Abstract of Ph. D. thesis

A Study of Circular and Elliptical Restricted Three Body Problems with Perturbations

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by

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In this study, Circular and Elliptical Restricted Three Body Problems are considered for obtaining two and three dimensional periodic orbits. The perturbing forces due to radiation and oblateness of the primaries are considered and their effects on various parameters of periodic orbits are analyzed. There are two major objectives of this work:

1. To refine and modify the existing technique of obtaining halo orbits to do more precise orbit determination.

2. To extend the method of Poincar´e Surface of Sections from CRTBP to ERTBP and compute different planar resonant periodic orbits in ERTBP framework.

The Lindstedt-Poincaré method, an analytical method, is used to find the fifth order approximate solution for getting an initial state vector of halo orbits around collinear Lagrangian points L_1, L_2 and L_3 in CRTBP. A comparison between the separations of the third, fourth and fifth order solutions show that the separation between the third and fourth order solution is more than the separation between the fourth and fifth order solution. This indicates that the halo orbits come closer and more accurate initial guess is obtained as the order of solution increases. The third, fourth and fifth order analytical solutions are revised using the numerical method of differential correction and the separation between these solutions are measured. The analysis shows that the separation between successive solutions decrease as the order of analytical solution increases. This confirms that the approximate solution converges to the exact solution as the order of solution increases. Further, the effects of perturbations due to radiation and oblateness of the primaries on various parameters of halo orbits around collinear Lagrangian points are analyzed.

Computation of Lagrangian points show that their locations depend on the mass factor of primaries. Further, the parameters of halo orbits are also affected due to variation in mass factor. The variations in locations, size, periods, amplitude and initial state vectors of collinear Lagrangian points L_1, L_2 and L_3 due to variation in mass factor of primaries are analyzed. For the analysis, different values of mass factor in the interval $[10^{-8}, 0.5]$ with a step size of 10^{-6} are considered. To verify the results, five different Sun-Planet systems are considered. It is observed that as the value of μ increases, Lagrangian point L_1 and halo orbits around it shift towards the more massive primary while Lagrangian

point L_2 and corresponding halo orbits recede from the second primary till $\mu_0 = 0.17894$ and for $\mu > \mu_0$, orbits move towards the second primary. Lagrangian point L_3 and halo orbits around it move towards the more massive primary.

The equations representing the motion of the infinitesimal body in ERTBP framework contain the independent variable explicitly. Due to this, the Jacobi constant does not exist and the computation of halo orbits become difficult. In this study, the motion of the infinitesimal body is considered with respect to the independent variable eccentric anomaly instead of true anomaly. The system representing the equations of motion is averaged with respect to the eccentric anomaly to obtain an autonomous system. In this system, halo orbits around collinear Lagrangian points are computed in the Sun-Mars system. The effects of eccentricity of orbit of the Mars on parameters of halo orbits are analyzed. Also, the Sun is considered as a source of radiation. The variations in the stability region of halo orbits due to solar radiation pressure are analyzed. From a graphical comparison between the orbits in CRTBP and ERTBP framework, it can be concluded that halo orbits in ERTBP framework are smaller.

In the averaged ERTBP framework, the Jacobi constant exists. This facilitates the computation of Poincaré Surface of Sections (PSS) in ERTBP framework. By extending the PSS technique from CRTBP to ERTBP, f-family orbits are computed in the photogravitational Sun-Saturn system. The effects of non-zero value of eccentricity of the orbit of the primaries on the maximum permissible value of Jacobi constant and on the length and location of excluded region are analyzed. It has been also observed that the primaries' eccentricity affect the locations, periods, size and diameters of these orbits. Due to radiation of the more massive primary, the excluded region expands and the maximum permissible value of Jacobi constant decreases. Further, orbits shift towards the second primary and contract due to increase in solar radiation pressure. It affects the stability of f-family orbits. Variations in parameters of f-family orbits due to variation in the value of Jacobi constant are studied. Regression analysis is used to find the estimator functions of the maximum permissible value of Jacobi constant, length of excluded region, locations and the diameter of orbits. Here, the eccentricity of the orbit of the primaries, e, is considered as the predictor variable.

In the Sun-Saturn ERTBP, the first order exterior resonant orbits are computed using the PSS method. These orbits lie on the right side of the f-family orbits. In this study, five different types of resonant orbits having 1, 2, 3, 4 and 5 interior loops are obtained. Due to increase in the value of e, these orbits shift towards the Sun, their period increase and semi-major axis decrease. So, interior resonant orbits shrink due to increase in e. With the increase the eccentricity of the orbit of the primaries, the eccentricity e_s of the orbit of the spacecraft decreases for $1:2$ resonant orbits while the reverse effect of observed on other $p:p+1$ resonant orbits, $p \in \{2,3,4,5\}$. The effects of solar radiation pressure and Jacobi constant on parameters of these orbits are similar in CRTBP and ERTBP.

The analysis of the first order interior resonant orbits in the photogravitaional Sun-Saturn ERTBP is done using the numerical method of PSS. The effects of eccentricity of the orbit of the primaries, solar radiation pressure and Jacobi constant on various parameters of these orbits are studied. In this case, the resonance ratio $p:s$ is such that $p > s$ and there are p external loops in the orbit of the spacecraft. Different values of $e \in [0, 0.09]$ are considered to analyze the variations in locations, periods, size, diameters, eccentricity and semi-major axis of interior resonant orbits. The orbits recede from the Saturn and shift towards the Sun, their periods, semi-major axis and eccentricity increase with the increase in e. The analysis of the size and shape of these orbits show that the orbits shrink while the loops enlarge. Due to solar radiation pressure, interior resonant orbits advance towards the Sun. Further, the orbits expand while loops contract with the increase in solar radiation pressure. By considering five different values of Jacobi constant in the range $[2.88, 2.92]$, variations in parameters of resonant orbits are noted.