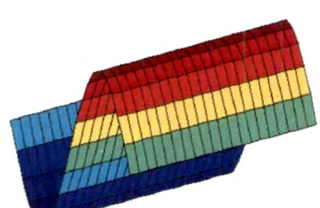
Chapter 6

ESTIMATION
OF
HOURLY AIR TEMPERATURES
BY
WILLIAM AND LOGAN MODEL,
DOUBLE FOURIER SERIES
AND
ARTIFICIAL NEURAL NETWORKS



6.1 INTRODUCTION

In crop simulation models hourly air temperature (HAT) plays an important role. The hourly air temperature is an average of the temperature in an hour measured at every second at normal height (i.e. at 1.25 meter above the earth surface.). At a few places automatic weather stations (Campbell, U S A) are installed which have instrument to get the hourly air temperature. But hourly air temperature data is not available directly at every research station, which is essential for the scientists to carry out the research. Usually, it is computed as the average of the maximum temperature (MaxAT) and minimum air temperature (MinAT) during a day.

The daily extreme temperatures are easily recorded from maximum and minimum thermometers installed in Stevenson screen (Epperson [29]). This instrument is available at every research station.

Our aim in this chapter is to predict the hourly air temperature (HAT). We estimate the hourly air temperatures in two situations, namely,

(i) When input variables are daily Maximum Air Temperature (MaxAT) and Minimum Air temperatures (MinAT).

(ii) When Input variables are Hourly Maximum Air Temperature (MaxHAT) and Hourly Minimum Air temperatures (MinHAT).

In case (i), two methods are proposed:

(a) Average of Daily Extreme Air Temperatures

and

(b) William and Logan Model

In case (ii) four methods are used:

(a) William and Logan Model ,

(b) Double Fourier Series

(c) Artificial Neural Networks

and

(d) Average of Hourly Extreme Air Temperatures

6.2 PROBLEM FORMULATION

The air temperature is a continuous function of time. However, we take it's values at discrete time point, that is at every hour. The hourly air temperature (HAT) function increases after 5 hour and after the occurrence of maximum temperature (nearly after 1400hr) it starts decreasing. This decrement is continuing up to 5 hr.

The figure 6.1 gives the hourly air temperature of a 27th March 2006. The figure indicates that hourly air temperature process has diurnal variations, namely the day and night variations, which can be described by two separate functions as shown in the figure 6.2 and 6.3.

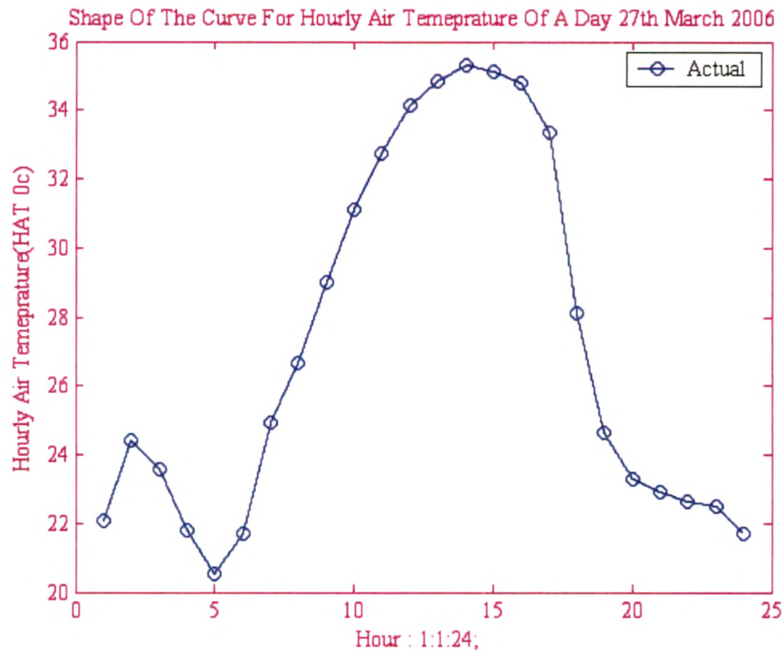

Shape Of The Curve For Hourly Air Temeprature Of A Day 27th March 2006
Actual
Hourly Air Temeprature(HAT 0c)
Hour : 1:1:24;
36
34
32
30
28
26
24
22
20
0
5
10
15
20
25

Fig: 6.1

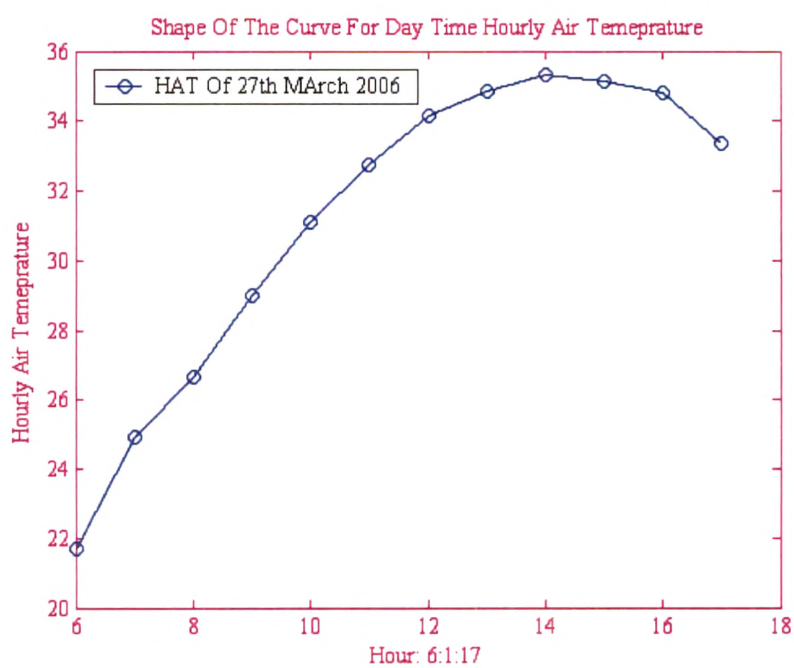

Shape Of The Curve For Day Time Hourly Air Temeprature
HAT Of 27th MArch 2006
Hourly Air Temeprature
Hour: 6:1:17
36
34
32
30
28
26
24
22
20
6
8
10
12
14
16
18

Fig: 6.2

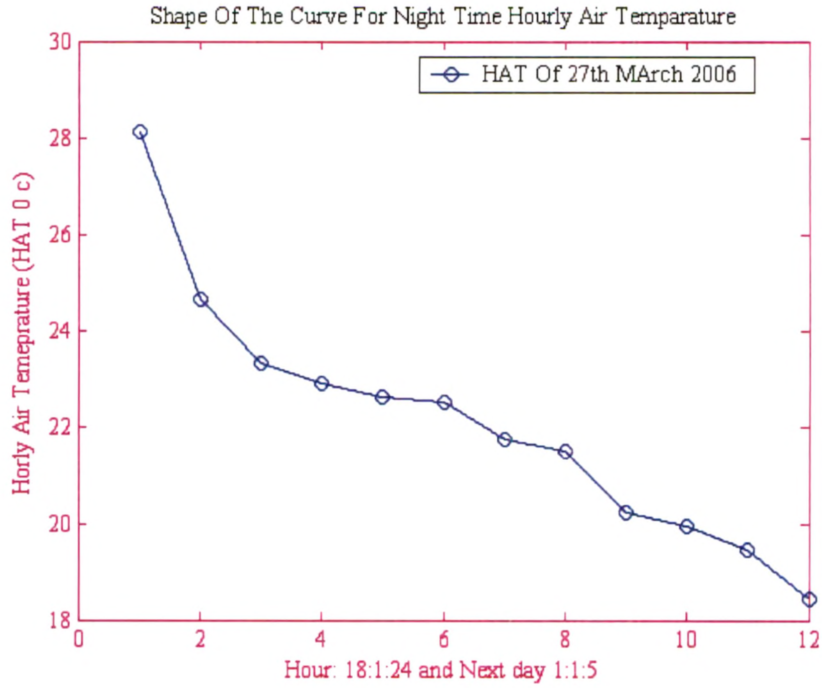


Fig: 6.3

There are various methods used to model the diurnal patterns using different functions matching the shape of the curve. These major methods are linear models (Sanders [133]), simple curve fitting models based on Fourier analysis (Walter [165]). Reicoksky *et al.* [128] have checked the accuracy of predicted hourly temperatures from extreme temperatures. They found that obtained temperatures by all the methods found significant except on non-clear days.

The two most frequently used methods for simulation of air temperatures are empirical models and energy-budget models (Goudriaan *et al.* [45]; Lemon *et al.* [98]; Myrup [111]). Energy budget models are difficult to use as they require various data inputs like, solar radiation, wind speed, dew point, air temperature etc. whereas the empirical model require only the maximum and minimum temperatures.

Diurnal patterns consist of some type of curve designed. Accuracy of the model depends on the matching of the daily temperature patterns and established theoretical relation by an equation for the estimation of air temperatures by proper selection of input variables.

William and Logan [121] have developed a mathematical model for diurnal variation of the Hourly Air Temeprautre (HAT) from the shape of the curve. They have used a truncated sine wave (Fig: 6.2) to predict the hourly air temperature for daytime and negative exponential function for nighttime temperature (Fig: 6.3).

Das *et al.* [27] have used the William and Logan model and estimated diurnal changes in soil and air temperatures.

Here, William and Logan mathematical model is applied for the Anand station to predict the hourly air temperature (HAT).

6.2.1 CASE (i) HAT BY EXTREME TEMPERATURES

METHOD (a) AVERAGE OF DAILY EXTREME AIR TEMPERATURE

The daily differences between mean temperatures obtained from maximum and minimum air temperatures recorded manually from liquid in glass thermometers in the observatory (D_{21}) and that obtained from the automatic weather station recorded at every hour (D_1) are shown in the figure 6.4(a). Three distinct periods of characteristic variation are discussed in this chapter. These distinct periods of the year 1992 are (i) 5th December to 26th May (340 to 147 J. days) (ii) 27th May to 22nd September (148 to 266 J.days) and (iii) 23rd September to 4th December (267 to 339 J.day).

During the first period, the mean temperatures obtained by mean of daily extremes (D_{21}) are higher than the those observed at automatic weather station, for most of the days except a few. These differences ranged from 1 ^{0}Cto 2^{0}C. In the second period, $D_{21} > D_1$, for almost all the days. In contrasts to these in the third period $D_1 > D_2$ except for a few.

After 32nd Julian day, the differences are less as compared to the previous days. This type of variation may be due to the addition or removal of the energy by advection. Biases in the manual measurement of temperature are also found to average about 0.5 ^{0}cto 1.5^{0}c by Epperson *et al.* [29]. Also, in the present study, the differences of these types amounted on an average, to 1-2^{0}c. This indicates that the mean temperatures obtained from the daily extremes are higher than the mean daily temperature obtained from hourly values of the day.

The algebraic summation of the differences for 350 days is depicted in the figure 6.4(b). It shows that the cumulative differences are in the increasing order, that is, $D_{21} > D_1$ for most of the days and the total summation came out to nearly 170^{0}c. This error associates itself in the calculation of the thermal time for growth and development of plant that would be used in the crop weather model.

Difference Between D_{21} (HAT) And D_1(HAT)

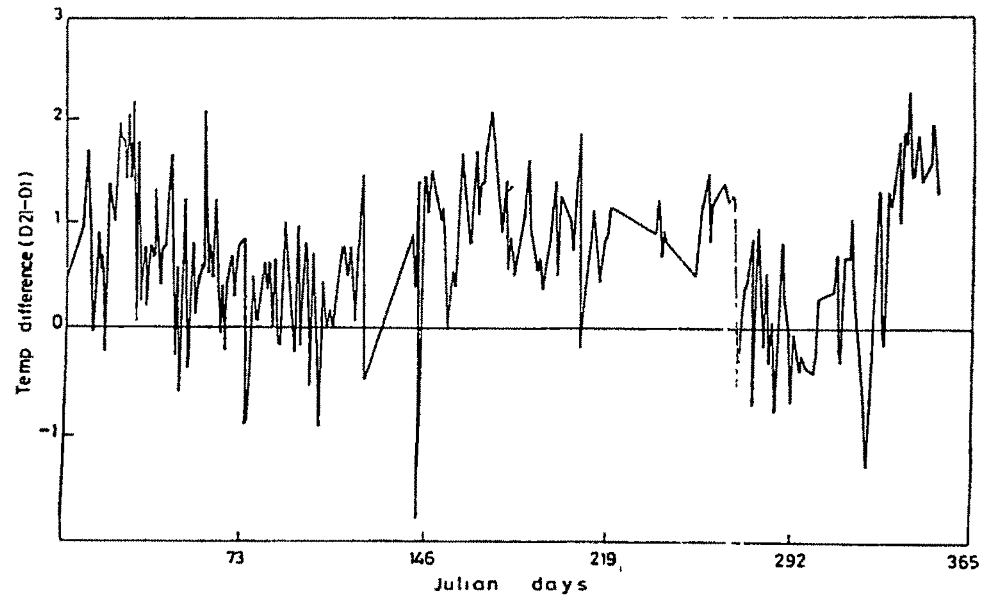


Fig: 6.4(a)

Cumulative Difference Between D_{21} (HAT) and D_1(HAT)

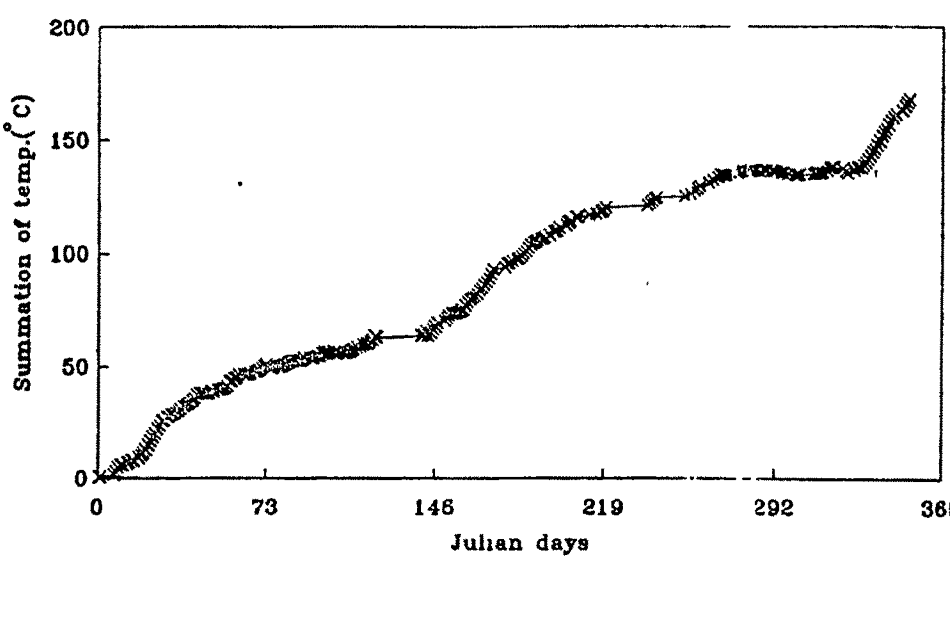


Fig: 6.4(b)

6.2.2 CASE (i) HAT BY EXTREME TEMPERATURES

METHOD (b): WILLIAM AND LOGAN MODEL

To get the true hourly air temperatures, William and Logan's model is used. They have included all the necessary input variables and parameter for estimation of hourly air temperature (HAT). They have taken maximum air temperature (MaxAT), minimum air temperature (MinAT), time variables hour and julian day (Table A-6.1), day length, night length etc.

This Model's assumptions are as below,

(i) The air maximum temperature is attended during the daytime hours.

(ii) The air minimum temperature occurs just before or after sunrise

Application of the Logan's model requires *'a'*, *'b'* and *'c'* parameters. These parameters value depends on the historical data series.

6.3 CASE (ii)

6.3.1 HAT BY HOURLY EXTREME TEMPERATURES

We have tried to find out the relation between hourly air temperatures (HAT) and Maximum Hourly Air Temperature (MaxHAT).

Figure 6.5 shows the relationship between Maximum Hourly Air Temperature (MaxHAT) and Hourly Air Temperature (HAT). From the figure linear relationship can be seen.

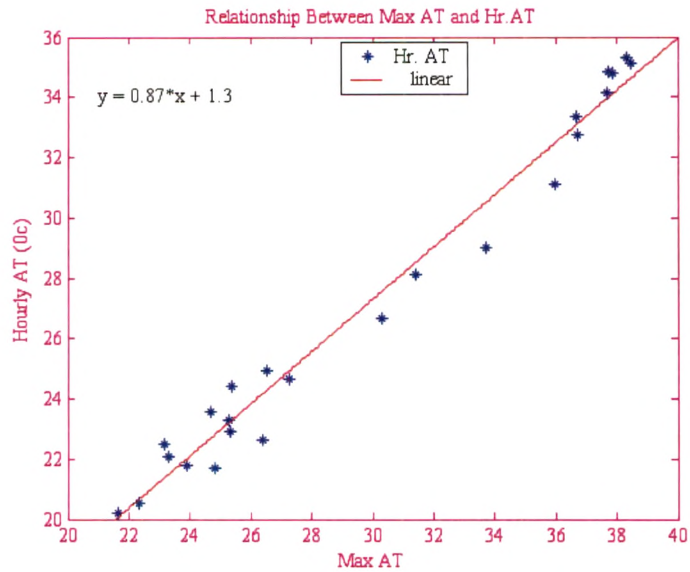


Fig: 6.5

Therefore, from the curve fitting linear equation is

$$HAT = 0.87 * (MaxHAT) + 1.33 \quad (6.1)$$

Using equation (6.1) HAT is obtained. Predicted (28th March 06) hourly air temperature is depicted in the figure 6.6. Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) by this linear equation are 0.8043 and 2.94% respectively. Hourly Air Temperature (HAT) of 13 & 23 April 2006 are also estimated by this equation (6.1) Their related RMSE and PAE are given in the Table 6.1.

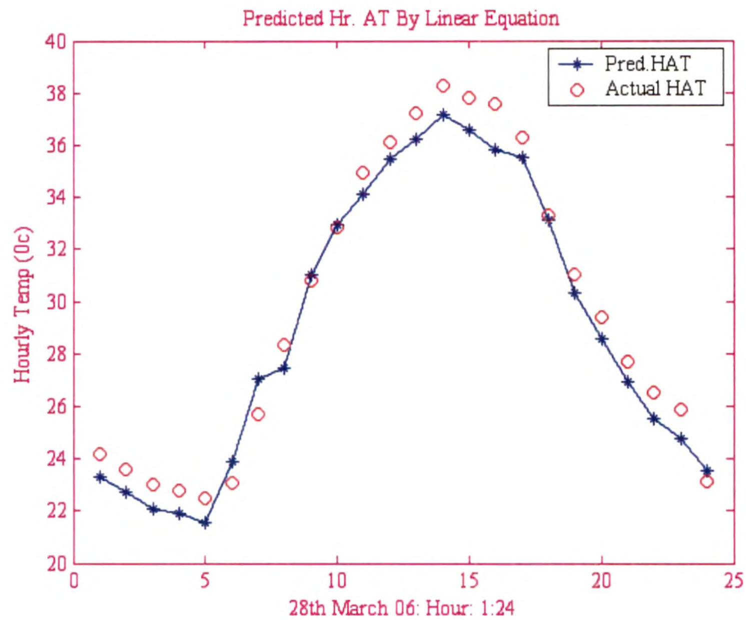


Fig: 6.6

TABLE: 6.1
RMSE &PAE DURING THE ESTIMATION OF "HAT" BY LINEAR EQUATION OF ONE VARIABLE " MaxHAT"

Sr. No	Predicting Julian Day/Day 2006	RMSE By Linear equation	PAE(%) By Linear equation
1	86ju./28 March	0.81	2.94
2	103ju.day/13 April	2.64	8.9
3	113ju day/23 April	1.19	4.39

Thus, results by linear equation of Maximum Hourly Air Temperature (MaxHAT) are significant but time that is day and hour is not included.

For estimation of Hourly Air Temperature (HAT) by time component, DS of Hourly Air Temperature (HAT) of 27th March 2006 is divided in two parts as mentioned above. One is HAT_1 from 5hr to 16hr and second one is HAT_2 17hr to 24hr and 1hr to 4hr. This time is selected as per the trend of the temperature.

Figure 6.7 and 6.8 shows the quadratic relation by curve fitting between hour (time) and HAT. These equations are as below.

$$HAT_1 = -0.15*x^2 + 4.5*x + 0.75; \quad (6.2)$$

$$HAT_2 = 0.19*x^2 - 9.3*x + 1.3 \times 10^2 \quad (6.3)$$

From these equations hourly air temperature of 28th March 2006 is estimated. Figure 6.9 shows the estimated hourly air temperature (* valued) by quadratic equation (6.2). Same way for other part of the time for hourly air temperature quadratic equation (6.3) is used and estimated hourly air temperatures are plotted in the figure 6.10 by * value. Their related Root Mean Square Error and Percentage of Average Error are given in the Table 6.2.

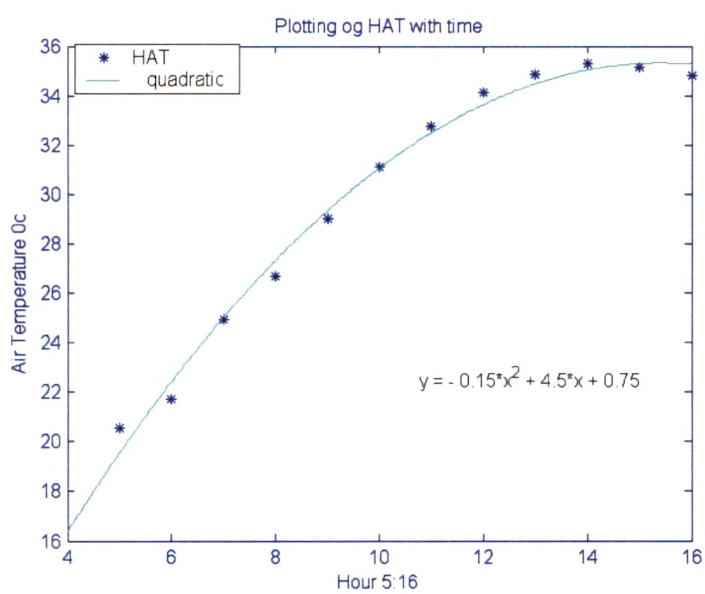


Fig: 6.7

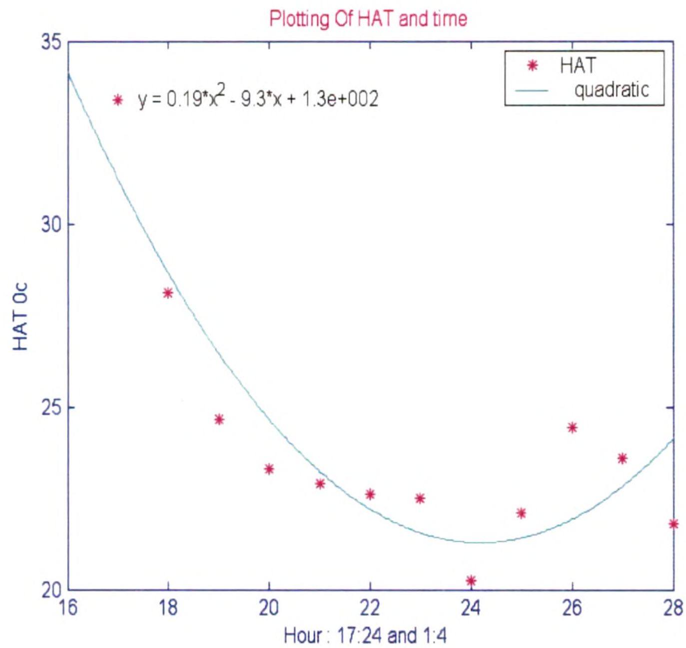

Plotting Of HAT and time
y = 0.19*x^2 - 9.3*x + 1.3e+002
HAT
quadratic
HAT 0c
Hour : 17:24 and 1:4
35
30
25
20
16
18
20
22
24
26
28

Fig: 6.8

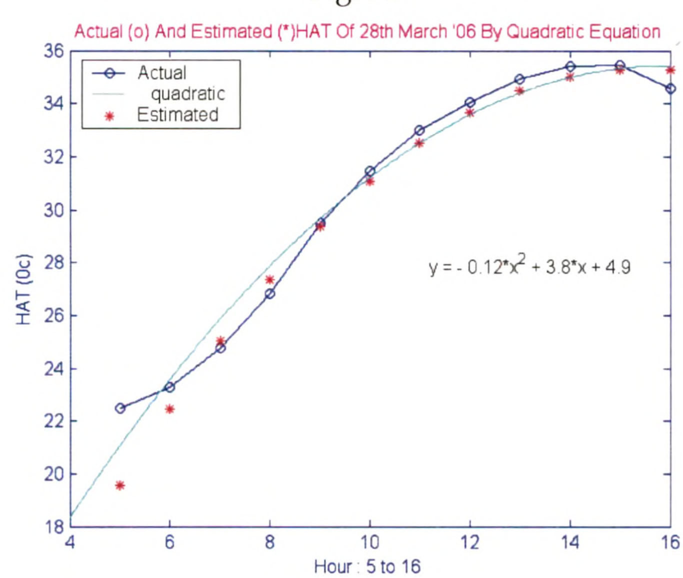

Actual (o) And Estimated (*)HAT Of 28th March '06 By Quadratic Equation
Actual
quadratic
Estimated
y = - 0.12*x^2 + 3.8*x + 4.9
HAT (0c)
Hour : 5 to 16
36
34
32
30
28
26
24
22
20
18
4
6
8
10
12
14
16

Fig: 6.9

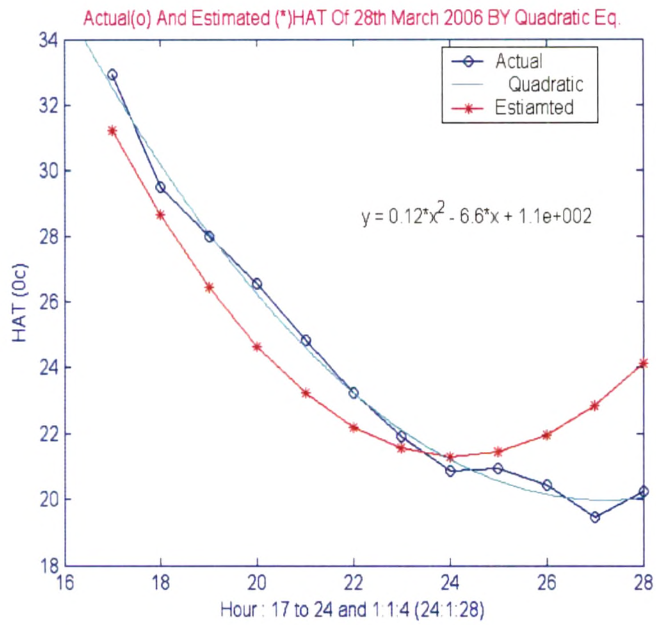


Fig: 6.10

TABLE: 6.2
RMSE & PAE DURING THE PREDICTION OF HAT BY QUADRATIC EQUATION WITH ONE VARIABLE "Time"

Sr. No	Predicting Julian Day/Day 2006	RMSE By Linear equation	PAE(%) By Linear equation
1	86ju. /28 March 5 to 16 hr	0.95	3.14
2	86ju. /28 March 17 to 24 hr and 1 to 4	1.89	7.85

Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) of second part of the time that is 17hr to 24hr and 1hr to 4hr have large value. Also, estimation of Hourly Air Temperature (HAT) for different days (1 to 365) for different season is found difficult as in this quadratic equation day variable is not included. Therefore, this curve fitting is not helpful for estimation of Hourly Air Temperature (HAT).

We have estimated hourly air temperature by applying William and Logan model with hourly extremes.

6.3.2 CASE (ii) (a) 'HAT' BY WILLIAM AND LOGAN MODEL.

Here the mathematical descriptions of day time and night time hourly air temperatures (HAT) given by William and Logan [121] model are reproduced here and used.

6.3.3 CASE (ii) (b) 'HAT' BY DOUBLE FOURIER SERIES (DFS)

Double Fourier Series (DFS) is used for estimation of hourly air temperature. Here two inputs are Maximum Hourly Air Temperature (MaxHAT) as x variable and Minimum Hourly Air Temperature (MinHAT) as y variable are used to estimate the hourly air temperature as z variable.

Their Root Mean Square Errors (RMSEs) and Percentage of Average Error (PAE) compare results by all these four methods'.

Estimated Hourly Air Temperature (HAT) (D_2) by the hourly extreme temperatures is compared with other three methods.

6.3.4 CASE (ii) (c) 'HAT' BY ARTIFICIAL NEURAL NETWORK (ANN)

Hecht [60] has used ANN with counter-propagation networks (CPN) to estimate the soil moisture. They have shown that ANN model is giving good results when trained and tested. It gives more accurate estimation of soil moisture.

Here, to avoid the difficulty of estimation of parameters like time of sunset, time of sunrise, day length, night length, lag coefficients for the time of maximum temperature etc. ANN with back propagation algorithm is used for estimation of hourly air temperature. Here, three inputs hour, Maximum Hourly Air Temperature (MaxHAT) and Minimum Hourly Air Temperature (MinHAT) are used.

6.4 DATA

The mean Hourly Air Temperature (HAT) recorded in the automatic weather station (Campbell USA) installed at the Agro meteorological Observatory of the Department of Agricultural Meteorology, B A College of Agriculture, Anand Agricultural University, Anand. (22.33^{0}N, 72.55′, msl 45m) for the year 1992 are used to derive the constants '*a*', '*b*' and '*c*' used in the mathematical equations given by William and Logan. Data series of the year March-April 2006 for the days 28th March (87 J. day), that is 13th April (102 J. day), 23rd April (113 j. day) of the year 2006. (Table 6.3) is used for comparisons with the results obtained by all the applied methods that are Mean temperatures (D_2=(MaxHAT+MinHAT/2)), William and Logan Model, ANN and DFS.

TABLE: 6.3

DS 2006 OF HAT, MaxHAT AND MinHAT :

28 March(hr)	AT °C	Max AT °C	Min AT °C	13 Apr (hr)	AT °C	Max AT °C	Min AT °C	23Apr (hr)	AT °C	Max AT °C	Min AT °C
1	21.5	23.15	21.62	1	24.19	25.37	24.56	1	26.7	27.63	26.62
2	20.22	21.65	20.52	2	23.6	24.7	23.96	2	26.35	27.25	26.67
3	19.93	20.92	20.54	3	23.04	23.99	23.22	3	25.64	26.85	25.84
4	19.44	20.55	19.52	4	22.75	23.8	23.16	4	25.34	26.52	25.93
5	18.45	19.59	18.62	5	22.48	23.38	23.03	5	24.59	25.83	25.2
6	20.23	22.71	18.58	6	23.05	26.06	23.53	6	24.83	29.53	24.88
7	22.47	23.61	22.09	7	25.7	29.68	25.54	7	27.8	32.5	27.62
8	23.29	25.21	23.23	8	28.31	30.17	28.97	8	30.29	33.76	32.07
9	24.75	26.96	24.78	9	30.77	34.27	31.54	9	32.6	36.17	32.94
10	26.84	31.72	26.15	10	32.85	36.52	33.46	10	34.29	38.13	35.25
11	29.48	32.89	30.35	11	34.92	37.87	35.77	11	35.28	39.12	36
12	31.48	36.04	32.3	12	36.14	39.41	37.59	12	36.82	40.69	38
13	32.99	36.5	33.98	13	37.24	40.33	37.68	13	37.63	40.91	38.29
14	34.04	37.29	35.54	14	38.26	41.39	39.21	14	37.53	41.07	38.89
15	34.93	37.82	35.61	15	37.81	40.72	38.98	15	37.63	40.19	39.49
16	35.4	38.66	37.17	16	37.56	39.81	38.72	16	37.24	40.01	39.2
17	35.45	38.5	36.98	17	36.3	39.53	36.79	17	35.73	39.34	35.22
18	34.56	37.64	36.01	18	33.29	36.72	33.85	18	31.03	34.13	30.79
19	32.94	35.88	33.13	19	31.03	33.52	31.54	19	27.95	31.09	28.08
20	30.98	32.79	31.84	20	29.41	31.48	29.67	20	26.39	28.46	26.63
21	29.98	31.73	30.75	21	27.69	29.59	27.82	21	24.86	27.37	25.46
22	28.55	30.29	28.94	22	26.54	27.93	27.01	22	24.25	26.77	24.23
23	25.83	27.97	26.05	23	25.85	27.1	25.99	23	24.86	25.07	24.19
24	24.25	26.55	23.81	24	23.11	25.68	22.72	24	23.25	24.52	23.22]

6.5 DETAILS OF THE METHODS AND RESULTS

6.5.1 WILLIAM AND LOGAN MODEL

The mathematical descriptions of daytime and nighttime temperatures recorded by William and Logan are used to estimate temperatures at specified hours. The relationships are reproduced here.

6.5.1.1 WILLIAM AND LOGAN MODEL FOR DAY TIME TEMPERATURE

A year is divided in 365 standard days (known as Julian days, Table A-6.1).

To find out the day time temperatures $(T_i)_d$, at a specified hour i, the sine function is used. This equation is

$$(T_i)_d = (T_X - T_N)\,\sin\left(\frac{\pi\, m}{(y+2a)}\right) + T_N\,; \tag{6.4}$$

where,

y = Day length in hour. That is total sunshine hours of a day.

T_X = Maximum Air Temperature of a specified Julian day.

T_N = Minimum Air Temperature of a specified Julian day.

m = The number of hours between the time of occurrence of minimum temperature and the time of sunset for a day.

Constant a = t_X-12.00, is the lag coefficient for the maximum air temperature for a day. That is difference between time of actual occurred maximum temperature for a day and time to be occurred maximum air temperature.

Here, constants 'a' is computed for 365 Julian days and there average is taken. For Anand station it is a= 2.69

6.5.1.2 WILLIAM AND LOGAN MODEL FOR NIGHT TIME TEMPERATURES

Consider $(T_1)_N$ be the i^{th} hourly air temperature to estimate in the night hours. It is predicted by equation (6.5) from the William and Logan model.

$$(T_i)_n = (T_N) + (T_{Se} - T_N)\ e^{\left(\frac{-bn}{z}\right)}; \tag{6.5}$$

Here $(T_1)_n$ is the air temperature to be estimated at i^{th} hour.

T_N = Minimum air temperature.

T_{Se} = Air temperature at time of sunset. It is found from equation (6.4)

n = Number of hours after time of sunset to the time of the minimum temperature.

z is a night length. $z = 24 - y$

Here, b is a constant. It is given by for a day by,

$$b = (\log_e[(T_{se} - T_N)/((T_1)_n - T_N)])\ (z/n);$$

b is computed for 365 Julian days from the observed hourly air temperatures at automatic weather station and there average is taken. This value is b= 1.423 for Anand station.

Constant $c = t_N - t_{sr}$; where, t_N is the time of occurrence of minimum temperature and t_{sr} is the time of sunrise. t_{sr} are noted from the related PANCHANG for 365 days.

Average value of c for Anand station is -1.785

❖ **CALCULATION OF z AND *y* GIVEN BY WILLIAM AND LOGAN**

Day length is a function of latitude and the day of the year. Day and night length given by William and Logan is as below,

$$y = 24\left(\frac{u}{\pi}\right);\ where\ ,u = \tan^{-1}\left(\frac{T_{m1}}{T_{m2}}\right);$$

$$T_{m1} = \left[1 - \left(-\tan\left(latitude\ \ in\ radian\ \ of\ a\ station\right)D\right)^2\right]^{\frac{1}{2}};$$

$$T_{m2} = \left[-\tan\left(latitude\ \ in\ radian\ \ of\ a\ station\right)\tan D\right];$$

$$D = (0.4014)\sin\left(2\pi\frac{(jday - 77)}{365}\right)\ and\ \ night\ \ length$$

$$z = 24 - y$$

❖ **CALCULATION OF *m* AND *n* INTERMS OF DAY LENGTH AND NIGHT LENGTH.**

Let $t_i = i$ *i.e. specified hour at which temperatur e has to be find out.*

$$t_b = 12 - \frac{y}{2} + c\ \ and\ \ t_e = 12 + \frac{y}{2}$$

Now ,if $t_b \le t_i \le t_e$ *then* $m = t_i - t_b$

Otherwise $m = y - c$

and $n = t_i - t_e$ *when* $t_i \ge t_e$

and

$n = (24 + t_e) + t_i$ *,when* $t_i \le t_b$

6.5.1.3 RSULTS AND DISCUSSION

6.5.1.3.1 CASE (i) (a) 'HAT' BY WILLIAM AND LOGAN MODEL USING DAILY EXTREME TEMPERATURE

Using equation 6.4 and 6.5, hourly air temperatures are estimated for day and night. The hourly mean temperature of 365 days recorded by the automatic weather station are used to estimate the lag coefficients *a, b* and *c*. The average values of all the respective coefficients are applied to find out the hourly air temperature, which are compared with the actual values as recorded by the automatic weather station. The comparison of hourly temperatures recorded by the automatic weather station with the predicted values for 13, 158 and 268 Julian days is graphically shown in figure 6.11. It shows that except for the period 05.00 hr to 14.00 hr the predicted values are consistent with the values of the automatic weather station. However, when hourly mean temperatures are used to obtain the daily means, these differences are negligible.

The differences between the daily mean of hourly temperatures as recorded by the automatic weather station and as predicted by the model are found statistically non-significant when tested by Student t-test and chi-square test.

The predicted hourly temperatures are used to obtain the daily mean temperature, which are compared with actual mean temperatures calculated from the recorded hourly air temperatures by the automatic weather station. This comparison is graphically shown in the figure 6.12.The differences between these two mean temperatures on daily basis are almost less than 1.5^{o}c. Thus it indicates that simulated daily mean temperatures are comparable with those obtained from the observed data and their respective differences are non-significant when tested by Student t-test. Similar result also reported by William and Logan [121].

Actual And Predicted HAT By Model

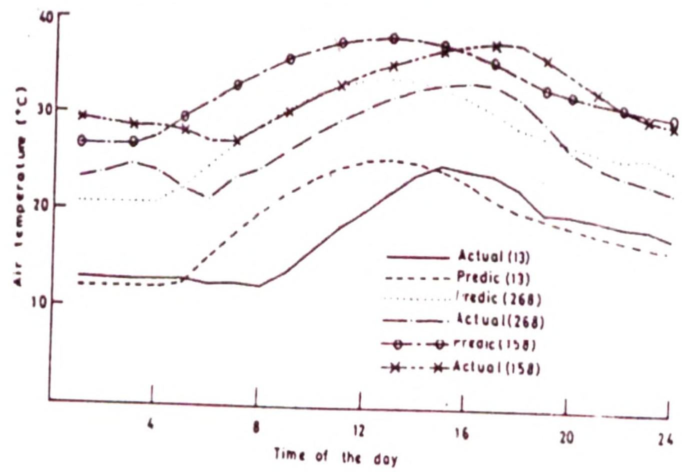


Fig: 6.11

Mean Temperatures By HAT (.) And Actual (*)

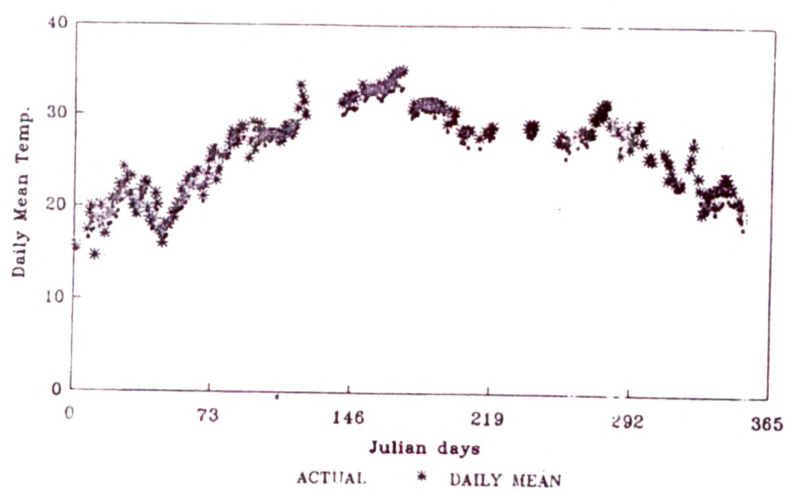


Fig: 6.12

6.5.1.3.2 CONCLUSION

CASE (i) (a)

Understanding of the temperature response of crop growth and development is important in building crop weather model. The thermal time concept is useful but leads to an error with the use of only maximum and minimum temperatures. The daily mean temperatures obtained from maximum and minimum temperatures (D_{21}) are found to be higher than those obtained from hourly records of automatic weather station (D_1). This would lead to an error in the thermal time calculation. So function model of the type suggested by William and Logan [121] could be used to obtain the hourly temperatures and thereby daily mean temperatures with more precision in working out the thermal time.

The input data required to estimated hourly temperature by the model are maximum and minimum temperatures recorded manually with the help of screen thermometer. The lag coefficients *a*, for daytime temperature, *b* for nighttime temperature, *c* for minimum temperature are also required. Temperature recorded for every minute in the automatic weather station is required for estimation of these coefficients. The values of the coefficients *a*, *b* and *c* so obtained are 2.69, 1.423 and -1.725 respectively. The result indicates that the estimated daily means values are in close agreement with the corresponding actual values recorded by automatic weather station.

6.5.1.3.3 RESULT AND DISCUSSION

CASE (ii) (a) HAT′ BY WILLIAM AND LOGAN MODEL USING HOURLY EXTREME TEMPERATURE.

The temperatures DS observed at the Agro meteorological observatory as well as the data recorded by the automatic weather station for the year 2006 are used to estimate Hourly Air Temperature (HAT) by the model.

Predicted HAT of the 86 Julian day or 27th March 2006 by the method of mean temperatures that is (MaxHAT + MinHAT)/2 , where Maximum Hourly Air Temperature (MaxHAT) and Minimum Hourly Air Temperature (MinHAT) are obtained by the liquid in glass thermometer in the observatory, is depicted in the figure 6.13 ('+'). This figure shows that these temperatures (D_2) are higher then the actual Hourly Air Temperature (HAT) (D_1, 'o') DS by the automatic weather station. That is $D_2 > D_1$.

Predicted Hourly Air Temperature (HAT) of 28th March 2006 by Parton's model is found significant to actual Hourly Air Temperature (HAT) as their Root Mean Square Error and Percentage of Average Error (PAE) are 0.9 % and 3.28 % (Table 6.3, R-II). These Hourly Air Temperature (HAT) are depicted in the figure 6.13.

Figure 6.14 shows the predicted Hourly Air Temperature (HAT) of 23rd April 2006 by William and Logan model. Their Root Mean Square Error (RMSE) (0.89) and Percentage of Average Error (PAE) (3.27) are showed in the Table 6.4 by R-II. There is a high difference between the actual and predicted Hourly Air

Temperature (HAT) during the 10 hr to 18 hrs. Model predicts the Hourly Air Temperature (HAT) over estimation in comparison to actual (Fig.6.13, Fig: 6.14) due to the facts that soil surface acts as a sink for the energy during this period. Which causes lag of time to attain the temperatures whereas estimated values are based on the mathematical relation and do not take into account such a factor.

Predicted Hourly Air Temperature (HAT) are tested by student t test for two tails and found significant.

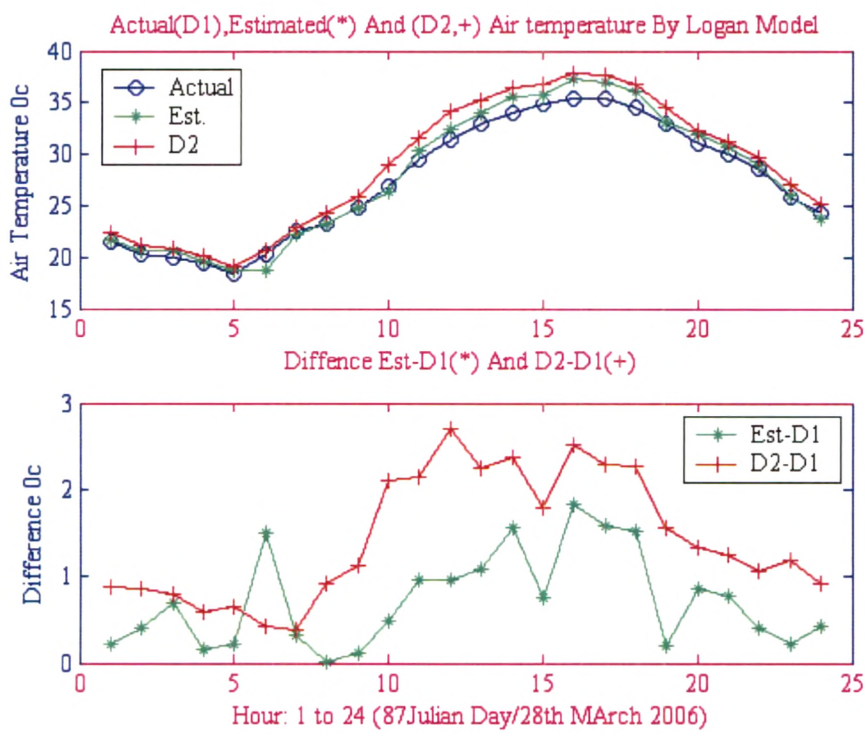


Fig: 6.13

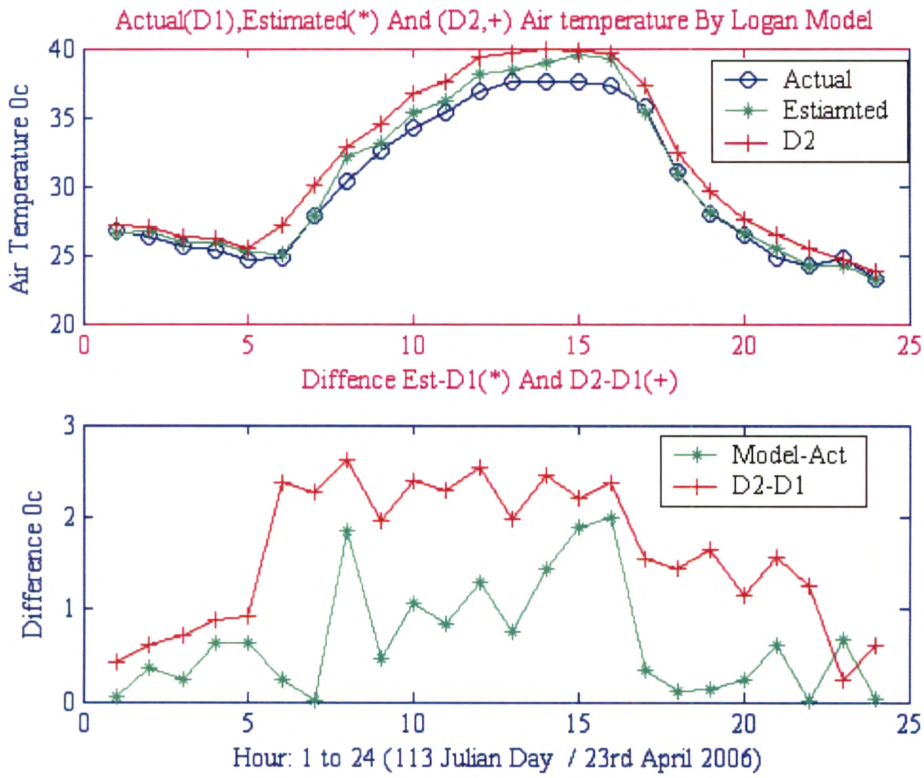


Fig: 6.14

6.5.2 CASE (ii) (C) DOUBLE FOURIER SERIES (DFS)

6.5.2.1 METHOD

Hourly Air temperature curves (Fig: 6.1) are continuous bounded and periodic. Its period is 24 hours.

To estimate hourly air temperatures by extreme temperatures, DFS can be used. In computing estimation of hourly air temperatures we use inputs are x and y and z is an out put. These variables are x= MaxHAT, y= MinHAT and z= HAT.

To estimate the hourly air temperatures of March 28, April 13 & 23 2006 first coefficients are found using DS of 27th March 2006. Selected value of m and n is 1 and therefore, DFS is

$$z = \begin{array}{l} \frac{1}{4}a_{00} + a_{11}\cos x * \cos y + b_{11}\sin x * \cos y + c_{11}\cos x * \sin y + d_{11}\sin x * \sin y + \\ \frac{1}{2}(a_{01}\cos y + c_{01}\cos y + a_{10}\cos x + b_{10}\sin x) \end{array} \quad \text{.......................(6.8)}$$

Coefficients matrix c= [a_{00} a_{11} b_{11} c_{11} d_{11} a_{01} c_{01} a_{10} b_{10}] is found by the DS of 27th March 2006. Coefficients of DFS can not be generalized for 365 days but it will be computing each time from the earlier observed value of the hourly air temperature. Equation (6.8) for k=2 number of observations can be written in matrix form by,

$$z = D * c$$

where,

$$D = \begin{bmatrix} \frac{1}{4} & \cos x_1 * \cos y_1 & \sin x_1 * \cos y_1 & \cos x_1 * \sin y_1 & \sin x_1 * \sin y_1 & \frac{1}{2}(\cos y_1 & \cos y_1 & \cos x_1 & \sin x_1) \\ \frac{1}{4} & \cos x_2 * \cos y_2 & \sin x_2 * \cos y_2 & \cos x_2 * \sin y_2 & \sin x_2 * \sin y_2 & \frac{1}{2}(\cos y_2 & \cos y_2 & \cos x_2 & \sin x_2) \end{bmatrix} \text{; and}$$

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Therefore, c= pinv(D)*z .

Using c vector, hourly air temperatures are predicted for the March 28th (Figure 6.15), April 13 and April 23 2006 (Table: 6.4, R-I). Estimated results are significant to actual hourly air temperatures by t -test.

6.5.2.2 RESULT AND DISCUSSION

All the hourly estimated temperatures by DFS for three days 28 March 2006, 13^{th} and 23^{rd} April 2006 are found significant by student t test. Estimated hourly air temperatures of 28^{th} March 2006 and difference with actual and $D_2(+) - D_1(o)$ are plotted in the figure 6.15 as a subplot. Here D_2 is hourly air temperatures by Mean of extreme temperature and D_1 is the actual observed hourly air temperatures. Figure shows the $D_2 - D_1$ have large differences (more than 2^0c) in comparison to difference by DFS(less than 1^0c). Also, both the methods have comparatively large difference in the peak hours of sunshine hours. Figure 6.15 shows the non-significant difference between actual and estimated hourly air temperatures of 28^{th} March 2006 by DFS. Hourly air temperatures of 13^{th} April 2006 are estimated and plotted in the figure 6.16 with the differences with actual hourly air temperatures. Subplot '+' shows the D_2-D_1 difference which is higher than the estimated D1. Here D_2 is the hourly air temperatures by mean of MaxHAT and MinHAT and D_1 is the actual hourly air temperatures.

Thus figure 6.15 and 6.16 show that D2 is over estimating hourly air temperatures, while hourly air temperatures by DFS are very near to actual hourly air temperatures.

Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) by DFS is obtained for the 28^{th} March, 13^{th} and 23^{rd} April 2006 (Table 6.4). March 28^{th} has largest PAE that is 2.82%.

Thus hourly air temperatures by DFS method are significant.

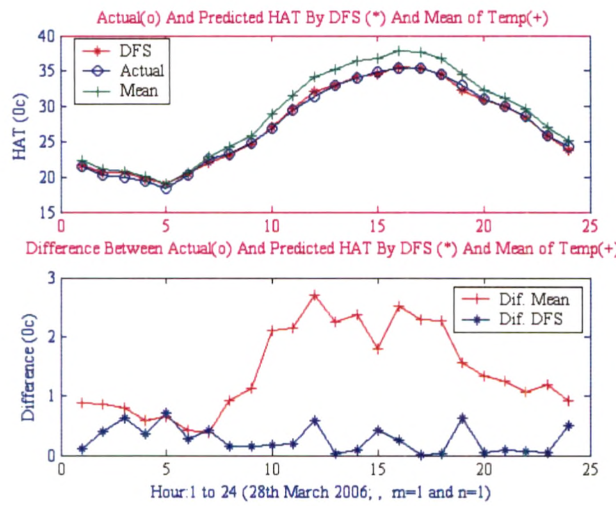


Fig: 6.15

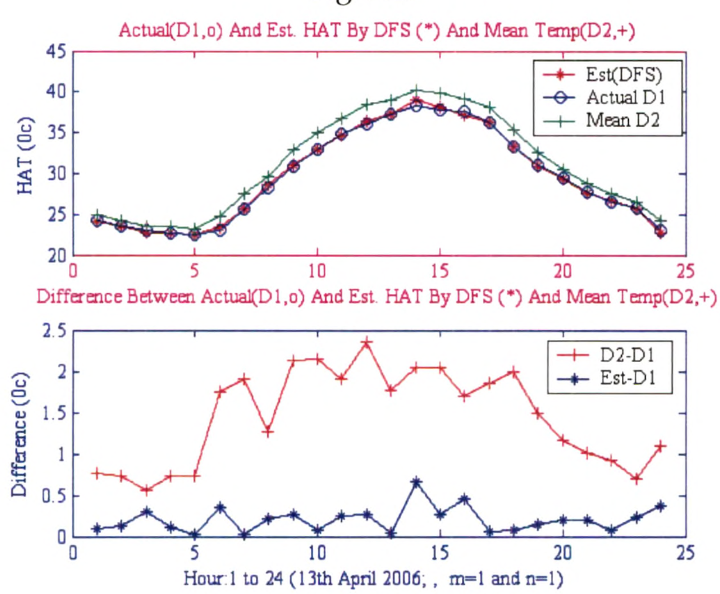


Fig: 6.16

6.5.3 CASE (ii) (c) ARTIFICIAL NEURAL NETWORK (ANN)

6.5.3.1 METHOD

Here, ANN is applied for estimation of hourly air temperatures with three inputs namely, hour 1 to 24 , maximum Hourly Air Temperature (MaxHAT) and Minimum Hourly Air Temperature (MinHAT) and hourly air temperatures (HAT) as the output. ANN is trained by historical data series (DS) of inputs and output. Hourly air temperatures are estimated for the March 28th , April12th and 23rd by training the NN for the 2006 of the days March 27th , April 11th and 22nd respectively.

6.5.3.2 TRAINING OF THE NETWORKS

Feed forward network is used for estimation of hourly air temperatures. This network is trained by supervised learning network and back propagation algorithm is used.

Used parameters during the training of the network are given in the Table 6.4. Error ratio is selected 1.04 with momentum 0.5.

Table: 6.4
DETAILS OF THE PARAMETER VALUES USED IN ANN TRAINING.

Sr. No.	Predicting Julian Day/Day 2006	Number of epochs used	Learn ing rate	Momen tum	No. of neurons	Error goal
1	87ju./28 March	59922	0 001	0 5	18	0.0001
2	102/12 April	237540	0 001	0.5	15	0.0001
3	113/23 April	76275	0 001	0.3	16	0.0003

6.5.3.3 STRUCTURE OF THE NEURAL NETWORK

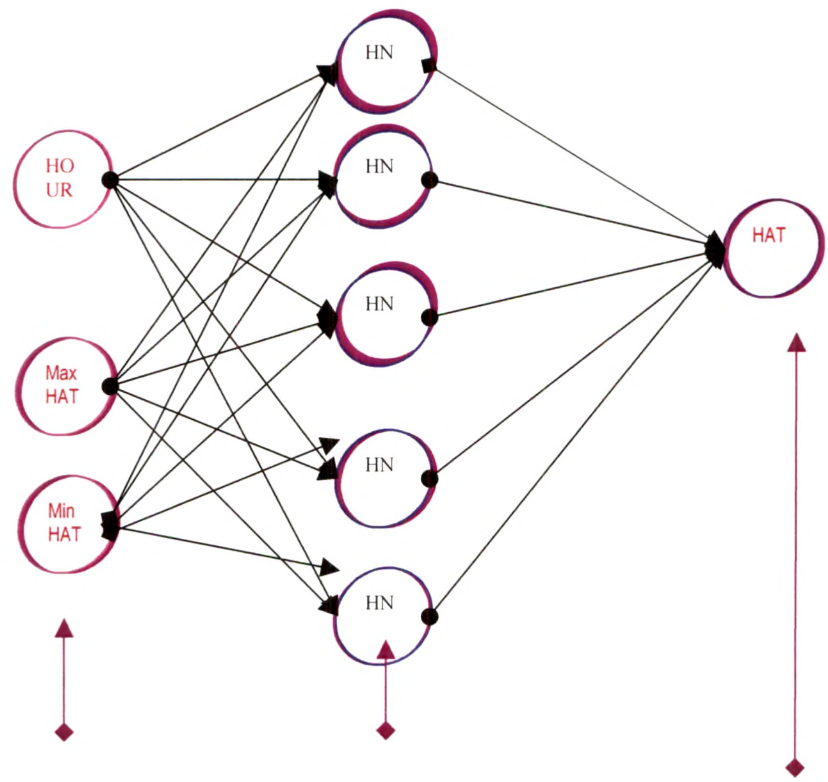


Fig: 6.17

Figure 6.17 shows the structure of the NN. Here network is considered to be multi-layered. These layers are input layer, hidden layer and output layer. Input layer has two neurons and output layer has one neuron. Number of hidden neurons (NH) can be adjusted to requirement of the analysis. The NH is more than the requirement, decreasing the accuracy in the final output. Therefore, to get significant results suitable NH is requiring.

6.5.3.4 RESULTS AD DISCUSSION

Estimation of hourly air temperatures by ANN is depicted in the figure 6.18 and 6.19. Hourly air temperatures are estimated for the days of March 28th, April 12th and 23rd. Figure 6.18 and 6.19 shows estimated hourly air temperatures for two days that is for March 28th and April 12th. Estimated hourly air temperatures by ANN have non-significant difference with actual hourly air temperatures (D_1). Hourly air temperatures (D_2) are higher than the estimated hourly air temperatures by ANN.

Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) computed during the analysis. These are given in the Table 6.5.

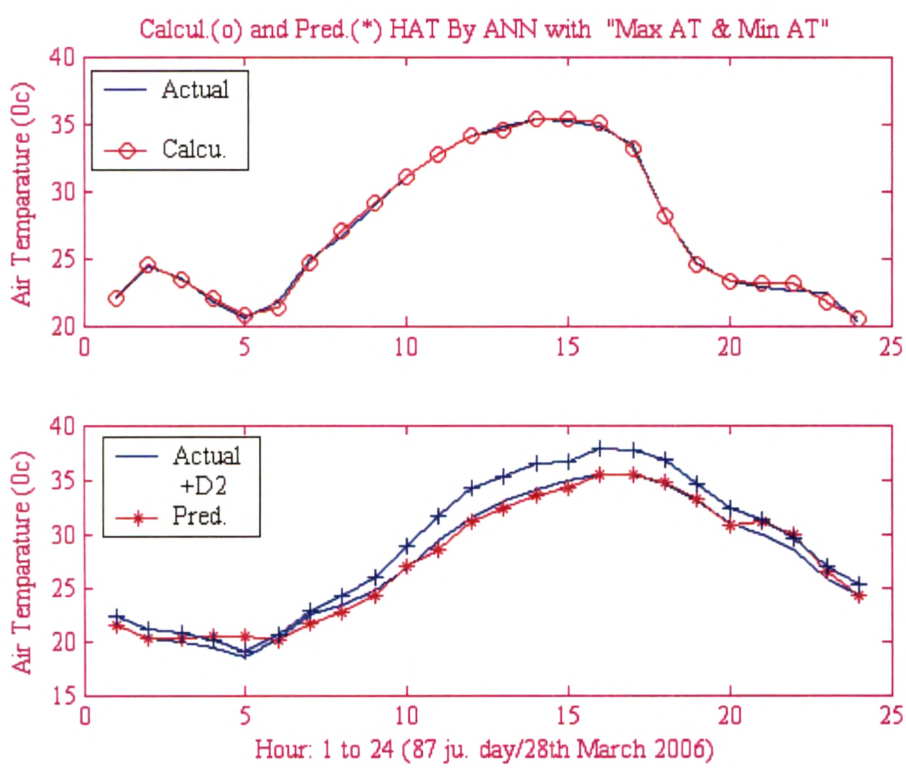


Fig: 6.18

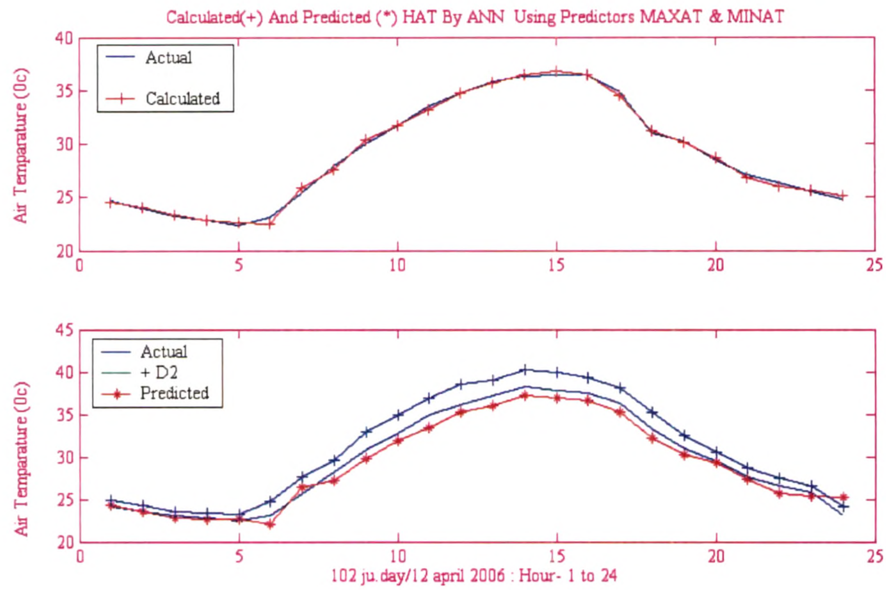


Fig: 6.19

6.6 COMPARISIONS OF THE USED FOUR METHODS

For estimation of hourly air temperatures of Anand station four methods are used. Their obtained Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) are shown in the table 6.5. Here columns are ranked for I to IV as per their values of Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) in ascending order. RMSE (R-IV) and PAE (R-IV) obtained during the use of the method Mean of the temperatures (D_2) are the highest. Percentage of Average Error (PAE) varies from 5.2 to 5.89%. While, RMSE (R-I) and PAE (R-I) by DFS method is lowest among all the methods. Percentage of Average Error (PAE) varies from 0.86 to 2.82%. RMSE (R-II) and PAE (R-II) found during the application of the method ANN have rank II.

TABLE: 6.5
DETAIL OF THE HAT OBTAINED BY FOUR METHODS

Sr. No	Predicting Julian Day/Day 2006	RMSE By DFS R-I	PAE(%) By DFS R-I	RMSE By ANNs R-II	PAE(%) By ANNs R-II	RMSE By Model R-III	PAE(%) By Model R-III	RMSE By (MaxHAT +Min AT)/2 R-IV	PAE(%) By (MaxHAT +MinHAT)/2 R-IV
1	87ju./28 March	0.77	2.82	0.72	2.65	0.89	3.27	1.59	5.83
2	102/12April	0.25	0.86	0.95	3.2	0.71	2.4	1.50	5.20
3	113/23April	0.57	1.91	1.02	3.43	0.89	3.2	1.76	5.89

Thus D_2 are overestimation of hourly air temperatures from actual hourly air temperatures with highest Percentage of Average Error (PAE). Root Mean Square Error (RMSE) and Percentage of Average Errors (PAE) by William and Logan model are higher than by DFS and ANN.

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