CHAPTER 3

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT ALLOWING PARTIAL PAYMENT FOR SINGLE SUPPLIER

CHAPTER 3

3.1. INTRODUCTION:

In this chapter, we have introduced the aspect of part payment. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods.

3.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model under inflation and permissible delay in payment allowing partial payment for single supplier is developed on the basis of the following assumptions.

(a) T_I is the time at which α ($0< \alpha <1$) fraction of total amount has to be paid.

(b) T ($T> T_I$) is the time at which remaining amount has to be cleared.

(c) T_{00} is the expected cycle time. T_I and T are known constants and T_{00} is a decision variable.

(d) U and V are indicator variables where

U= 0 if part payment is done at T_I

=1 otherwise

V= 0 if the balanced amount is cleared at T

=1 otherwise

In this chapter, we assume that supplier allows a fixed period T_I during which α fraction of total amount has to be paid and at time T remaining amount has to be cleared. Hence up to time period T_I no interest is charged for α fraction, but beyond that period, interest will be charged upon not doing promised payment of α fraction. Similarly for

(1- α) fraction no interest will be charged up to time period T but beyond that period interest will be charged. However, customer can sell the goods and earn interest on the sales revenue during the period of admissible delay.

Interest earned and interest charged is as follows.

(i) Interest earned on the entire amount up to time period T_1 is $dce^{Rt_1}T_{00}T_1Ie$

(ii) Interest earned on (1-α) fraction during the period $(T-T_1)$ is

$$(1-\alpha)dce^{Rt_1}(T-T_1)T_{00}\,Ie$$

(iii) If part payment is not done at T_1 then interest will be earned over α fraction for period $(T-T_1)$ but interest will also be charged for α fraction for $(T-T_1)$ period.

Interest earned= $dce^{Rt_1}\,\alpha(T-T_1)T_{00}\,Ie$

Interest charged= $dce^{Rt_1}\,\alpha(T-T_1)T_{00}\,Ic$

To discourage not doing promised payment, we assume that Ic is quite larger than Ie.

(iv) Interest earned over the amount $dce^{Rt_1}T_{00}T_1Ie$ over the period $(T-T_1)$ is

$$dce^{Rt_1}\,T_{00}\,T_1\,Ie\ \ (T-T_1)Ie$$

(v) If the remaining amount is not cleared at T then interest will be earned for the period $(T_{00}-T)$ for (1-α) fraction. Simultaneously interest will be charged on the same amount for the same period.

Interest earned= $(1-\alpha)dce^{Rt_1}(T_{00}-T)T_{00}\,Ie$

Interest charged= $(1-\alpha)dce^{Rt_1}(T_{00}-T)T_{00}\,Ic$

Total interest earned and charged is as follows.

$$\begin{aligned}
&dce^{Rt_1}T_{00}T_1Ie+(1-\alpha)dce^{Rt_1}(T-T_1)T_{00}\,Ie+\{dce^{Rt_1}\,\alpha(T-T_1)T_{00}\,Ie\\
&-dce^{Rt_1}\,\alpha(T-T_1)T_{00}\,Ic\}+dce^{Rt_1}\,T_{00}\,T_1\,Ie(T-T_1)Ie\\
&+V[(1-\alpha)dce^{Rt_1}(T_{00}-T)T_{00}\,Ie+\ dce^{Rt_1}\,T_{00}\,T_1\,Ie(T-T_1)Ie(T_{00}-T)Ie\\
&+dce^{Rt_1}\,T_{00}\,T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie(T_{00}-T)Ie\\
&+\{dce^{Rt_1}\,\alpha T_{00}\,Ie(T-T_1)Ie-dce^{Rt_1}\,\alpha(T_{00}-T)T_{00}\,Ic\}\\
&-(1-\alpha)dce^{Rt_1}(T_{00}-T)T_{00}\,Ic]
\end{aligned}$$

3.3. OPTIMAL POLICY DECISION FOR THE MODEL:

We use the same policy as discussed in chapter 2.

Lemma 3.3.1: $C_{10}(r)$ = expected cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is obtained as

$$C_{10}(r)=\frac{1}{\mu^2}e^{\frac{-\mu r}{(d+\theta)}}\,e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}\left(\mu r-(d+\theta)\right)+\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right]+\frac{\theta ce^{Rt_1}}{\mu}.$$

Then,

$$\begin{aligned}C_{00}&=A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-dce^{Rt_1}T_{00}T_1Ie-(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\\&-Udce^{Rt_1}\alpha T_{00}(T-T_1)Ie+Udce^{Rt_1}\alpha T_{00}(T-T_1)Ic-dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie\\&-V[(1-\alpha)dce^{Rt_1}T_{00}(T_{00}-T)Ie+dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie(T_{00}-T)Ie\\&+dce^{Rt_1}T_{00}\,T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\ (T_{00}-T)Ie]\\&-V[U\ \{dce^{Rt_1}\alpha T_{00}\,Ie(T-T_1)(T_{00}-T)Ie\}]\\&+V[U\{dce^{Rt_1}\alpha T_{00}\,Ic(T_{00}-T)+(1-\alpha)dce^{Rt_1}T_{00}\,Ic(T_{00}-T)\}]\end{aligned}$$

$P_{01}\left(\frac{q}{d+\theta}\right)=P_1-P_1e^{\frac{-(\lambda+\mu)q}{d+\theta}}$, and $P_1=\frac{\lambda}{\lambda+\mu}$.

Proof: Conditioning on the state of the system when inventory drops to r, we obtain

$$\begin{aligned}C_{00}&=A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-dce^{Rt_1}T_{00}T_1Ie-(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\\&-Udce^{Rt_1}\alpha T_{00}(T-T_1)Ie+Udce^{Rt_1}\alpha T_{00}(T-T_1)Ic-dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie\\&-V[(1-\alpha)dce^{Rt_1}T_{00}(T_{00}-T)Ie+dce^{Rt_1}T_{00}\,T_1Ie(T-T_1)Ie(T_{00}-T)Ie\\&+dce^{Rt_1}T_{00}\,T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\ (T_{00}-T)Ie]\\&-V[U\ \{dce^{Rt_1}\alpha T_{00}\,Ie(T-T_1)(T_{00}-T)Ie\}]\\&+V[U\{dce^{Rt_1}\alpha T_{00}\,Ic(T_{00}-T)+(1-\alpha)dce^{Rt_1}T_{00}\,Ic(T_{00}-T)\}]\ \ \}\end{aligned}$$

This follows because when inventory drops to r, the state will be 0 (ON) with probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and 1(OFF) with probability $P_{01}\left(\frac{q}{d+\theta}\right)=1-P_{00}\left(\frac{q}{d+\theta}\right)$. If the state is ON, the cost incurred is $A(q, r, \theta)$ which is weighted by the probability $P_{00}\left(\frac{q}{d+\theta}\right)$ of this event If on the other hand, the state is OFF when inventory drops to r, the expected cost is $A(q, r, \theta) + C_{10}(r)$ which is weighted by the probability $P_{01}\left(\frac{q}{d+\theta}\right)$ of the corresponding event. The transition probability $P_{00}\left(\frac{q}{d+\theta}\right)$ and $P_{01}\left(\frac{q}{d+\theta}\right)$ are obtained by CTMC, Bhat, U.N.[1984] for these states. They are given by

$$P_{00}\left(\frac{q}{d+\theta}\right) = P_0 + P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}}, \quad P_{01}\left(\frac{q}{d+\theta}\right) = P_1 - P_1 e^{\frac{-(\lambda+\mu)q}{d+\theta}},$$

$$P_1 = 1 - P_0 = \frac{\lambda}{\lambda+\mu}, \text{ and } P_0 = \frac{\mu}{\lambda+\mu}$$

which are the steady state probabilities for the OFF and ON states respectively. Now, referring to Figure 3.1, of chapter 2 the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of the next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + hy[r-(d+\theta)]e^{Rt_1} + \theta cye^{Rt_1}, \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\cdot\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cre^{Rt_1}}{(d+\theta)}, \qquad y \geq \frac{r}{d+\theta}$$

so that

$$C_{10}(r) = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta)e^{Rt_1} + he^{Rt_1}y\big(r - y(d+\theta)\big) + \theta ce^{Rt_1}y\right\}\mu e^{-\mu y}dy.$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cre^{Rt_1}}{(d+\theta)}\right\}\mu e^{-\mu y}dy$$

$$=\frac{1}{\mu^2}e^{\frac{-\mu r}{d+\theta}}e^{Rt_1}\left[he^{\frac{\mu r}{d+\theta}}(\mu r-(d+\theta))+\pi\mu d+h(d+\theta)+\hat{\pi}-\theta c\mu\right]+\frac{\theta ce^{Rt_1}}{\mu}.$$

Proposition 3.3.2: The Average cost objective function under inflation and permissible delay in payments allowing partial payment is given by

$AC(q,r,\theta)=\frac{C_{00}}{T_{00}}$, where T_{00} is the same expression as in lemma (2.3.2) of chapter 2.

C_{00} is given by

$$\begin{aligned}C_{00} &= A(q,r,\theta)+P_{01}\left(\frac{q}{d+\theta}\right)C_{10}(r)-dce^{Rt_1}T_{00}T_1Ie-(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\\
&-Udce^{Rt_1}\alpha T_{00}(T-T_1)Ie+Udce^{Rt_1}\alpha T_{00}(T-T_1)Ic-dce^{Rt_1}T_{00}T_1Ie(T-T_1)Ie\\
&-V[(1-\alpha)dce^{Rt_1}T_{00}(T_{00}-T)Ie+dce^{Rt_1}T_{00}T_1Ie(T-T_1)Ie(T_{00}-T)Ie\\
&+dce^{Rt_1}T_{00}T_1Ie(T_{00}-T)Ie+(1-\alpha)dce^{Rt_1}T_{00}(T-T_1)Ie\ (T_{00}-T)Ie]\\
&-V[U\ \{dce^{Rt_1}\alpha T_{00}Ie(T-T_1)(T_{00}-T)Ie\}]\\
&+V[U\{dce^{Rt_1}\alpha T_{00}Ic(T_{00}-T)+(1-\alpha)dce^{Rt_1}T_{00}Ic(T_{00}-T)\}]\ \}\end{aligned}$$

Proof: Proof follows using Renewal Reward Theorem (RRT). The optimal solution for q and r are obtained by using Newton Rapson method in R programming.

3.4. NUMERICAL EXAMPLE:

There are four patterns of payments:

1. U=0, V=0 i.e. promise of doing part payment at time T_1 and clearing the remaining amount at time T both are satisfied.

2. U=0, V=1 i.e. promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time T.

3. U=1, V=0 i.e. part payment is not done at time T_1 but all the amount is cleared at time T.

4. U=1, V=1 i.e. part payment is not done at time T_1 and also the amount is not cleared at time T.

Case-I: Inflation rate is less than interest charged.

In this section we verify the results by a numerical example. We assume that k = Rs. 10/order, c = Rs.5/unit, d = 20/units, h = Rs. 5/unit/time, π = Rs. 250/unit, $\hat{\pi}$=Rs. 25/unit/time, θ=5/unit/time, $Ic = 0.15$, $Ie = 0.08$, $T_1 = 0.3$, T=0.6, α=0.5, $R = 0.05$, $t_1 = 6$, $\lambda = 0.25$, $\mu = 2.5$. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda = 4$, and $1/\mu = 0.4$ respectively. The long run probabilities are obtained as $P_0 = 0.909$ and $P_1 = 0.091$.

The optimal solution for the above numerical example based on the above four patterns of payment is obtained as

Patterns	q	r	AC
U=0,V=0	16.19804	15.01994	261.6373
U=0,V=1	17.8344	14.41302	260.9979
U=1,V=0	16.19804	15.01994	263.0547
U=1,V=1	13.2388	16.16954	263.247

Conclusion:

From the above numerical example we conclude that cost is minimum if part payment is done at T_1 but account is not cleared at T and the cost is maximum if part payment is not done at T_1 and also account is not cleared at T, this implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period.

Case-II: Inflation rate is greater than interest charged.

In this section we verify the results by a numerical example. We assume that k = Rs. 10/order, c = Rs.5/unit, d = 20/units, h = Rs. 5/unit/time, π = Rs. 250/unit, $\hat{\pi}$=Rs. 25/unit/time, θ=5/unit/time, $Ic = 0.15$, $Ie = 0.08$, $T_1 = 0.3$, T=0.6, α=0.5, $R = 0.35$, $t_1 = 6$, $\lambda = 0.25$, $\mu = 2.5$. The last two parameters indicate that the expected lengths of the ON and OFF periods are $1/\lambda = 4$, and $1/\mu = 0.4$ respectively. The long run probabilities are obtained as $P_0 = 0.909$ and $P_1 = 0.091$.

The optimal solution for the above numerical example based on the above four patterns of payment is obtained as

Patterns	q	r	AC
U=0,V=0	8.786195	18.02806	1489.404
U=0,V=1	10.49453	17.29658	1496.151
U=1,V=0	8.786195	18.02806	1497.978
U=1,V=1	6.289428	19.13815	1473.322

Conclusion:

In this case we observe that average cost is minimum if part payment is not done at T_1 and also account is not cleared at T which implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation.

3.5. SENSITIVITY ANALYSIS:

We study below in the Sensitivity analysis, the effect of change in the parameter on the following four patterns of payment.

Case-I: Inflation rate is less than interest charged.

3.5.1. Sensitivity Analysis for λ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.1
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=0, V=0)

	R	q	r	AC
λ=0.1	0.03	17.1787	6.111374	185.6628
	0.05	16.48399	6.36741	207.4757
	0.08	15.49699	6.73732	245.374
	0.1	14.87357	6.97477	274.596
	0.13	13.9873	7.31738	325.4052
λ=0.15	0.03	17.0773	9.99281	209.2525
	0.05	16.38612	10.24894	232.2091
	0.08	15.40427	10.61895	277.2296
	0.1	14.7842	10.85633	308.0017
	0.13	13.90276	11.19893	368.4761
λ=0.2	0.03	16.9786	12.69945	224.1986
	0.05	16.29084	12.95568	249.0663
	0.08	15.31407	13.3257	295.1825
	0.1	14.69726	13.56314	330.7744
	0.13	13.82053	13.90564	392.6875

From the above table we see that when both the promises are fulfilled of doing payment, average cost increases when inflation rate R increases and λ increases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.2
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=0, V=1)

	R	q	r	AC
λ=0.1	0.03	18.81541	5.5221	184.985
	0.05	18.11683	5.77123	206.874
	0.08	17.12575	6.13076	244.926
	0.1	16.50092	6.36113	274.293
	0.13	15.61408	6.693096	325.377
λ=0.15	0.03	18.03791	9.642713	208.873
	0.05	18.02028	9.64908	231.594
	0.08	16.35819	10.25937	277.004
	0.1	16.41433	10.23841	307.679
	0.13	14.85316	10.82981	368.4328
λ=0.2	0.03	17.93968	12.34732	223.8133
	0.05	17.9261	12.35221	248.4391
	0.08	16.9475	12.7111	294.707
	0.1	16.3299	12.9414	330.4343
	0.13	15.4543	13.27214	392.614

We see that as inflation rate R increases and λ increases average cost increases, when promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time T.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.3
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=1, V=0)

	R	q	r	AC
λ=0.1	0.03	17.1787	6.111374	186.9198
	0.05	16.48399	6.36741	208.8931
	0.08	15.4969	6.737318	247.066
	0.1	14.87357	6.97477	276.5097
	0.13	13.9873	7.31738	327.6958
λ=0.15	0.03	17.0773	9.99281	211.0484
	0.05	16.38612	10.24894	233.6265
	0.08	15.40427	10.6189	279.6537
	0.1	14.78426	10.85633	309.9149
	0.13	13.90276	11.19893	371.7051
λ=0.2	0.03	16.9786	12.69945	225.9945
	0.05	16.29084	12.95568	250.4836
	0.08	15.31407	13.3257	296.8794
	0.1	14.69726	13.56314	332.6876
	0.13	13.8205	13.90564	394.9781

In this situation also average cost increases when inflation rate R increases and λ increases, when promise of doing part payment at time T_1 is not satisfied but remaining amount is cleared at time T.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=1 and V=1. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.1.4
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=1, V=1)

	R	q	r	AC
λ=0.1	0.03	14.1953	7.23641	187.3716
	0.05	13.53545	7.494338	209.0099
	0.08	12.59908	7.86609	246.5389
	0.1	12.00875	8.103967	275.4387
	0.13	11.17122	8.446078	325.6036
λ=0.15	0.03	13.71008	11.27458	211.4674
	0.05	13.43388	11.38354	233.7692
	0.08	12.13923	11.90223	278.902
	0.1	11.91404	11.99377	308.8793
	0.13	10.73621	12.47908	369.0934
λ=0.2	0.03	13.6082	13.98946	226.4385
	0.05	13.33503	14.09784	250.6515
	0.08	12.4068	14.4704	296.4133
	0.1	11.82176	14.70873	331.6866
	0.13	10.99174	15.0514	392.9696

When both the promises of doing payment are not satisfied, impact of increase in inflation rate R and λ results in increase in average cost.

3.5.2. Sensitivity Analysis for μ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.2.1
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=0)

	R	q	r	AC
μ=0.5	0.03	25.64692	84.83821	819.447
	0.05	24.63514	85.3299	922.9856
	0.08	23.19264	86.03272	1103.495
	0.1	22.27871	86.47913	1243.159
	0.13	20.97547	87.11686	1486.665
μ=1.5	0.03	19.46766	27.08709	344.3221
	0.05	18.69011	27.41263	386.6923
	0.08	17.58339	27.88095	460.4715
	0.1	16.88332	28.18018	517.494
	0.13	15.88654	28.6102	616.816
μ=2.5	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359

When both the promises of doing payment are fulfilled by the businessman we find that as inflation rate R increases and μ increases, average cost decreases. This may be because unavailability of supplier is for less period of time and hence it is not necessary to stock more items which results in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.2.2
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=1)

	R	q	r	AC
μ=0.5	0.03	36.55733	79.61103	814.1644
	0.05	35.7292	80.00257	917.2922
	0.08	34.57371	80.55032	1097.127
	0.1	33.85915	80.88998	1236.297
	0.13	32.86669	81.36282	1478.994
μ=1.5	0.03	22.24361	25.94828	342.7546
	0.05	21.47833	26.25856	385.1115
	0.08	20.39274	26.70352	458.8971
	0.1	19.70851	26.9868	515.9462
	0.13	18.7378	27.39258	615.3507
μ=2.5	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406

We observe that when promise of doing part payment at T_1 is satisfied but clearing the remaining amount at T is not fulfilled, impact of increase in inflation rate R and μ results in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.2.3
Sensitivity Analysis Table by varying the parameter values of R & μ when pattern of payment is (U=1, V=0)

	R	q	r	AC
μ=0.5	0.03	25.6469	84.83821	820.704
	0.05	24.63514	85.3299	924.403
	0.08	23.19264	86.03272	1105.192
	0.1	22.27871	86.47913	1245.072
	0.13	20.97547	87.11686	1488.955
μ=1.5	0.03	19.46766	27.08709	345.5792
	0.05	18.69011	27.41263	388.1097
	0.08	17.58339	27.88095	462.1684
	0.1	16.88332	28.18018	519.4073
	0.13	15.88654	28.6102	619.1065
μ=2.5	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264

We see that as inflation rate R increases and μ increases, average cost decreases when promise of doing part payment at T_1 is not satisfied but clearing the remaining amount at T is fulfilled.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed where U=1 and V=1. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.2.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=1)

	R	q	r	AC
μ=0.5	0.03	14.32158	90.39558	826.235
	0.05	13.5173	90.79397	929.952
	0.08	12.39144	91.35254	1110.664
	0.1	11.69119	91.70023	1250.405
	0.13	10.71192	92.18692	1493.917
μ=1.5	0.03	14.87902	29.04959	347.4107
	0.05	14.14924	29.3708	389.7167
	0.08	13.11527	29.83011	463.3103
	0.1	12.46443	30.12178	520.1362
	0.13	11.54302	30.53792	619.0256
μ=2.5	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583

We observe that as inflation rate R increases and μ increases, average cost decreases when both the promises of doing payment are not satisfied.

3.5.3. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.1
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=0)

	R	q	r	AC
k=10	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359
k=15	0.03	19.43265	13.83951	240.2752
	0.05	18.63733	14.12252	268.499
	0.08	17.5092	14.53204	317.5309
	0.1	16.79783	14.79516	355.3481
	0.13	15.78782	15.1753	421.0925
k=20	0.03	21.49331	13.1275	246.1465
	0.05	20.60615	13.43032	274.6147
	0.08	19.34934	13.86894	324.03
	0.1	18.55767	14.15115	362.1161
	0.13	17.43471	14.5594	428.2823

For the above pattern we see that increase in inflation rate R and ordering cost k results in increase in average cost. However order quantity q increases but the reorder quantity r decreases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=0 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.3.2
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=0, V=1)

	R	q	r	AC
k=10	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406
k=15	0.03	21.08817	13.26511	239.0319
	0.05	20.28637	13.54083	267.2845
	0.08	19.15108	13.93917	316.3951
	0.1	18.43626	14.19484	354.2939
	0.13	17.42327	14.56361	420.2155
k=20	0.03	23.17008	12.57033	244.4731
	0.05	22.27303	12.866	272.9367
	0.08	21.00418	13.29378	322.3759
	0.1	20.20617	13.56868	360.502
	0.13	19.07622	13.96576	426.7775

We see that as inflation rate R increases and k increases, value of q increases and the value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.3.3
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=0)

	R	q	r	AC
k=10	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264
k=15	0.03	19.43265	13.83951	241.5323
	0.05	18.63733	14.12252	269.9164
	0.08	17.5092	14.53204	319.2278
	0.1	16.79783	14.79516	357.2613
	0.13	15.78782	15.1753	423.3831
k=20	0.03	21.49331	13.1275	247.4036
	0.05	20.60615	13.43032	276.0321
	0.08	19:34934	13.86894	325.7276
	0.1	18.55767	14.15115	364.0294
	0.13	17.43471	14.5594	430.5728

Here we see that impact of increase in inflation rate R and ordering cost k, results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and ordering cost k, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.3.4
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=1)

	R	q	r	AC
k=10	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583
k=15	0.03	16.32302	14.97289	243.3403
	0.05	15.56404	15.26058	271.5019
	0.08	14.48802	15.67602	320.3556
	0.1	13.81005	15.94238	357.9842
	0.13	12.84854	16.32624	423.3143
k=20	0.03	18.28978	14.24768	250.2498
	0.05	17.44311	14.55632	278.7377
	0.08	16.24353	15.00282	328.1113
	0.1	15.48803	15.28964	366.1084
	0.13	14.417	15.70376	432.0264

In this situation also increase in inflation rate R and ordering cost k, results in increase in q and decrease in r which results in increase in average cost.

3.5.4. Sensitivity Analysis for θ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.4.1
Sensitivity Analysis Table by varying the parameter values of R & θ when pattern of payment is (U=0, V=0)

	R	q	r	AC
θ=5	0.03	16.88233	14.7637	233.6852
	0.05	16.19804	15.01994	261.6373
	0.08	15.22611	15.3901	310.2421
	0.1	14.61246	15.62752	347.7608
	0.13	13.74027	15.96999	413.0359
θ=7	0.03	17.75518	15.27922	256.1078
	0.05	17.03662	15.55068	286.8706
	0.08	16.01591	15.9426	340.3739
	0.1	15.37125	16.1939	381.6815
	0.13	14.45485	16.55638	453.5606
θ=10	0.03	19.02515	15.95638	289.0128
	0.05	18.25677	16.25005	323.9032
	0.08	17.16488	16.67385	384.6002
	0.1	16.47531	16.94548	431.4721
	0.13	15.49453	17.3371	513.0501

We observe that with increase in inflation rate R and increase in rate of deterioration θ, average cost increases even if both the promises of doing payment are fulfilled.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.4.2
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=1)

	R	q	r	AC
θ=5	0.03	18.52109	14.1643	232.9749
	0.05	17.8344	14.41302	260.9979
	0.08	16.86092	14.77166	309.7512
	0.1	16.24746	15.00135	347.4029
	0.13	15.37742	15.33197	412.9406
θ=7	0.03	19.38058	14.67849	255.5029
	0.05	18.66003	14.9425	286.3457
	0.08	17.63801	15.32323	340.0127
	0.1	16.99379	15.56698	381.4646
	0.13	16.07963	15.91792	453.6254
θ=10	0.03	20.63381	15.35375	288.5467
	0.05	19.86382	15.64016	323.5288
	0.08	18.7713	16.05299	384.4094
	0.1	18.08221	16.31729	431.4405
	0.13	17.10397	16.69774	512.8098

We see that as inflation rate R increases and θ increases, value of q increases and the value of reorder quantity r increases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.4.3

Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=0)

	R	q	r	AC
θ=5	0.03	16.88233	14.7637	234.9423
	0.05	16.19804	15.01994	263.0547
	0.08	15.22611	15.3901	311.939
	0.1	14.6124	15.62752	349.674
	0.13	13.74027	15.96999	415.3264
θ=7	0.03	17.75518	15.27922	257.3648
	0.05	17.03662	15.55068	288.2879
	0.08	16.01591	15.9426	342.0708
	0.1	15.37125	16.19395	383.5947
	0.13	14.45485	16.55638	455.8512
θ=10	0.03	19.02515	15.95638	290.2699
	0.05	18.25677	16.25005	325.3206
	0.08	17.16488	16.67385	386.2971
	0.1	16.47531	16.94548	433.3853
	0.13	15.49453	17.3371	515.3406

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase which results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=1. Inflation rate R is assumed to take values 0.03, 0.05, 0.08, 0.1, 0.13 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.4.4
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=1)

	R	q	r	AC
	0.03	13.89057	15.91055	235.4601
	0.05	13.2388	16.16954	263.247
θ=5	0.08	12.31452	16.54256	311.5025
	0.1	11.7319	16.7811	348.7066
	0.13	10.90561	17.124	413.3583
	0.03	14.74002	16.44295	257.7117
	0.05	14.05276	16.71732	288.2974
θ=7	0.08	13.07758	17.11251	341.432
	0.1	12.46279	17.3652	382.4109
	0.13	11.59046	17.72846	453.6438
	0.03	15.97927	17.14276	290.3865
	0.05	15.24029	17.43963	325.0835
θ=10	0.08	14.19148	17.86711	385.3853
	0.1	13.52994	18.14043	431.9093
	0.13	12.59087	18.53334	513.325

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase and hence average cost increases.

Conclusion:

The comparative study of the above sensitivity analysis of case-I that is when inflation rate is less than interest charged is summarized below:
Average cost is least for pattern (U=0, V=1) and highest for pattern (U=1, V=1).

AC (U=0, V=1) < AC (U=0, V=0) < AC (U=1, V=0) < AC (U=1, V=1).
It is always beneficial to keep promises especially first one. The option of part payment is very useful for enhancing business and encouraging the small entrepreneurs.

Case-II: Inflation rate is greater than interest charged.

3.5.5. Sensitivity Analysis for λ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.5.1
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=0, V=0)

	R	q	r	AC
λ=0.1	0.15	13.42719	7.536989	364.6168
	0.2	12.12649	8.056274	485.4891
	0.25	10.95544	8.535088	647.9415
	0.3	9.90025	8.975768	866.4483
	0.35	8.94875	9.380658	1160.54
λ=0.15	0.15	13.34581	11.41842	409.7328
	0.2	12.05238	11.93749	546.419
	0.25	10.888	12.4159	730.2247
	0.3	9.83904	12.85605	977.5636
	0.35	8.893186	13.26035	1310.584
λ=0.2	0.15	13.26658	14.12509	440.4957
	0.2	11.9801	14.64394	587.9732
	0.25	10.8224	15.1219	786.3524
	0.3	9.77936	15.56166	1053.371
	0.35	8.839035	15.96537	1412.965

We see that as inflation rate R increases and λ increases, value of q decreases and the value of reorder quantity r increases and hence average cost increases, when both the promises are fulfilled of doing payment.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.5.2
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=0, V=1)

	R	q	r	AC
λ=0.1	0.15	15.05468	6.905507	364.8249
	0.2	13.75917	7.406539	486.5188
	0.25	12.5981	7.866458	650.241
	0.3	11.55748	8.287616	870.6585
	0.35	10.6249	8.672205	1167.569
λ=0.15	0.15	14.9775	10.782	409.9142
	0.2	13.6909	11.28211	547.4114
	0.25	12.53827	11.74098	732.473
	0.3	11.5055	12.1608	981.7035
	0.35	10.58041	12.54409	1317.517
λ=0.2	0.15	14.9022	13.48393	440.651
	0.2	13.6243	13.983	588.9292
	0.25	12.47977	14.4409	788.5504
	0.3	11.45476	14.85957	1057.442
	0.35	10.53698	15.24143	1419.805

We see that as inflation rate R increases and λ increases, average cost increases when promise of doing part payment at time T_1 is satisfied but remaining amount is not cleared at time T.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.5.3
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=1, V=0)

	R	q	r	AC
λ=0.1	0.15	13.4272	7.536989	367.1994
	0.2	12.12649	8.056274	488.9752
	0.25	10.95545	8.535088	652.6472
	0.3	9.900255	8.975768	872.8004
	0.35	8.94875	9.38065	1169.115
λ=0.15	0.15	13.3458	11.4184	412.3154
	0.2	12.05238	11.9374	549.9051
	0.25	10.88806	12.4159	734.9305
	0.3	9.83904	12.85605	983.9157
	0.35	8.89318	13.26035	1319.158
λ=0.2	0.15	13.266	14.12509	443.0783
	0.2	11.98016	14.64394	591.4593
	0.25	10.8224	15.12197	791.0582
	0.3	9.77936	15.56166	1059.723
	0.35	8.839035	15.9653	1421.54

In this situation also average cost increases when inflation rate R increases and λ increases, when promise of doing part payment at time T_1 is not satisfied but remaining amount is cleared at time T.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of ON period λ keeping other parameter values fixed where U=1 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of ON period λ is assumed to take values 0.1, 0.15, 0.2. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.5.4
Sensitivity Analysis Table by varying the parameter values of R & λ when pattern of payment is (U=1, V=1)

	R	q	r	AC
λ=0.1	0.15	10.64318	8.664582	364.2633
	0.2	9.421672	9.178524	483.2027
	0.25	8.329594	9.64799	642.6581
	0.3	7.353962	10.07542	856.6479
	0.35	6.48322	10.46331	1144.07
λ=0.15	0.15	10.5547	12.55482	409.4269
	0.2	9.339238	13.06881	544.1954
	0.25	8.252826	13.53809	725.0229
	0.3	7.282574	13.96508	967.8677
	0.35	6.416958	14.3523	1294.246
λ=0.2	0.15	10.46866	15.27013	440.2361
	0.2	9.25895	15.78422	585.8109
	0.25	8.178065	16.2533	781.2304
	0.3	7.21297	16.67991	1043.777
	0.35	6.352394	17.06647	1396.756

When both the promises of doing payment are not satisfied, impact of increase in inflation rate R and λ, results in increase in average cost.

3.5.6. Sensitivity Analysis for μ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=0 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.6.1
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=0, V=0)

	R	q	r	AC
μ=0.5	0.15	20.14965	87.52194	1675.081
	0.2	18.22546	88.46774	2257.823
	0.25	16.48615	89.32529	3044.119
	0.3	14.91364	90.10249	4105.151
	0.35	13.49172	90.80671	5537.12
μ=1.5	0.15	15.25575	28.88473	693.5999
	0.2	13.78838	29.53055	930.8138
	0.25	12.46484	30.12157	1250.464
	0.3	11.27009	30.66194	1681.334
	0.35	10.19126	31.15538	2262.274
μ=2.5	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404

When both the promises of doing payment are fulfilled by the businessman we find that as inflation rate R increases and μ increases, average cost decreases. This may be because unavailability of supplier is for less period of time and hence it is not necessary to stock more items which result in decrease in average cost.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.6.2
Sensitivity Analysis Table by varying the parameter values of R & μ when pattern of payment is (U=0, V=1)

	R	q	r	AC
	0.15	32.25563	81.65452	1682.245
	0.2	30.89473	82.30591	2264.322
μ=0.5	0.25	29.75355	82.85393	3049.063
	0.3	28.80833	83.30901	4107.211
	0.35	28.0364	83.68135	5534.236
	0.15	18.12623	27.65052	695.41
	0.2	16.71267	28.25346	931.116
μ=1.5	0.25	15.45126	28.79946	1250.197
	0.3	14.32767	29.29204	1682.27
	0.35	13.32927	29.73465	2265.116
	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
μ=2.5	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151

We observe that when promise of doing part payment at T_1 is satisfied but clearing the remaining amount at T is not fulfilled, impact of increase in inflation rate R and μ results in decrease in average cost.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and length of OFF period μ keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.6.3
Sensitivity Analysis Table by varying the parameter values of R & μ when pattern of payment is (U=1, V=0)

	R	q	r	AC
	0.15	20.14965	87.52194	1677.66
	0.2	18.2254	88.46774	2261.309
μ=0.5	0.25	16.48615	89.32529	3048.824
	0.3	14.91364	90.10249	4111.503
	0.35	13.49172	90.80671	5545.575
	0.15	15.25575	28.8847	696.1825
	0.2	13.78838	29.53055	934.2999
μ=1.5	0.25	12.46484	30.12157	1255.169
	0.3	11.27009	30.66194	1687.686
	0.35	10.19126	31.15538	2270.848
	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
μ=2.5	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978

We see that as inflation rate R increases and μ increases, average cost decreases when promise of doing part payment at T_1 is not satisfied but clearing the remaining amount at T is fulfilled.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and length of OFF period μ keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and length of OFF period μ is assumed to take values 0.5, 1.5, 2.5. We resolve the problem to find optimal values of *q*, *r* and *AC.*

Table 3.5.6.4
Sensitivity Analysis Table by varying the parameter values of *R* & μ when pattern of payment is (U=1, V=1)

	R	q	r	AC
μ=0.5	0.15	10.10361	92.48949	1666.819
	0.2	8.725648	93.17555	2247.877
	0.25	7.531373	93.77078	3032.146
	0.3	6.497442	94.28654	4090.728
	0:35	5.603545	94.73265	5519.602
μ=1.5	0.15	10.96332	30.80171	692.225
	0.2	9.627772	31.41513	929.829
	0.25	8.441602	31.96649	1248.263
	0.3	7.390016	32.46037	1675.188
	0.35	6.459664	32.90118	2250.115
μ=2.5	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322

We observe that as inflation rate R increases and μ increases, average cost decreases when both the promises of doing payment are not satisfied.

3.5.7. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and ordering cost k, keeping other parameter values fixed where U=0 and V=0. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.7.1
Sensitivity Analysis Table by varying the parameter values of R & k when pattern of payment is (U=0, V=0)

	R	q	r	AC
k=10	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404
k=15	0.15	15.15034	15.41925	471.8318
	0.2	13.67226	15.99694	628.2439
	0.25	12.34417	16.53048	838.4713
	0.3	11.14946	17.02225	1121.251
	0.35	10.07356	17.47471	1501.865
k=20	0.15	16.72656	14.82172	479.3162
	0.2	15.08645	15.44387	636.5154
	0.25	13.61499	16.01966	847.6086
	0.3	12.29256	16.5515	1131.341
	0.35	11.10301	17.04159	1513.004

For the above pattern we see that increase in inflation rate R and ordering cost k results in increase in average cost. However order quantity q increases but the reorder quantity r decreases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and ordering cost k, keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.7.2
Sensitivity Analysis Table by varying the parameter values of R & k when pattern of payment is (U=0, V=1)

	R	q	r	AC
k=10	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151
k=15	0.15	16.78515	14.79988	473.6709
	0.2	15.30952	15.35803	628.6383
	0.25	13.98951	15.87152	839.3724
	0.3	12.80799	16.34258	1123.736
	0.35	11.75012	16.77359	1506.763
k=20	0.15	18.3647	14.22062	482.8003
	0.2	16.72121	14.82372	638.9079
	0.25	15.25239	15.38	848.0316
	0.3	13.93839	15.89166	1132.612
	0.35	12.76223	16.36103	1516.425

We see that as inflation rate R increases and k increases, value of q increases and the value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and ordering cost k, keeping other parameter values fixed where U=1 and V=0. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.7.3
Sensitivity Analysis Table by varying the parameter values of R & k when pattern of payment is (U=1, V=0)

	R	q	r	AC
k=10	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978
k=15	0.15	15.15034	15.41925	474.4144
	0.2	13.67226	15.99694	631.73
	0.25	12.34417	16.53048	843.1771
	0.3	11.14946	17.0222	1127.603
	0.35	10.07356	17.47471	1510.439
k=20	0.15	16.72656	14.82172	481.8987
	0.2	15.08645	15.44387	640.0016
	0.25	13.61499	16.01966	852.3144
	0.3	12.29256	16.5515	1137.693
	0.35	11.10301	17.04159	1521.578

Here we see that impact of increase in inflation rate R and ordering cost k results in increase in average cost.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and ordering cost k, keeping other parameter values fixed where U=1 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and ordering cost k is assumed to take values 10, 15, 20. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.7.4
Sensitivity Analysis Table by varying the parameter values of *R* & k when pattern of payment is (U=1, V=1)

	R	q	r	AC
k=10	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322
k=15	0.15	12.24252	16.57191	471.1172
	0.2	10.84128	17.15092	628.1384
	0.25	9.588645	17.6816	836.465
	0.3	8.46937	18.16621	1115.454
	0.35	7.470183	18.60712	1490.281
k=20	0.15	13.74212	15.96924	477.9238
	0.2	12.18185	16.59665	635.588
	0.25	10.78702	17.17366	847.5113
	0.3	9.54012	17.7024	1128.499
	0.35	8.426102	18.18515	1505.017

In this situation also increase in inflation rate R and ordering cost k results in increase in value of q and decrease in the value of reorder quantity r which results in increase in average cost.

3.5.8. Sensitivity Analysis for θ:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=0. Inflation rate

R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.8.1
Sensitivity Analysis Table by varying the parameter values of R & θ when pattern of payment is (U=0, V=0)

	R	q	r	AC
θ=5	0.15	13.18918	16.18941	463.4471
	0.2	11.90975	16.70801	618.9821
	0.25	10.75833	17.1857	828.2443
	0.3	9.72121	17.62486	1109.961
	0.35	8.786195	18.02806	1489.404
θ=7	0.15	13.87577	16.78852	509.0805
	0.2	12.53098	17.33704	680.4105
	0.25	11.32044	17.84203	910.9765
	0.3	10.22971	18.30616	1221.431
	0.35	9.246411	18.73199	1639.645
θ=10	0.15	14.87466	17.58784	576.0727
	0.2	13.43478	18.17996	770.6005
	0.25	12.13826	18.72478	1032.456
	0.3	10.96982	19.22517	1385.12
	0.35	9.915986	19.68414	1860.281

We observe that with increases in inflation rate R and increases in rate of deterioration θ, average cost increases even if both the promises of doing payment are fulfilled.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R and rate of deterioration θ, keeping other parameter values fixed where U=0 and V=1. Inflation rate R is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of q, r and AC.

Table 3.5.8.2
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=0, V=1)

	R	q	r	AC
θ=5	0.15	14.82867	15.54353	463.5771
	0.2	13.55915	16.04184	619.9026
	0.25	12.42268	16.49856	830.3931
	0.3	11.40532	16.91599	1113.965
	0.35	10.49453	17.29658	1496.151
θ=7	0.15	15.50304	16.14231	509.3848
	0.2	14.1681	16.67102	681.547
	0.25	12.9721	17.15566	913.3938
	0.3	11.90049	17.59871	1225.77
	0.35	10.94051	18.0027	1646.812
θ=10	0.15	16.4867	16.94095	576.6058
	0.2	15.05659	17.514	772.0204
	0.25	13.7741	18.03921	1035.225
	0.3	12.62372	18.51949	1389.898
	0.35	11.59204	18.95757	1867.997

We see that as inflation rate R increases and θ increases, value of q increases and the value of reorder quantity r increases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=0. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.8.3
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=0)

	R	q	r	AC
θ=5	0.15	13.18918	16.18941	466.0297
	0.2	11.9097	16.70801	622.4682
	0.25	10.75833	17.1857	832.95
	0.3	9.72121	17.62486	1116.313
	0.35	8.786195	18.02806	1497.978
θ=7	0.15	13.87577	16.78852	511.6631
	0.2	12.53098	17.33704	683.8966
	0.25	11.32044	17.84203	915.6823
	0.3	10.22971	18.30616	1227.783
	0.35	9.246411	18.73199	1648.22
θ=10	0.15	14.87466	17.58784	578.6553
	0.2	13.43478	18.17996	774.0867
	0.25	12.13826	18.72478	1037.162
	0.3	10.96982	19.22517	1391.472
	0.35	9.915986	19.68414	1868.856

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase and hence average cost increases.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate *R* and rate of deterioration θ, keeping other parameter values fixed where U=1 and V=1. Inflation rate *R* is assumed to take values 0.15, 0.2, 0.25, 0.3, 0.35 and deterioration rate θ is assumed to take values 5, 7, 10. We resolve the problem to find optimal values of *q*, *r* and *AC*.

Table 3.5.8.4
Sensitivity Analysis Table by varying the parameter values of *R* & θ when pattern of payment is (U=1, V=1)

	R	q	r	AC
θ=5	0.15	10.38479	17.34289	463.2328
	0.2	9.180637	17.85713	616.8797
	0.25	8.10508	18.32605	823.2002
	0.3	7.145135	18.75224	1100.467
	0.35	6.289428	19.13815	1473.322
θ=7	0.15	11.04048	17.96032	508.6105
	0.2	9.768346	18.50508	678.0063
	0.25	8.631103	19.00202	905.577
	0.3	7.615318	19.4538	1211.52
	0.35	6.709058	19.86311	1623.074
θ=10	0.15	11.99861	18.78409	575.2568
	0.2	10.62766	19.37339	767.7875
	0.25	9.400965	19.91109	1026.574
	0.3	8.304131	20.40015	1374.641
	0.35	7.324433	20.84353	1843.043

We see that as inflation rate R increases and θ increases, value of q and the value of reorder quantity r increase which results increase in average cost.

Conclusion:

The comparative study of the above sensitivity analysis of case-II that is when inflation rate is higher than interest charged is summarized below:
Average cost is least for pattern (U=1, V=1) and highest for pattern (U=1, V=0).
This implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is that once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation.

3.6. CONCLUSION:

By comparing two cases that is when inflation rate is less than interest charged we conclude that cost is minimum if part payment is done at T_I but account is not cleared at T and the cost is maximum if part payment is not done at T_I and also account is not cleared at T. This implies that we encourage the small businessmen to do the business by allowing partial payment and simultaneously we want to discourage them for not clearing the account at the end of credit period. However when inflation rate is higher than interest charged we observe that average cost is minimum if part payment is not done at T_I and also account is not cleared at T which implies that businessmen are advised not to settle the account at the end of the credit period but settle the account at the end of the cycle period. The reason for this is that once the inflation rate is greater than the interest rates charged, we actually see our debt wiped out by inflation. Debtors are benefitted by inflation due to the reduction of real value of debt burden.