter values on the optimal solution we have conducted sensitivity analysis, by varying  $\mu_1$ ,  $\lambda_2$ ,  $h$  and  $k$  on the following seven patterns of payment.

### 6.5.1. Sensitivity Analysis for $\mu_1$ :

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  and clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , both are satisfied. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.1.1**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=0$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	3.1989	31.742	31.195	1.6671	6.8755
3	3.25	30.764	30.153	1.0378	6.5665
3.4	3.289	30.178	29.58	0.7459	6.406
4.4	3.3954	28.947	28.514	0.2633	6.1107
4.8	3.4374	28.539	28.201	0.1312	6.0228

We see that increasing  $\mu_1$  i.e. decreasing expected length of OFF period for 1<sup>st</sup> supplier, results in decrease in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC:



**Table 6.5.1.2**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=1$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	2.7897	31.375	30.797	1.5588	6.9929
3	2.8875	30.408	29.729	0.9462	6.6734
3.4	2.9496	29.824	29.144	0.6646	6.5076
4.4	3.094	28.586	28.058	0.2048	6.2026
4.8	3.1492	28.174	27.741	0.0805	6.1116

We see that as  $\mu_1$  increases i.e. expected length of OFF period for 1<sup>st</sup> supplier decreases, average cost decreases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_i=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{li}$  is not satisfied but all the amount is cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.1.3**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=1, U_2=1, V_1=0, V_2=0$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	3.2628	31.715	31.168	1.6931	6.8421
3	3.3091	30.739	30.131	1.0607	6.5335
3.4	3.3466	30.154	29.561	0.7667	6.3732
4.4	3.4478	28.927	28.534	0.2796	6.0783
4.8	3.4883	28.521	28.189	0.1459	5.9912

We see that increasing  $\mu_1$  i.e. decreasing expected length of OFF period for 1<sup>st</sup> supplier, results in decrease in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_i=0$ ,  $V_1=0$  and  $V_2=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and clearing the remaining amount at time  $T_1$  for 1<sup>st</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.1.4**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=0$ ,  $U_2=0$ ,  $V_1=0$ ,  $V_2=1$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	2.894807	31.459918	30.889826	1.589321	6.95821
3	2.9883564	30.495828	29.835451	0.9743761	6.640378
3.4	3.048766	29.91408	29.25791	0.690931	6.475395
4.4	3.1923548	28.685818	28.187664	0.2262752	6.171573
4.8	3.2463929	28.277973	27.876554	0.1000576	6.080889

We see that as  $\mu_1$  increases i.e. expected length of OFF period for 1<sup>st</sup> supplier decreases, average cost decreases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_1$  for 1<sup>st</sup> supplier, however remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_i=0$ ,  $V_1=1$  and  $V_2=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and promise of clearing the remaining amount at time  $T_2$  for 2<sup>nd</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.1.5**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=0$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	3.04146	31.596199	31.037312	1.627847	6.916945
3	3.108438	30.625285	29.98824	1.003636	6.604978
3.4	3.155037	30.04058	29.41159	0.714835	6.443119
4.4	3.2711593	28.806351	28.336233	0.2394774	6.14556
4.8	3.3159297	28.395869	28.020877	0.109958	6.056989

We see that as  $\mu_1$  increases i.e. expected length of OFF period for 1<sup>st</sup> supplier decreases, average cost decreases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_2$  for 2<sup>nd</sup> supplier, however remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_1=0, U_2=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.1.6**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=0, U_2=1, V_1=0, V_2=0$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	$r$	AC
2.4	3.237649	31.725811	31.17901	1.682961	6.855258
3	3.284455	30.749471	30.14012	1.051278	6.546867
3.4	3.322036	30.16482	29.5696	0.757816	6.386726
4.4	3.4229698	28.936749	28.5068	0.2719976	6.09227
4.8	3.463274	28.529834	28.195133	0.1387996	6.004668

We see that increasing  $\mu_1$  i.e. decreasing expected length of OFF period for 1<sup>st</sup> supplier, results in decrease in average cost, when part payment at time  $T_{11}$  is done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\mu_1$  and keeping other parameter values fixed where  $U_1=1$ ,  $U_2=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is not satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$  are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.1.7**  
**Sensitivity Analysis Table by varying the parameter values of  $\mu_1$**   
**when patterns of payment is ( $U_1=1$ ,  $U_2=0$ ,  $V_1=0$ ,  $V_2=0$ )**

$\mu_1$	$q_0$	$q_1$	$q_2$	R	AC
2.4	3.223528	31.731817	31.184855	1.677208	6.86253
3	3.274242	30.753851	30.143832	1.047325	6.553339
3.4	3.31408	30.16817	29.5723	0.75489	6.392686
4.4	3.420041	28.93784	28.50754	0.271084	6.096975
4.8	3.462193	28.530216	28.195383	0.138487	6.008877

We see that increasing  $\mu_1$  i.e. decreasing expected length of OFF period for 1<sup>st</sup> supplier, results in decrease in average cost, when part payment at time  $T_{11}$  is not done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

#### 6.5.2. Sensitivity Analysis for $\lambda_2$ :

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  and clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$

supplier where  $i=1, 2$ , both are satisfied. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.1**  
Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$   
when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=0$ )

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.318411	30.617133	29.833753	0.4008284	6.39465
0.43	3.303763	30.395773	29.708538	0.5773873	6.401486
0.45	3.28921	30.1781	29.58231	0.745911	6.406121
0.47	3.2767399	29.965521	29.450673	0.9071424	6.408698
0.49	3.264202	29.756447	29.756447	1.061605	6.409636

We see that increasing  $\lambda_2$  i.e. decreasing expected length of ON period for 2<sup>nd</sup> supplier, results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.2**  
Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$   
when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=1$ )

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	2.9996152	30.281891	29.409401	0.3235322	6.490541
0.43	2.9742574	30.050881	29.2783	0.4980911	6.500238
0.45	2.949612	29.8241	29.1442	0.664632	6.507126
0.47	2.9255607	29.601849	29.009157	0.8239513	6.513199
0.49	2.9020726	29.383642	28.872557	0.9765192	6.517024

We see that as  $\lambda_2$  increases i.e. expected length of ON period for 2<sup>nd</sup> supplier decreases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_i=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is not satisfied but all the amount is cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.3**  
**Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$**   
**when patterns of payment is ( $U_1=1, U_2=1, V_1=0, V_2=0$ )**

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.3723279	30.593768	29.815691	0.4209089	6.36274
0.43	3.3591025	30.372193	29.690152	0.5978603	6.369118
0.45	3.3466	30.154	29.561	0.7667	6.3732
0.47	3.3349783	29.941525	29.431656	0.9283884	6.375412
0.49	3.323915	29.732249	29.30012	1.083232	6.37589

We see that increasing  $\lambda_2$  i.e. decreasing expected length of ON period for 2<sup>nd</sup> supplier, results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_i=0, V_1=0$  and  $V_2=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ii}$  is satisfied for both suppliers and clearing the remaining amount at time  $T_I$  for 1<sup>st</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.4**  
Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$   
when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=1$ )

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.0876767	30.362482	29.514204	0.3471289	6.461766
0.43	3.0678308	30.13612	29.387407	0.5230215	6.469714
0.45	3.048766	29.91408	29.25791	0.690931	6.475395
0.47	3.030395	29.696262	29.126538	0.851541	6.479113
0.49	3.012644	29.48258	28.9939	1.005435	6.481126

We see that as  $\lambda_2$  increases i.e. expected length of ON period for 2<sup>nd</sup> supplier decreases, average cost increases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_1$  for 1<sup>st</sup> supplier, however remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_i=0, V_1=1$  and  $V_2=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and promise of clearing the remaining amount at time  $T_2$  for 2<sup>nd</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.5**  
Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$   
when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=0$ )

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.200164	30.495511	29.680571	0.373045	6.427482
0.43	3.177307	30.266018	29.547438	0.5479518	6.43641
0.45	3.155037	30.04058	29.41159	0.714835	6.443119
0.47	3.1332771	29.819173	29.273825	0.8743763	6.447911
0.49	3.111951	29.60169	29.13481	1.027171	6.451045

We see that as  $\lambda_2$  increases i.e. expected length of ON period for 2<sup>nd</sup> supplier decreases, average cost increases, when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_2$  for 2<sup>nd</sup> supplier, however remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_1=0$ ,  $U_2=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.2.6**  
**Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$**   
**when patterns of payment is ( $U_1=0$ ,  $U_2=1$ ,  $V_1=0$ ,  $V_2=0$ )**

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.350144	30.603112	29.822842	0.412714	6.375137
0.43	3.3356959	30.38188	29.697632	0.5892718	6.382061
0.45	3.322036	30.16482	29.56969	0.757816	6.386726
0.47	3.3090783	29.951878	29.439786	0.9190188	6.389437
0.49	3.296745	29.74292	29.30857	1.073475	6.390453

We see that increasing  $\lambda_2$  i.e. decreasing expected length of ON period for 2<sup>nd</sup> supplier, results in increase in average cost, when part payment at time  $T_{11}$  is done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value  $\lambda_2$  and keeping other parameter values fixed where  $U_1=1$ ,  $U_2=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is not satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ ,



the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$  are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.2.7**  
Sensitivity Analysis Table by varying the parameter values of  $\lambda_2$   
when patterns of payment is ( $U_1=1, U_2=0, V_1=0, V_2=0$ )

$\lambda_2$	$q_0$	$q_1$	$q_2$	$r$	AC
0.41	3.3401819	30.607429	29.82618	0.4090033	6.382346
0.43	3.3267296	30.385702	29.700613	0.5859545	6.388642
0.45	3.31408	30.16817	29.5723	0.75489	6.392686
0.47	3.3021433	29.954737	29.442052	0.916488	6.394781
0.49	3.290846	29.74532	29.31048	1.071338	6.395187

We see that increasing  $\lambda_2$  i.e. decreasing expected length of ON period for 2<sup>nd</sup> supplier, results in increase in average cost, when part payment at time  $T_{11}$  is not done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

### 6.5.3. Sensitivity Analysis for h:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  and clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , both are satisfied. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.3.1**  
Sensitivity Analysis Table by varying the parameter values of  $h$   
when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=0$ )

$h$	$q_0$	$q_1$	$q_2$	$r$	AC
5	3.289	30.178	29.58	0.7459	6.406
5.2	3.2362706	29.854723	29.221549	0.6022174	6.545112
5.4	3.1855529	29.54831	28.8808	0.4640401	6.680748
5.6	3.137486	29.257809	28.556833	0.331034	6.813115
5.8	3.0918631	28.981893	28.248148	0.2028219	6.942342

We see that increasing holding cost, results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ti}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and  $AC$ .

**Table 6.5.3.2**  
**Sensitivity Analysis Table by varying the parameter values of  $h$**   
**when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=1$ )**

$h$	$q_0$	$q_1$	$q_2$	$r$	$AC$
5	2.94963	29.8241	29.14412	0.6646	6.5076
5.2	2.9175047	29.526532	28.814907	0.5252968	6.643017
5.4	2.8863705	29.243731	28.500818	0.3911335	6.77517
5.6	2.8561698	28.974583	28.201073	0.2618246	6.904264
5.8	2.8268737	28.717979	27.914578	0.137043	7.030412

We see that as holding cost  $h$  increases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_i=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{Ti}$  is not satisfied but all the amount is cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and  $AC$ .

**Table 6.5.3.3**  
**Sensitivity Analysis Table by varying the parameter values of h**  
**when patterns of payment is ( $U_1=1, U_2=1, V_1=0, V_2=0$ )**

h	$q_0$	$q_1$	$q_2$	r	AC
5	3.34667	30.1548	29.561	0.766756	6.37329
5.2	3.290283	29.831422	29.203236	0.6223978	6.512695
5.4	3.237025	3.237025	29.525489	0.483581	6.648723
5.6	3.1866242	29.235481	28.539221	0.3499659	6.781464
5.8	3.1388387	28.960003	28.230933	0.2211845	6.911049

We see that increasing holding cost  $h$ , results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_i=0, V_1=0$  and  $V_2=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and clearing the remaining amount at time  $T_1$  for 1<sup>st</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.3.4**  
**Sensitivity Analysis Table by varying the parameter values of h**  
**when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=1$ )**

h	$q_0$	$q_1$	$q_2$	r	AC
5	3.048766	29.91408	29.25791	0.690931	6.475395
5.2	3.011016	29.610617	28.921667	0.5502039	6.611792
5.4	2.974681	29.32252	28.60153	0.414787	6.744952
5.6	2.939686	29.048488	28.296173	0.2843193	6.875
5.8	2.905965	28.78737	28.00445	0.158462	7.00205

We see that as holding cost  $h$  increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_1$  for 1<sup>st</sup> supplier, however remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_i=0$ ,  $V_1=1$  and  $V_2=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is satisfied for both suppliers and promise of clearing the remaining amount at time  $T_2$  for 2<sup>nd</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.3.5**  
**Sensitivity Analysis Table by varying the parameter values of  $h$**   
**when patterns of payment is ( $U_1=0$ ,  $U_2=0$ ,  $V_1=1$ ,  $V_2=0$ )**

$h$	$q_0$	$q_1$	$q_2$	$r$	AC
5	3.155037	30.04058	29.41159	0.714835	6.443119
5.2	3.1108146	29.72814	29.065332	0.5728694	6.580717
5.4	3.068563	29.4319	28.73604	0.436307	6.715007
5.6	3.028145	29.150475	28.422401	0.304777	6.846118
5.8	2.989451	28.8827	28.12302	0.177931	6.97417

We see that as holding cost  $h$  increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_2$  for 2<sup>nd</sup> supplier, however remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_1=0$ ,  $U_2=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.3.6**  
**Sensitivity Analysis Table by varying the parameter values of h**  
**when patterns of payment is ( $U_1=0, U_2=1, V_1=0, V_2=0$ )**

h	$q_0$	$q_1$	$q_2$	r	AC
5	3.322036	30.16482	29.56969	0.757816	6.386726
5.2	3.266879	29.84124	29.210885	0.6137228	6.526002
5.4	3.214758	29.53511	28.87036	0.475192	6.66186
5.6	3.1654016	29.244897	28.546563	0.3418463	6.79444
5.8	3.118576	28.96922	28.23815	0.21333	6.92387

We see that increasing holding cost  $h$ , results in increase in average cost, when part payment at time  $T_{11}$  is done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value holding cost  $h$  and keeping other parameter values fixed where  $U_1=1, U_2=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is not satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$  are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.3.7**  
**Sensitivity Analysis Table by varying the parameter values of h**  
**when patterns of payment is ( $U_1=1, U_2=0, V_1=0, V_2=0$ )**

h	$q_0$	$q_1$	$q_2$	r	AC
5	3.31408	30.16817	29.5723	0.75489	6.392686
5.2	3.2592393	29.844537	29.213475	0.6108684	6.531904
5.4	3.207414	29.53836	28.87292	0.472404	6.667706
5.6	3.1583346	29.248112	28.54909	0.3391224	6.800232
5.8	3.111767	28.97239	28.24064	0.210667	6.92961

We see that increasing holding cost  $h$ , results in increase in average cost, when part payment at time  $T_{11}$  is not done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

#### 6.5.4. Sensitivity Analysis for $k$ :

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.4.1**  
**Sensitivity Analysis Table by varying the parameter values of  $k$**   
**when patterns of payment is ( $U_1=0, U_2=0, V_1=0, V_2=0$ )**

$k$	$q_0$	$q_1$	$q_2$	$r$	AC
4.5	3.111032	29.671291	29.006302	0.737815	6.279806
5	3.28943	30.17821	29.5867	0.745923	6.40678
5.5	3.4598931	30.669571	30.131509	0.7515865	6.525952
6	3.6221036	31.145901	30.66168	0.7551955	6.640325
6.5	3.777383	31.609071	31.173463	0.757111	6.749874

We see that increasing ordering cost  $k$ , results in increase in average cost when the businessmen settle all the account for both the suppliers at the respective time.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_i=0$  and  $V_i=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied but remaining amount is not cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.4.2**  
**Sensitivity Analysis Table by varying the parameter values of k**  
**when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=1$ )**

k	$q_0$	$q_1$	$q_2$	r	AC
4.5	2.8054216	29.35804	28.614707	0.6604661	6.370295
5	2.9496	29.824	29.144	0.6646	6.5076
5.5	3.085467	30.273758	29.651465	0.6670386	6.638618
6	3.214123	30.708484	30.138066	0.6678945	6.763939
6.5	3.3364355	31.129981	30.606673	0.6675279	6.884337

We see that as ordering cost k increases, average cost increases when part payment is done for both the suppliers at the given time, but remaining amount is not cleared at the respective time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost k and keeping other parameter values fixed where  $U_i=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{i1}$  is not satisfied but all the amount is cleared at time  $T_i$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ . We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.4.3**  
**Sensitivity Analysis Table by varying the parameter values of k**  
**when patterns of payment is ( $U_1=1, U_2=1, V_1=0, V_2=0$ )**

k	$q_0$	$q_1$	$q_2$	r	AC
4.5	3.1652013	29.646864	28.987165	0.7589051	6.248338
5	3.3466	30.154	29.561	0.7667	6.3732
5.5	3.5190507	30.646433	30.113229	0.7721612	6.49187
6	3.6834178	31.12346	30.6439	0.7754561	6.605076
6.5	3.840672	31.587317	31.156156	0.777025	6.713537

We see that increasing ordering cost  $k$ , results in increase in average cost when part payment is not done for both the suppliers at the given time, but remaining amount is cleared at the respective time given by both the suppliers.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_i=0$ ,  $V_1=0$  and  $V_2=1$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and clearing the remaining amount at time  $T_1$  for 1<sup>st</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.

**Table 6.5.4.4**  
**Sensitivity Analysis Table by varying the parameter values of  $k$**   
**when patterns of payment is ( $U_1=0$ ,  $U_2=0$ ,  $V_1=0$ ,  $V_2=1$ )**

$k$	$q_0$	$q_1$	$q_2$	$r$	AC
4.5	2.894766	29.43805	28.71733	0.685312	6.341442
5	3.048766	29.91408	29.25791	0.690931	6.475395
5.5	3.194302	30.37343	29.7753	0.694534	6.602912
6	3.3324755	30.817963	30.272255	0.6964828	6.724851
6.5	3.464168	31.24919	30.751	0.697085	6.841897

We see that as ordering cost  $k$  increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_1$  for 1<sup>st</sup> supplier, however remaining amount is not cleared at time  $T_2$  for 2<sup>nd</sup> supplier.

(v) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_i=0$ ,  $V_1=1$  and  $V_2=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{1i}$  is satisfied for both suppliers and promise of clearing the remaining amount at time  $T_2$  for 2<sup>nd</sup> supplier is satisfied, but remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and AC.



**Table 6.5.4.5**  
**Sensitivity Analysis Table by varying the parameter values of k**  
**when patterns of payment is ( $U_1=0, U_2=0, V_1=1, V_2=0$ )**

k	$q_0$	$q_1$	$q_2$	r	AC
4.5	2.990814	29.55069	28.85612	0.708247	6.312748
5	3.155037	30.04058	29.41159	0.714835	6.443119
5.5	3.310505	30.51383	29.94361	0.719203	6.567096
6	3.458317	30.97222	30.454996	0.721744	6.685543
6.5	3.599366	31.41726	30.94793	0.722792	6.799144

We see that as ordering cost  $k$  increases, results in increase in average cost when part payment is done for both the suppliers at the given time and the remaining amount is cleared at time  $T_2$  for 2<sup>nd</sup> supplier, however remaining amount is not cleared at time  $T_1$  for 1<sup>st</sup> supplier.

(vi) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_1=0, U_2=1$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_b$ , the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$ , are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0, q_1, q_2, r$  and AC.

**Table 6.5.4.6**  
**Sensitivity Analysis Table by varying the parameter values of k**  
**when patterns of payment is ( $U_1=0, U_2=1, V_1=0, V_2=0$ )**

k	$q_0$	$q_1$	$q_2$	r	AC
4.5	3.141795	29.65712	28.99513	0.749866	6.26124
5	3.322036	30.16482	29.56969	0.757816	6.38672
5.5	3.493264	30.65622	30.12091	0.763269	6.50589
6	3.656581	31.13297	30.65136	0.766664	6.61960
6.5	3.812865	31.59656	31.16344	0.768352	6.728543

We see that increasing ordering cost  $k$ , results in increase in average cost, when part payment at time  $T_{11}$  is done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is not cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

(vii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value ordering cost  $k$  and keeping other parameter values fixed where  $U_1=1$ ,  $U_2=0$  and  $V_i=0$  where  $i=1, 2$  that is promise of doing part payment at time  $T_{11}$  is not satisfied for 1<sup>st</sup> supplier but promise of doing part payment at time  $T_{12}$  is satisfied for 2<sup>nd</sup> supplier however clearing the remaining amount at time  $T_i$  the time period given by  $i^{\text{th}}$  supplier where  $i=1, 2$  are satisfied for both the suppliers. We resolve the problem to find optimal values of  $q_0$ ,  $q_1$ ,  $q_2$ ,  $r$  and  $AC$ .

**Table 6.5.4.7**  
**Sensitivity Analysis Table by varying the parameter values of  $k$**   
**when patterns of payment is  $(U_1=1, U_2=0, V_1=0, V_2=0)$**

$k$	$q_0$	$q_1$	$q_2$	$r$	$AC$
4.5	3.133973	29.66065	28.99789	0.74682	6.267006
5	3.31408	30.16817	29.5723	0.75489	6.392686
5.5	3.485211	30.65937	30.12339	0.760469	6.51203
6	3.6484766	31.135938	30.653712	0.7639863	6.625898
6.5	3.804734	31.59936	31.16566	0.765793	6.734975

We see that increasing ordering cost  $k$ , results in increase in average cost, when part payment at time  $T_{11}$  is not done for 1<sup>st</sup> supplier but part payment at time  $T_{12}$  is cleared for 2<sup>nd</sup> supplier however remaining amount is cleared at the respective time given by both the suppliers.

## 6.6. CONCLUSION:

From this we conclude that the cost is minimum if part payment is not done at  $T_{11}$  but account is cleared at  $T_i$  and the cost is maximum if part payment is done at  $T_{11}$  but account is not cleared at  $T_i$ , this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.