PART- I

PERISHABLE PRODUCTS STOCHASTIC INVENTORY MODELS FOR SINGLE SUPPLIER

CHAPTER 2

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT FOR SINGLE SUPPLIER

CHAPTER 2

2.1. INTRODUCTION:

In most inventory models it is implicitly assumed that the product to be ordered is always available (i.e., continuous supply availability), that is when an order is placed it is either received immediately or after a deterministic or perhaps random lead time. However if the product is purchased from outside supplier he can cut off the supply at random times for duration of random length, or the product may be unavailable as in the case of equipment breakdowns, labour strikes or other unpredictable circumstance, then the production/inventory manager would need to know how much to produce or purchase when the supply is fully available.

At any time, the state of the system can be ON or OFF. We use 0 to denote the ON state and 1 to denote the OFF state. If the supplier is available we call it ON period and if he is not available call it OFF period. Also we specifically assume that the ON and OFF periods are exponentially distributed with parameters λ and μ respectively.

Deterioration/Perishability of an item in the inventory is defined as loss of its utility. It is reasonable to note that product may be understood to have life time which ends when its utility reaches zero. There is a great deal of interest in the analysis of perishable inventory models.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm's limited capital funds. The effects of inflation are not usually considered when an inventory system is analyzed because most people think that the inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed in many developing countries, especially in politically turmoil countries such as united Germany, Russia and Iraq; so it is necessary to consider the effects of inflation on the inventory system.

The primary benefit of taking trade credit is that one can have savings in purchase cost and opportunity cost, which become quite relevant for deteriorating items.

In such cases, one has to procure more units than required in the given cycle to account for the deteriorating effect. In particular, when unit purchase cost is high and decay is continuous, the saving due to delayed payment appears to be more significant than when the decay is continuous but without delayed payment. In order to boost up competitive spirit of the business, the small entrepreneurs are to be encouraged by giving some privileges to them. This may feasibly make them available in the business in spite of their limited finance resources.

Inventory model for non-deteriorating and deteriorating items with future supply uncertainty considering demand rate as d for single supplier was developed by Gujarathi and Kandpal [2003]. In this chapter we therefore consider a more realistic case of demand, by considering rate of demand d $\geq$ 1 for a single supplier and developed stochastic inventory model for perishable products where the effect of inflation and permissible delay in payment was considered.

2.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model under inflation and permissible delay in payment for single supplier is developed on the basis of the following assumptions.

(a) Demand rate d is deterministic and it is $d \geq 1$.

(b) The status of the system is initially ON.

(c) We define X and Y to be the random variables corresponding respectively to the lengths of ON and OFF periods of the supplier. We specifically assume that $X \sim \exp(\lambda)$ and $Y \sim \exp(\mu)$. Further, X and Y are independently distributed.

(d) Ordering cost is Rs. k/order.

(e) Holding cost is Rs. h/unit/unit time.

(f) Shortage cost is Rs. π/unit.

(g) Time dependent part of the backorder cost is Rs. $\hat{\pi}$ /unit/time.

(h) q = order up to level.

(i) r = reorder level; q, r are decision variables.

(j) θ is the rate of deterioration which is constant fraction of on hand inventory. The deteriorated units can neither be replaced nor repaired during cycle period.

(k) Purchase cost is Rs. c/unit.

(l) T_0 is credit period which is a known constant and T_{00} is cycle period which is a decision variable.

(m) r_1 = discount rate representing the time value of money.

(n) f = inflation rate

(o) $R = f - r_1$ = present value of the nominal inflation rate.

(p) t_1 = time period with inflation

(q) c_0 = present value of the inflated price of an item Rs. /unit = $ce^{(f-r_1)t_1} = ce^{Rt_1}$

(r) Ie = interest rate earned; Ic = interest rate charged.

(s) δ = indicator variable = 0, if account is settled completely at T_0,

= 1, otherwise.

(t) $Ie(1)$ = Interest earned over period (0 to T_0) = $dce^{Rt_1}\, T_0 T_{00}\, Ie$

(u) $Ie(2)$ = Interest earned over period (T_0 to T_{00}) upon interest earned ($Ie(1)$) previously.

$$= [dce^{Rt_1}\, T_{00} + Ie(1)]\left(T_{00} - T_0\right) Ie.$$

(v) Ic =Interest charged by the supplier $= \delta\, dc\, e^{Rt_1}\, Ic(T_{00} - T_0)$, clearly ($Ic > Ie$).

$A(q,r,\theta) =$ (cost of ordering) + (cost of holding inventory) + (cost of item that deteriorate during a single interval that starts with an inventory of $(q + r)$ units and ends with r units with inflation rate);

$$= k + \frac{1}{2} \cdot \frac{hq^2 e^{Rt_1}}{(d+\theta)} + \frac{hrqe^{Rt_1}}{(d+\theta)} + \frac{\theta cqe^{Rt_1}}{(d+\theta)}$$

$P_{ij}(t) = P$(being in state j at time t/starting in state i at time 0); $i, j = 0, 1$.

P_i = long run probabilities, $i = 0, 1$.

$C_{10}(r)$ = Expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle.

In this chapter we assume that

(i) A Supplier allows a fixed period 'T_0' to settle the account. During this fixed period no interest is charged by the supplier but beyond this period, interest Ic is charged by the supplier under the terms and conditions agreed upon.

(ii) During the fixed credit period T_0, revenue from sales is deposited in an interest bearing account.

(iii) The account is settled completely either at the end of the credit period or at the end of the cycle period.
(iv) Interest charged is usually higher than interest earned. Here we settle the account completely at T_0 as revenue generated till period T_0 may be presumably sufficient for settlement of the account completely as selling cost is greater than the purchase cost.

For inflation rate f, the continuous time inflation factor for the time period t_1 is e^{ft_1} which means that an item that costs Rs. c at time $t_1 = 0$, will cost ce^{ft_1} at time t_1. For discount rate r_1, representing the time value of money, the present value factor of an amount at time t_1 is $e^{-r_1 t_1}$. Hence the present value of the inflated amount ce^{ft_1} (net inflation factor) is $ce^{ft_1} e^{-r_1t_1}$. For an item with initial price c (Rs./unit), at time $t_1 = 0$, the present value of the inflated price of an item is given by $c_0 = ce^{(f-r_1)t_1} = ce^{Rt_1}$ where $R = f - r_1$ in which c is inflated through time t_1 to ce^{ft_1}, $e^{-r_1t_1}$ is the factor deflating the future worth to its present value and R is the present value of the inflation rate.

2.3. OPTIMAL POLICY DECISION FOR THE MODEL:

We use Renewal Reward Theorem (RRT) to model a stochastic inventory problem with supply interruptions where the supplier may be unavailable, since it has found wide applicability in queuing models and stochastic inventory models as exemplified in the works of Ross [1983] and Tijms [1986].

As explained in Ross [1983], RRT is a powerful tool used in optimization of stochastic systems. Once a regenerative cycle of a stochastic process is identified, one can form an average cost objective function as a ratio of the expected cycle cost to the expected cycle time.

For the inventory model under consideration the policy to be used is as follows. When inventory drops to 'r' and if the period is ON an order for 'q' units is placed which increases the inventory to the level $(q + r)$, i.e., (q, r) policy is used. When inventory drops to r and period is OFF, then the decision maker has to wait till the supplier becomes available. Upon his availability an order can be placed for number of units which increases the inventory to the level $(q + r)$ units. Hence in the OFF period possibility of shortages is also there. Cycle is defined to be period when inventory is replenished. Cycle is also shown in Fig.2.1. For this policy the inventory level and the status process is depicted in Fig.2.1.

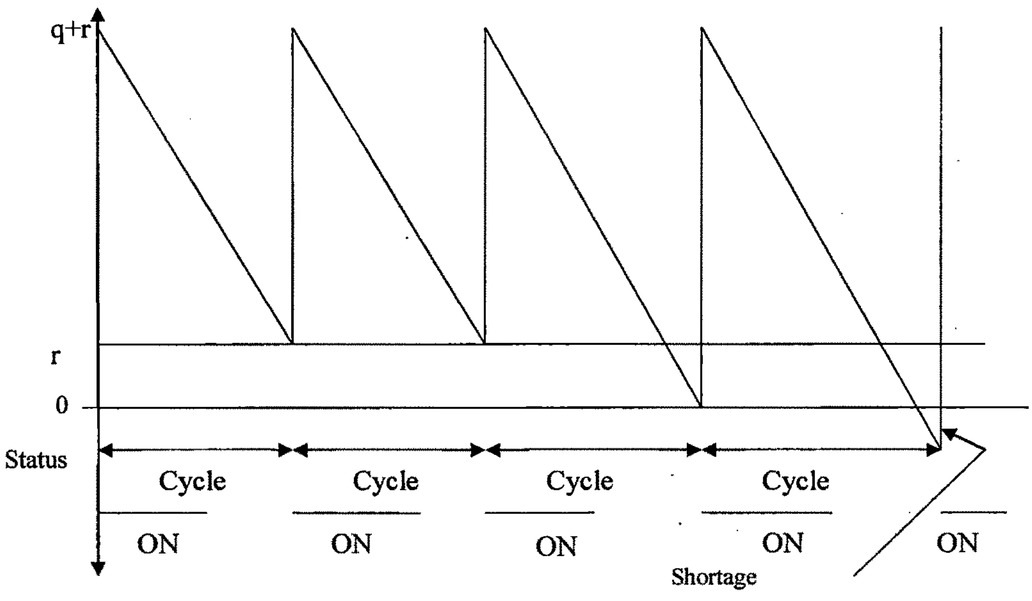

Fig.2.1 Inventory level and the status process for single supplier

Referring to Fig.2.1, we see that the cycles of this process start when the inventory goes up to a level of $(q + r)$ units. Once the cycle is identified we construct the average cost objective function as mentioned below.

$AC(q,r,\theta)$ = Average cost objective function.

$= \frac{C_{00}}{T_{00}}$, where $C_{00} = E$ (cost per cycle); $T_{00} = E$ (length of a cycle);

Now to make use of RRT we prove the following:

Lemma 2.3.1: $C_{10}(r)$ = expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is obtained as

$$C_{10}(r) = \frac{1}{\mu^2} e^{\frac{-\mu r}{(d+\theta)}} e^{Rt_1} \left[h e^{\frac{\mu r}{d+\theta}} \left(\mu r - (d+\theta)\right) + \pi \mu d + h(d+\theta) + \hat{\pi} - \theta c \mu \right] + \frac{\theta c e^{Rt_1}}{\mu}.$$

then,

$$C_{00} = A(q,r,\theta) + P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r) - [Ie(1) + Ie(2)] + Ic$$

$$P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}, \text{ and } P_1 = \frac{\lambda}{\lambda+\mu}.$$

Proof: Conditioning on the state of the system when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q}{d+\theta}\right)A(q,r,\theta) + P_{01}\left(\frac{q}{d+\theta}\right)[A(q,r,\theta) + C_{10}(r)] - [Ie(1) + Ie(2)] + Ic. \quad (2.3.1)$$

This follows because when inventory drops to r, the state will be 0 (ON) with probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and 1 (OFF) with probability $P_{01}\left(\frac{q}{d+\theta}\right) = 1 - P_{00}\left(\frac{q}{d+\theta}\right)$. If the state is ON, the cost incurred is $A(q,r,\theta)$ which is weighted by the probability $P_{00}\left(\frac{q}{d+\theta}\right)$ of this event If on the other hand, the state is OFF when inventory drops to r, the expected cost is $A(q,r,\theta) + C_{10}(r)$ which is weighted by the probability $P_{01}\left(\frac{q}{d+\theta}\right)$ of the corresponding event. The transition probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and $P_{01}\left(\frac{q}{d+\theta}\right)$ are obtained by CTMC, Bhat, U.N.[1984] for these states. They are given by

$$P_{00}\left(\frac{q}{d+\theta}\right) = P_0 + P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}, \quad P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}$$

$P_1 = 1 - P_0 = \frac{\lambda}{\lambda+\mu}$, and $P_0 = \frac{\mu}{\lambda+\mu}$ which are the steady state probabilities for the OFF and ON states respectively.

Now, referring to Fig.2.1, the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta)e^{Rt_1}+hy[r-(d+\theta)]e^{Rt_1}+\theta cye^{Rt_1}, \qquad y<\frac{r}{d+\theta}$$

$$\frac{1}{2}\cdot\frac{hr^2e^{Rt_1}}{(d+\theta)}+\pi e^{Rt_1}\left(y-\frac{r}{(d+\theta)}\right)d+\frac{\hat{\pi}e^{Rt_1}}{2}\left(y-\frac{r}{(d+\theta)}\right)^2+\frac{\theta cre^{Rt_1}}{(d+\theta)}, \quad y\geq\frac{r}{d+\theta}$$

so that

$$C_{10}(r)=\int_0^{r/(d+\theta)}\left\{\frac{1}{2}hy^2(d+\theta)e^{Rt_1}+he^{Rt_1}y\big(r-y(d+\theta)\big)+\theta ce^{Rt_1}y\right\}\mu e^{-\mu y}dy.$$

$$+\int_{r/(d+\theta)}^{\infty}\left\{\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)}+\pi e^{Rt_1}\left(y-\frac{r}{(d+\theta)}\right)d+\frac{\hat{\pi}e^{Rt_1}}{2}\left(y-\frac{r}{(d+\theta)}\right)^2+\frac{\theta cre^{Rt_1}}{(d+\theta)}\right\}\mu e^{-\mu y}dy$$

$$=\frac{1}{\mu^2}e^{\frac{-\mu r}{d+\theta}}e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}\big(\mu r-(d+\theta)\big)+\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right]+\frac{\theta ce^{Rt_1}}{\mu}$$

Lemma 2.3.2: Expected cycle length is given by

$$T_{00}=\frac{q}{d+\theta}+\frac{1}{\mu}P_{01}\left(\frac{q}{d+\theta}\right).$$

Proof: Using a conditioning argument similar to the one in Lemma (2.3.1), we obtain

$$T_{00}=\frac{q}{d+\theta}P_{00}\left(\frac{q}{d+\theta}\right)+P_{01}\left(\frac{q}{d+\theta}\right)\left(\frac{q}{d+\theta}+T_{10}\right) \tag{2.3.2}$$

where $T_{10}=E$(Time to reach the beginning of the next cycle when inventory drops to r and state is OFF). Clearly $T_{10} = 1/\mu$, since the OFF duration Y is distributed exponentially with parameter μ. Substituting for T_{10} in (2.3.2) and solving for T_{00} gives the desired expression for T_{00}.

Lemma 2.3.3: The function $C_{10}(r)$ is strictly convex and it is minimized at

$$\bar{r}=\frac{(d+\theta)e^{Rt_1}}{\mu}\log\left[\frac{\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu}{h(d+\theta)}\right]$$

Proof: The first derivative of $C_{10}(r)$ is obtained as

$$\frac{d}{dr}C_{10}(r)=\frac{e^{Rt_1}}{\mu}\left[h-\frac{e^{-\mu r/(d+\theta)}}{d+\theta}\left(\pi\mu\, d+h(d+\theta)+\hat{\pi}-\theta\, c\mu\right)\right]$$

Putting $\frac{d}{dr}C_{10}(r)$=0 and solving for r gives

$$\bar{r}=\frac{(d+\theta)e^{Rt_1}}{\mu}\log\left[\frac{\pi\mu\, d+h(d+\theta)+\hat{\pi}-\theta\, c\mu}{h(d+\theta)}\right]$$

The second derivative is

$$\frac{d^2}{dr^2}C_{10}(r)=\frac{e^{Rt_1}\, e^{-\mu r/(d+\theta)}}{(d+\theta)^2}\left(\pi\mu\, d+h(d+\theta)+\hat{\pi}-\theta\, c\mu\right)$$

which is always positive, hence $C_{10}(r)$ is strictly convex.

Proposition 2.3.1: The Average cost objective function is given by

$$AC(q,r,\theta)=\frac{C_{00}}{T_{00}}=\frac{A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-(Ie(1)+Ie(2))+Ic}{\frac{q}{d+\theta}+\frac{1}{\mu}P_{01}\left(\frac{q}{d+\theta}\right)}. \qquad (2.3.3)$$

Proof: Proof follows using Renewal Reward Theorem (RRT). The optimal solution for q and r are obtained by using Newton Rapson method in R programming.

2.4. NUMERICAL EXAMPLE:

Case-I: Taking δ=1 i.e. account is not settled at time period T_0.

In this section we verify the results by a numerical example. We assume that k=Rs. 10/order, c=Rs. 5/unit, d= 20/units, h=Rs. 5/unit/time, π=Rs. 250/unit, R=0.05, $\hat{\pi}$=Rs. 25/unit/time, θ=5/unit/time, δ=1, Ic=0.15, Ie=0.08, T_0=0.6, t_1=6, λ=0.25, μ=2.5. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda=4$, and $1/\mu=0.4$ respectively. The long run probabilities are obtained as $P_0=0.909$ and $P_1=0.091$. The optimal solution is obtained as

q=16.198, r=15.02 and $AC=\frac{C_{00}}{T_{00}}=266.575$

Case-II: Taking δ=0 i.e. account is settled at time period T_0.

Keeping other parameters as it is, we consider $\delta=0$ i.e. account is settled at time period T_0. The optimal solution is obtained as

q= 18.56644, r= 14.14799 and $AC=\frac{C_{00}}{T_{00}}=260.3604$.

Conclusion:

From the above numerical example, we conclude that the cost is minimum when account is settled at credit time given by the supplier.

2.5. SENSITIVITY ANALYSIS:

Case-I: Taking δ=1 i.e. account is not settled at time period T_0.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and purchase cost c keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking $\delta=1$ i.e. account is not settled at time period T_0 and the purchase cost c is assumed to take values 5, 15, 25. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.1.

Table 2.5.1
Sensitivity Analysis Table by varying the parameter values of *R* & *c*

	R	q	r	AC
c=5	0.05	16.198	15.02	266.575
	0.1	14.6125	15.6275	354.425
	0.15	13.1892	16.1894	472.443
	0.2	11.9098	16.708	631.126
	0.25	10.7584	17.1857	844.637
	0.3	9.22122	17.6249	1132.09
c=15	0.05	23.1296	14.9202	333.394
	0.1	19.34682	15.5277	444.622
	0.15	14.6865	16.0896	594.196
	0.2	12.92145	16.6082	795.475
	0.25	10.97276	17.0859	1066.481
	0.3	9.64643	17.5251	1431.55
c=25	0.05	26.8942	14.8194	400.206
	0.1	25.3913	15.4269	534.81
	0.15	18.6024	15.9888	715.937
	0.2	14.98498	16.5074	959.807
	0.25	11.54741	16.9851	1288.31
	0.3	9.894619	17.4243	1730.98

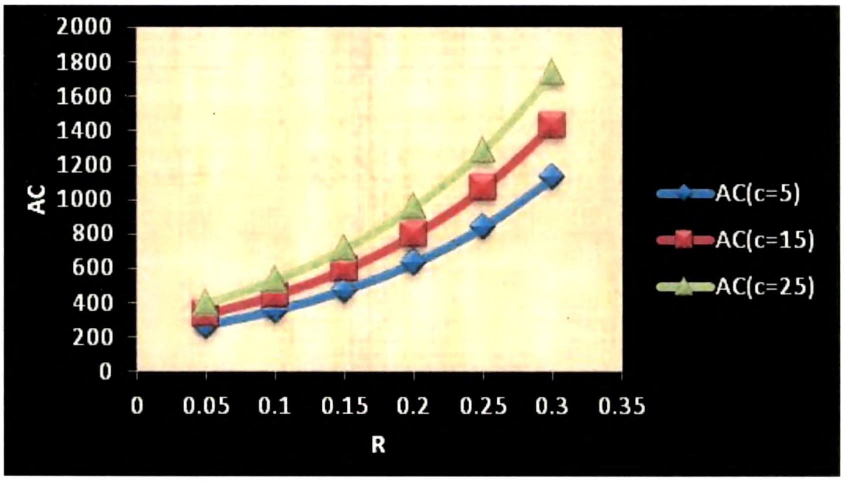

Fig. 2.5.1 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying purchase cost *c*

From the above table we see that taking inflation rate R=0.05 and increasing the value of purchase cost c i.e. c=5, 15, 25, value of q increases but the value of reorder quantity r decreases and hence average cost increases. Similarly when inflation rate R is increased for various values of c, we find that average cost increases.

(ii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking $\delta=1$ i.e. account is not settled at time period T_0 and the length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.2.

Table 2.5.2
Sensitivity Analysis Table by varying the parameter values of R & λ

	R	q	r	AC
λ=0.1	0.05	16.484	6.3674	212.413
	0.1	14.8736	6.97477	281.261
	0.15	13.4272	7.5369	373.613
	0.2	12.1265	8.05632	497.633
	0.25	10.9554	8.53512	664.334
	0.3	9.90025	8.97578	888.575
λ=0.15	0.05	16.3861	10.2489	237.146
	0.1	14.7843	10.8563	314.666
	0.15	13.3458	11.4184	418.729
	0.2	12.0524	11.9375	558.563
	0.25	10.8881	12.4159	746.617
	0.3	9.83904	12.8561	999.691
λ=0.2	0.05	16.2908	12.9557	254.004
	0.1	14.6973	13.5631	337.439
	0.15	13.2665	14.1251	449.492
	0.2	11.9802	14.644	600.117
	0.25	10.8224	15.122	802.745
	0.3	9.77939	15.5616	1075.5

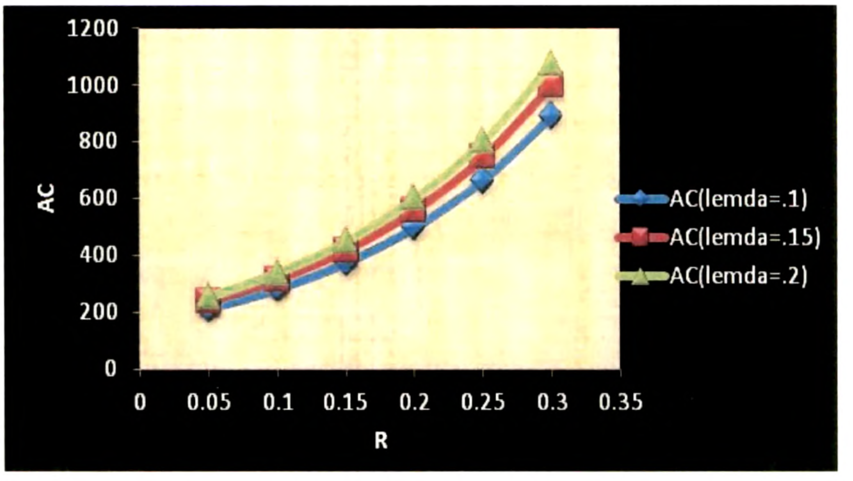

Fig. 2.5.2 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of ON period

We see that as inflation rate R increases and λ increases i.e. expected length of ON period decreases, value of q decreases to a smaller extent but the value of reorder quantity r increases to a larger extent and hence average cost increases.

(iii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and holding cost keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking $\delta=1$ i.e. account is not settled at time period T_0 and the holding cost h is assumed to take values 5, 15, 20. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.3.

Table 2.5.3
Sensitivity Analysis Table by varying the parameter values of *R* & *h*

	R	q	r	AC
h=5	0.05	16.198	15.02	266.575
	0.1	14.6125	15.6275	354.425
	0.15	13.1892	16.1894	472.443
	0.2	11.9098	16.708	631.126
	0.25	10.7584	17.1857	844.637
	0.3	9.72122	17.6249	1132.09
h=15	0.05	11.1341	6.23921	479.323
	0.1	10.0597	6.69113	639.231
	0.15	9.09146	7.10621	854.276
	0.2	8.21817	7.48688	1143.67
	0.25	7.43008	7.83553	1533.32
	0.3	6.71855	8.15446	2058.21
h=20	0.05	10.1017	3.8934	545.393
	0.1	9.12928	4.30994	727.643
	0.15	8.25229	4.69198	972.76
	0.2	7.46088	5.04189	1302.677
	0.25	6.7463	5.362	1746.92
	0.3	6.10101	5.65451	2345.4

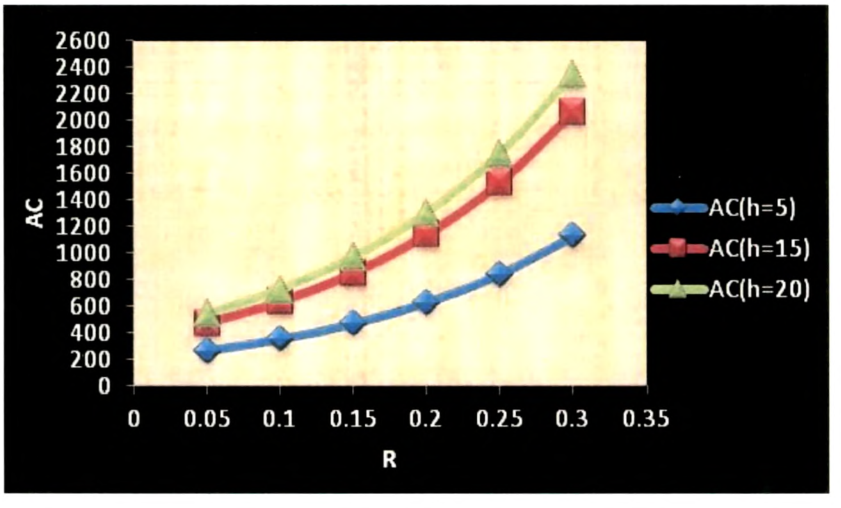

Fig. 2.5.3 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate *R* and for varying holding cost

We see that as inflation rate R increases and holding cost h increases, value of q as well as the value of reorder quantity r decreases, but average cost increases.

(iv) We have also conducted Sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking $\delta=1$ i.e. account is not settled at time period T_0 and the length of OFF period μ is assumed to take values 3.5, 4.5, 5.5. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.4.

Table 2.5.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ

	R	q	r	AC
μ=3.5	0.05	8.437622	12.13917	204.1676
	0.1	7.153977	12.67328	269.7916
	0.15	6.124131	13.11492	358.0273
	0.2	5.28179	13.48485	476.7953
	0.25	4.581602	13.7983	636.7777
	0.3	3.991702	14.06654	852.3858
μ=4.5	0.05	7.526421	9.132862	172.0836
	0.1	6.4529	9.56431	226.3303
	0.15	5.577113	9.92831	299.2271
	0.2	4.849717	10.2389	397.3014
	0.25	4.236719	10.50649	529.3563
	0.3	3.713965	10.73892	707.2677
μ=5.5	0.05	6.906468	7.224054	151.4933
	0.1	5.963587	7.590872	198.4088
	0.15	5.18608	7.904698	261.4186
	0.2	4.533863	8.175977	346.1517
	0.25	3.979334	8.412433	460.1992
	0.3	3.502762	8.619933	613.8017

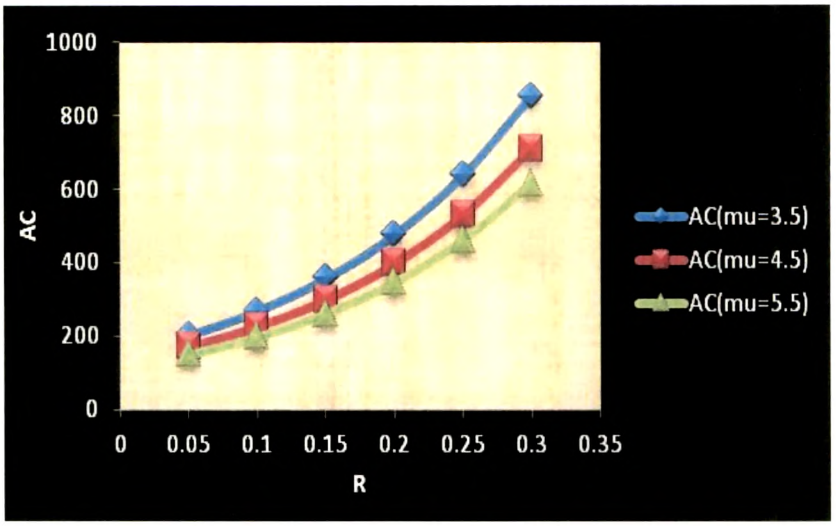

Fig. 2.5.4 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of OFF period

We see that as inflation rate R increases and μ increases i.e. expected length of OFF period decreases, value of q decreases and the value of reorder quantity r also decreases, as a consequence average cost also decreases.

Case-II: Taking δ=0 i.e. account is settled at time period T_0.

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and purchase cost c keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the purchase cost c is assumed to take values 5, 15, 25. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.5.

Table 2.5.5
Sensitivity Analysis Table by varying the parameter values of *R* & *c*

	R	q	r	AC
c=5	0.05	18.56644	14.14799	260.3604
	0.1	16.18106	15.02638	348.3072
	0.15	14.18595	15.79423	466.0568
	0.2	12.49104	16.47081	624.0147
	0.25	11.0338	17.07047	836.2077
	0.3	9.769292	17.60432	1121.553
c=15	0.05	25.87896	11.6108	311.7744
	0.1	20.60682	13.33029	424.5098
	0.15	16.80065	14.6943	574.0795
	0.2	13.92145	15.79855	773.703
	0.25	11.67276	16.70569	1041.066
	0.3	9.874643	17.45956	1399.94
c=25	0.05	38.99424	7.968189	356.6691
	0.1	27.59133	10.99124	497.1973
	0.15	20.60244	13.23097	680.3026
	0.2	15.84982	14.95113	922.6037
	0.25	12.46741	16.27976	1245.682
	0.3	9.994619	17.30766	1678.288

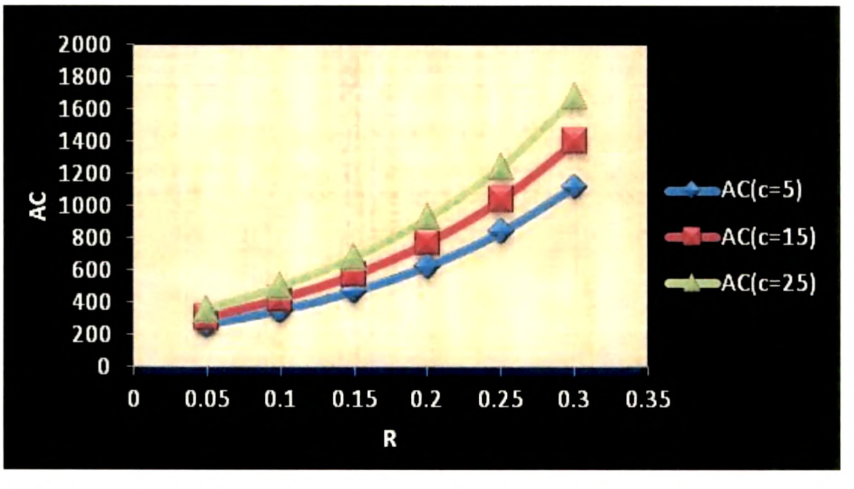

Fig. 2.5.5 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying purchase cost *c*

We see that as inflation rate *R* increases and purchase cost c increases, value of q increases but the value of reorder quantity *r* decreases and hence average cost increases.

(ii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.6.

Table 2.5.6
Sensitivity Analysis Table by varying the parameter values of *R* & λ

	R	q	r	AC
λ=0.1	0.05	18.91048	5.488464	206.4616
	0.1	16.54903	6.343288	275.7315
	0.15	14.57433	7.08979	368.3015
	0.2	12.89612	7.747436	492.3067
	0.25	11.45203	8.330734	658.7104
	0.3	10.1972	8.850874	882.295
λ=0.15	0.05	18.79284	9.37233	231.1044
	0.1	16.42319	10.23511	308.9339
	0.15	14.44147	10.98883	413.0475
	0.2	12.75745	11.65294	552.6226
	0.25	11.30862	12.24187	740.0292
	0.3	10.05017	12.76677	991.9486
λ=0.2	0.05	18.67823	12.0813	247.874
	0.1	16.30061	12.95201	331.5106
	0.15	14.312	13.7129	443.4523
	0.2	12.62245	14.3832	593.582
	0.25	11.16925	14.97765	795.2222
	0.3	9.90748	15.5072	1066.338

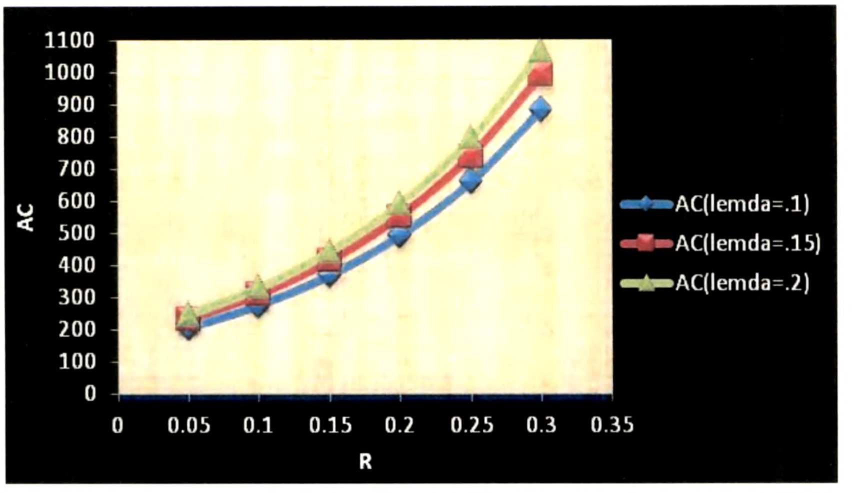

Fig. 2.5.6 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of ON period

We see that as inflation rate R increases and λ increases i.e. expected length of ON period decreases, value of q decreases to a smaller extent but the value of reorder quantity r increases to a larger extent and hence average cost increases.

(iii) We have also conducted Sensitivity analysis by varying the value of inflation rate R and holding cost keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking δ=0 i.e. account is settled at time period T_0 and the holding cost h is assumed to take values 5, 15, 20. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.7.

Table 2.5.7
Sensitivity Analysis Table by varying the parameter values of *R* & *h*

	R	q	r	AC
h=5	0.05	18.56644	14.14799	260.3604
	0.1	16.18106	15.02638	348.3072
	0.15	14.18595	15.79423	466.0568
	0.2	12.49104	16.47081	624.0147
	0.25	11.0338	17.07047	836.2077
	0.3	9.769292	17.60432	1121.553
h=15	0.05	7.030882	11.97079	381.3605
	0.1	6.040024	12.41903	508.4446
	0.15	5.227961	12.79203	679.6592
	0.2	4.552189	13.10624	910.4462
	0.25	3.982188	13.37394	1221.642
	0.3	3.495849	13.60424	1641.367
h=20	0.05	11.83737	5.948259	475.0154
	0.1	10.52584	6.49393	634.6434
	0.15	9.382288	6.980734	849.05
	0.2	8.378483	7.41655	1137.362
	0.25	7.492708	7.807643	1525.319
	0.3	6.707541	8.15942	2047.697

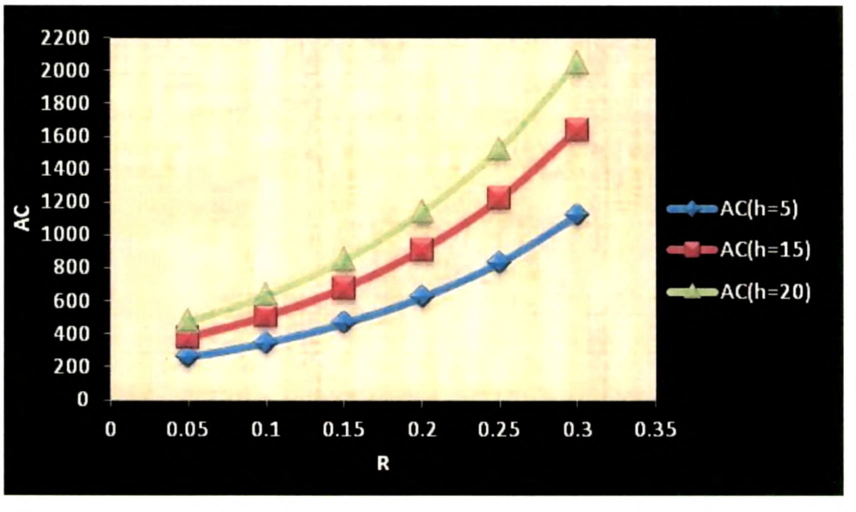

Fig. 2.5.7 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate *R* and for varying holding cost

We see that as inflation rate R increases and holding cost h increases, value of q as well as the value of reorder quantity r decreases but the average cost increases.

(iv) We have also conducted Sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed. Inflation rate R is assumed to take values 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 taking $\delta=0$ i.e. account is settled at time period T_0 and the length of OFF period μ is assumed to take values 3.5, 4.5, 5.5. We resolve the problem to find optimal values of q, r and AC. The optimal values of AC and R are plotted in Fig. 2.5.8.

Table 2.5.8
Sensitivity Analysis Table by varying the parameter values of *R* & μ

	R	q	r	AC
μ=3.5	0.05	18.67569	8.462394	233.5384
	0.1	15.82504	9.371443	313.8484
	0.15	13.5768	10.14325	420.7643
	0.2	11.74742	10.80899	563.6429
	0.25	10.22504	11.38977	755.0646
	0.3	8.937003	11.90059	1011.973
μ=4.5	0.05	15.13281	6.511063	175.8006
	0.1	13.29046	7.08039	232.7234
	0.15	11.73491	7.592297	308.5391
	0.2	10.40158	8.055119	409.8156
	0.25	9.245652	8.475048	545.4002
	0.3	8.234697	8.856968	727.2189
μ=5.5	0.05	14.20465	4.849949	156.7112
	0.1	12.48422	5.341539	206.4619
	0.15	11.03189	5.787196	272.5594
	0.2	9.787081	6.193134	360.6749
	0.25	8.707786	6.563986	478.4458
	0.3	7.763559	6.903423	636.164

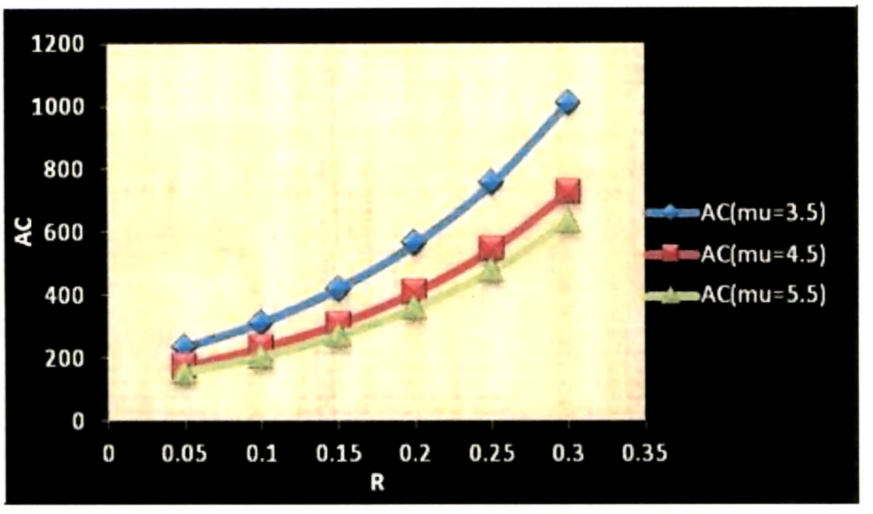

Fig. 2.5.8 Sensitivity Analysis Graph for Average cost with respect to varying inflation rate R and for varying length of OFF period

We see that as inflation rate R increases and μ increases i.e. expected length of OFF period decreases, value of q decreases and the value of reorder quantity r also decreases as a consequence average cost also decreases.

2.6. CONCLUSION:

In this chapter, on comparing the average cost value for various sensitivity analysis done by varying the various parameter values, we find that the cost is minimum if payment is done at T_0 i.e. account is settled at time period T_0 which is credit period given by supplier. This implies that we encourage the small businessmen to do the business by giving them a loan and simultaneously we want to discourage them from not clearing the account at the end of credit period.