Chapter 3

Pair Production at Finite Temperature

3.1 Introduction

Following the discussion in the last chapter on pair production from vacuum in the presence of an oscillating external chromo-electric field , in this chapter we will discuss the effect of heat bath on such a process.

Before going into the details of the calculation we will elaborate, on the physical situation relevant for this computation. In particular we will try to show that ,whether the time scale of reduction of the external field due to pair creation process is long enough for the system to come to thermal equilibrium.

It is worth emphasising here that, though in the earlier chapter , the vacuum chromo-electric field in the flux tube was taken to be oscillating in time, (since it followed from the solutions of the vacuum Yang-Mills (YM) equations), one need not assume the same, in the presence of a heat bath. To determine the nature of the chromo-electric field in the presence of the plasma, one has to solve the YM equations with a plasma source term. Although for a realistic study, one should compute the spontaneous pair production rate in the presence of such a field, as an approximation to the more realistic case we will restrict ourselves to a constant external chromo-electric field, as a proper investigation of this process has not been performed before.

Estimation Of The Characteristic Time Scales

In this section we first estimate the time scale for the production of pairs. If we recall, the expression for the spontaneous pair production rate from vacuum / unit time /unit volume is given by

$$W_{\dot{p}} = \frac{\alpha_s E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{\frac{-n\pi m^2}{gE}}$$

From this expression, one can crudely estimate the time scale of production of $q\bar{q}$ pairs in an unit volume, and it comes out as

$$t_p \sim \frac{\pi^2}{\alpha_s E^2} e^{\frac{\pi m^2}{gE}} \tag{3.1}$$

In the above expression we have assumed that just one pair is being produced , so we have neglected the sum over n in equation (3.1). After estimating the time scale of production of pairs, we will estimate next, the time scale of depletion of the external field. It is worth mentioning here that this time scale has been estimated by Gyulassy et.al[1] and Gatoff et.al[2] before. Gyulassy had estimated it assuming abelian dominance approximation for pair production rate and Gatoff et.al had estimated it using hydrodynamic equations. We will however estimate the same, from the principle of energy conservation , essentially following the argument of reference (3), assuming Schwinger picture for pair production to hold good.

Since in this model pairs are produced with zero longitudinal momentum but all possible values of the transverse momentum, $p_\perp$, the amount energy loss with the production of a pair, where each one of the produced particles is having average energy $\langle\sqrt{m^2 + p_\perp{}^2}\rangle$, is $2\langle\sqrt{m^2 + p_\perp{}^2}\rangle$, so after producing n such pairs, the total energy lost by the external field is $2n\langle\sqrt{m^2 + p_\perp{}^2}\rangle$. Since the production probability for a pair is given by $\frac{\alpha_s E^2}{\pi^2} e^{\frac{-\pi m^2}{gE}}$, the associated energy loss by the field can be written as

$$\frac{d\varepsilon(t)}{dt} = -2\langle\sqrt{m^2 + p_\perp{}^2}\rangle \frac{\alpha_s E^2}{\pi^2} e^{\frac{-\pi m^2}{gE}} \tag{3.2}$$

where $\varepsilon(t) = \frac{E^2}{2}$ is the field energy. From the solution of this differential equation, one arrives at the time required for the electric field to decay to $\frac{1}{e}$ th of its original value as

$$t_d = C\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.3}$$

Here $C = \frac{\pi^2}{2\alpha_s\langle\sqrt{p_\perp^2 + m^2}\rangle}$, $y_{max} = \frac{\pi m^2}{gE_{max}}$, $gE_{max} = gE(t=0) = gE$, and $y_{min} = \frac{\pi m^2}{g(E/e)}$ and E_1 represents the exponential integral. So one needs to know the average value of $\sqrt{m^2 + p_\perp{}^2}$, for the proper estimation of the depletion time.The distribution of particles in the momentum interval $p_\perp$ to $p_\perp + dp_\perp$ can be computed from the solution of the Dirac equation in presence of the external field and it is (Ref Kerman[4], Nussinov[5]);

$$\frac{dN}{dp_\perp{}^2} \propto ln\left[1 - e^{\frac{-(m^2+p_\perp{}^2)}{gE}}\right]$$

From this equation one can compute the average energy of each produced particle to be

$$\langle\sqrt{m^2 + p_\perp{}^2}\rangle \simeq k\frac{\sqrt{gE}}{2}$$

where

$$k = O(1)$$

Hence

$$t_d = \frac{\pi^2}{k\alpha_s\sqrt{gE}}\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.4}$$

Finally from these relations one arrives at the ratio of depletion time to production time as

$$\frac{t_d}{t_p} = \left(\frac{1}{k(4\pi\alpha_s)}\right)(gE)^{\frac{3}{2}}e^{\frac{-\pi m^2}{gE}}\left[E_1\left(y_{max}\right) - E_1\left(y_{min}\right)\right] \tag{3.5}$$

With m = 0.2 GeV, $\alpha_s = 0.3$ and gE = 1 GeV^2 we get $t_d \simeq 5fm/c$ and $\frac{t_d}{t_p} \simeq 30$

One can see from these relationships that, as the ratio of field strength to mass square increases the time scale of reduction of the electric field also increases. Since the strength of the electric field is proportional to the mass number($A_P{}^{\frac{1}{6}}A_T{}^{\frac{1}{6}}$) of the colliding nuclei, for heavier nuclei, one can expect the external field to last for a time longer than the production time of the pairs.

Since these produced pairs come almost with a Boltzmann like distribution in momentum space[4] , (both in the case of constant as well as the time varying external field), they will come close to thermal equilibrium very fast through collision with each other. Moreover, other than the collisional processes, the joule heating of the plasma generated because of the conduction current produced by the external chromo-electric field will also contribute towards the thermalisation of the system. A quantitative estimate of momentum equlibration time, in a parton cascade model, has been obtained by Biro et. al.[15] who get a value of 0.31 fm/c. This value is essentially the same as the thermalization time $\sim$ 0.3 fm/c for gluons at RHIC energies estimated by Shuryak[16]. For quarks the thermalization time $\sim$ 1-2 fm/c. In brief, since the depletion time t_d is greater than the production time t_p, and the thermalisation time is smaller than the depletion time, one might be justified in assuming the existence of the external field in the thermally equilibrated plasma.

Though it is not very clear whether, initially the temperature of the system will be the same everywhere, but if the time scale of thermalisation is faster than the speed of separation of the two color charged, Lorentz contracted receding nuclei , one can expect the temperature to remain constant in the space, between them. Thus, in our view, it is pertinent to study the process of pair production at finite temperature, in RHIC.

As we go along we will see that because of the presence of heat bath, the rate of spontaneous creation of $q\bar{q}$ pairs in the presence of external field, is no more homogeneous in space ;rather it decreases towards the center.As a result of this differential rate of pair production, after all the field energy is exhausted in producing pairs, there will be an anisotropy in the temperature (global) distribution of the produced plasma.In our view, the following hydrodynamic evolution of the plasma will bear a signature of this anisotropic temperature distribution.

Having motivated the physical situation, we discuss the organisation of the chapter.In section 2 we will review the basics of finite temperature field theory .In section 3 we will be computing the finite temperature pair production rate in presence of external electric field following which we will conclude the chapter by discussing possible extension of our work to improve of our result.

3.2 Introduction To Thermal Effective Action:

In this section we will be introducing thermal field theory and the concept of thermal effective action.It is a well known fact that there are two different ways of introducing temperature in Quantum field theory.One of them being the imaginary time formalism[6] of Matsubara and the other is the real time[7] finite temperature field theory or thermo field dynamics.The real time formalism has distinct advantages over the former, in terms of computation of dynamical quantities.However as far the thermodynamic quantities are concerned the two formalisms give identical results.

The objective of the present work is to find out the rate of $q\bar{q}$ production at finite temperature in the presence of a static external chromo-electric field.One can compute this quantity either by evaluating the (reference(8)) thermal S matrix in presence of the external field or computing the imaginary part of the effective potential which is essentially the free energy[9] density of the system.In our work, we calculate the free energy density or the effective Lagrangian of the system.

3.3 Effective Action

Now let us recall that the expression for the partition function Z is given as

$$Z = Tre^{-\beta H} \equiv \sum_a \langle \phi_a \mid e^{-\beta H} \mid \phi_a \rangle \tag{3.6}$$

The first task in finite temperature studies of field theory is to write down the partition function in field theory as a functional integral involving Lagrangian density expressed in terms of the dynamical fields present in the theory. More precisely, given a theory, defined in Minkowski space, how does one compute the partition function Z, in relativistic quantum field theory. In order to illustrate the basic ideas , for the moment, we consider the case of a scalar quantum field theory with field operators (in Heisenberg picture), $\phi(t,x)$ with momenta $\pi(t,x)$ the Lagrangian density L and Hamiltonian density H.

If $\phi(\vec{x},0)$ is the Schrodinger-picture field operator having eigen states $\mid \phi_a\rangle$ and$\mid \phi_b\rangle$, with eigenvalues $\phi_a(x)$ and $\phi_b(x)$ then the transition amplitude for the system to go from the state $\phi_a(x)$ at t=0, to the state $\phi_b(x)$ at $t = t_f$ is

$$\langle \phi_b \mid e^{-iHt} \mid \phi_a \rangle = N' \int_{\phi_a}^{\phi_b} [d\phi] exp\left(-\int_0^{t_f} d\tau \int d^3x L\left(\phi, \dot{\phi}\right)\right) \tag{3.7}$$

Here N' as a normalisation constant, and the functional integral is defined over classical fields $\phi(t,x)$.

As it has been shown in number of places (see reference(6) and the references therein) one can use functional integral form of equation (3.7) to obtain a functional integral form for Z by a series of steps. (i) Choose the initial state and the final state to be the same,(ii) change the time coordinate t over to a variable defined as $\tau = it$ with the limits of integration varying between 0 to β and (iii) lastly as a consequence of the trace operation, perform the functional integration over the fields $\phi(\tau,x)$ with a periodic boundary conditions in τ i.e $\phi(0,x) = \phi(\beta,x)$.

For a system with no conserved charge one can equivalently show that,

$$Z = N' \int_{periodic} [d\phi] exp\left(\int_0^\beta d\tau \int d^3x L\left(\phi, \dot{\phi}\right) \right) \tag{3.8}$$

For a Lagrangian that is quadratic in the field variables, one can compute the partition function Z exactly by expanding the field variables $\phi(t, x)$ as

$$\phi(\beta, x) = \frac{1}{\beta} \sum_n \int \frac{d^3p}{(2\pi)^3} e^{-i(\omega_n \tau - p.x)} \phi(\tilde{\omega_n}, p) \tag{3.9}$$

and performing the Gaussian integration over the field variables. Here $\omega_n = \frac{2\pi n}{\beta}(n = -\infty \text{ to } \infty)$ are the (Matsubara) frequencies for bosons and have been defined to agree with the periodic boundary conditions of the field variables. The finite temperature Green functions defined as

$$G_\beta(\bar{x}_1, \bar{x}_2 .... \bar{x}_j) = \frac{Tre^{-\beta H}(T\phi(\bar{x}_1)....\phi(\bar{x}_j))}{Tre^{-\beta H}} \tag{3.10}$$

can be shown to be coming from

$$G_\beta(\bar{x}_1, \bar{x}_2 .... \bar{x}_j) = \frac{\delta^j Z(J)}{\delta J(\bar{x}_j).....\delta J(\bar{x}_1)} |_{J=0} \tag{3.11}$$

where

$$Z = \frac{\int_{periodic} [D\phi] exp\left(\int_0^\beta d\tau \int d^3x L(\phi, \partial_\mu \phi) + J\phi \right)}{\int_{periodic} [D\phi] exp\left(\int_0^\beta d\tau \int d^3x L(\phi, \partial_\mu \phi) \right)} \tag{3.12}$$

The generating functional for the connected Green function is defined through

$$W^\beta(J) = lnZ^\beta(J) \tag{3.13}$$

An effective action $\overline{\Gamma}(\phi_c)$ is defined in terms of the Legendre transform[10] of $W^\beta(J)$ as

$$\overline{\Gamma}(\phi_c) = W^\beta[J] - \int d\bar{x} \phi_c(\bar{x}) J(x) \tag{3.14}$$

where $\phi_c(\bar{x})$ is the classical field defined as

$$\phi_c(\bar{x}) = \frac{\delta W^\beta[J]}{\delta J[\bar{x}]} \tag{3.15}$$

and the source *J(x)* is given by

$$J(\bar{x}) = -\frac{\delta\Gamma[\phi_c]}{\delta\phi_c[\bar{x}]} \tag{3.16}$$

Here the vector $\bar{x} = (-i\tau, \vec{x})$.

The quantity $\overline{\Gamma}(\phi_c)$, evaluated semiclassically about some field configuration $\phi_c(\bar{x})$, gives the free energy of the system in that configuration. Usually $V_{eff}(\phi)$, the effective potential, the first term in a derivative expansion of $\overline{\Gamma}(\phi)$, is just the free energy density in a background constant field configuration. The quantity effective lagrangian L_{eff}, is defined[11] to be, L_{eff} = - V_{eff} This quantity is used for determining not only the thermal ground state energy of the system but also for determining the phase transition, symmetry breaking etc. In the case of a first order phase transition, a system can be trapped temporarily in a meta stable state leading to non-equilibrium phenomena. The rate of decay for such a system is determined from the imaginary part of its free energy (reference (9): Affleck, Langer). Though we have outlined the formalism for scalar bosons, it has been generalised for the case of fermions and gauge bosons too. For fermions one has to take the anti-periodic boundary condition because of anti commutation relation satisfied by the fermions. For vector bosons, other than periodic boundary conditions one also has to take care of the extra degrees of freedom carried by the gauge bosons. We are not going to elaborate on this point any further here. All the details can be found in reference (12).

3.4 Computation of Effective Lagrangian From The Fermionic Determinant

In this section we compute the effective action for a system of fermions with SU(2) color symmetry, in an external chromo-electric field.In our calculation we assume the plasma to consist of equal number of quarks and antiquarks, so the net baryon number as well as the chemical potential are zero. Further we do not include the dynamics of the gluon fields, though in a more realistic case one ought to do so.

It is also worth mentioning, that this problem has been studied earlier, in the context of quantum electrodynamics U(1) symmetry, by Loewe and Rojas[8] using real time thermal field theory and also by Cox, Helmann and Yildiz[13]. Unfortunately there is no agreement between the results obtained by them. Cox et al find no effect of temperature on pair production rate, whereas the authors of reference(8) do find a finite temperature contribution to pair production rate. In fact their result also shows that it increases dramatically with temperature. Our calculation, when performed with an U(1) symmetry, agrees qualitatively with the findings of reference(8) but it disagrees in other aspects.

For instance, in contrast to the result of Loewe and Rojas, we find that the finite temperature pair production rate has two distinct pieces in it, one being the vacuum contribution and the other the finite temperature contribution having a sign difference. The finite temperature contribution,unlike the vacuum contribution is a space dependent quantity implying that the pair production rate as well as all the thermodynamic quantities vary in space, in particular along the longitudinal direction. In fact, this striking result is due to shielding of the electric field by the polarised plasma in between. Consequently, as one moves away from the source, the field strength decreases, giving rise to a differential rate in pair production. Since the rate of pair production varies in space, the number density of produced particles will also vary in space leading to a similar behaviour of the thermodynamic quantities like pressure, entropy, temperature etc. Since we are interested in investigating the pair production rate, we will not discuss the thermodynamic quantities here.

3.4.1 Computation of Effective Lagrangian

We start from the "partition function" in Minkowski space defined by

$$Z[A] = \frac{\int D\bar{\psi} D\psi e^{i\int L d^4x}}{\int D\bar{\psi} D\psi e^{i\int L_o d^4x}} \tag{3.17}$$

where $L = \bar{\psi}\left(i\gamma_\mu \partial^\mu - g\gamma_\mu A_a{}^\mu \tau_a\right)\psi - m\bar{\psi}\psi$ is the fermionic Lagrangian in the presence of external vector field $A_\mu{}^a$, τ_a's are the Pauli matrices and $L_o = \bar{\psi}\left(i\gamma_\mu \partial^\mu - m\right)\psi$ is the free fermionic Lagrangian, such that $Z[0] = 1$.
Since we are interested in evaluating the effective action, in the presence of external chromo-electric field only, we choose $A_0{}^a = -E^a z$ and other components of A to be equal to zero. Following standard prescriptions (see reference(6) and (10)), we obtain the finite temperature partition function in terms of the Euclidean action S_β defined as,

$$S_\beta = \frac{1}{\beta^2} \sum_{n=-\infty}^{\infty} \int \bar{\psi}_n(x) \left[\left(\omega_n \gamma^0 + g A^a{}_o \tau^a \gamma^0\right) + i\gamma^j \partial_j - m\right] \psi_n(x) d^3x \tag{3.18}$$

and compactify the time direction by putting antiperiodic boundary condition to get

$$Z[A] = \frac{\prod_{n=-\infty}^{\infty} \int D\bar{\psi}_n D\psi_n e^{-S_\beta}}{\prod_{n=-\infty}^{\infty} \int D\bar{\psi}_n D\psi_n e^{-S_{o\beta}}} \tag{3.19}$$

Here ψ 's are the fermion fields in the fundamental representation of SU(2) defined as $\psi(x) = \frac{1}{\beta}\sum_n \int \frac{d^3x}{2\pi^3} e^{-i(\omega_n\tau - px)}\tilde{\psi}_n(\omega_n, p)$ and

$$S_{o\beta} = \frac{1}{\beta^2}\sum_{n=-\infty}^{\infty} \int \bar{\psi}_n(x)\left[\omega_n\gamma^0 + i\gamma^j\partial_j - m\right]\psi_n(x)d^3x$$

It should be noted that in Eq(3.19) γ^0 and $A_0{}^a$ are quantities in Euclidean space. On integrating over the fermion fields one arrives at

$$Z[A] = \prod_{n=-\infty}^{\infty} Det\left[\frac{(\gamma^0\omega_n + g\gamma^0 A_o^a\tau^a) + i\gamma^j\partial_j - m}{(\omega_n\gamma^0 + i\gamma^j\partial_j - m)}\right] \tag{3.20}$$

This determinant is defined over color, spinor as well as the coordinate space.

Using well known techniques[14] one can further write it as

$$Z[A] = \prod_{n=-\infty}^{\infty} Det\left[\frac{(\omega_n + gA_o^a\tau^a)^2 - g\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0\end{pmatrix}E^a\tau_a - \partial_k^2 + m^2}{\omega_n^2 - \partial_k^2 + m^2}\right]^{1/2} \tag{3.21}$$

The determinant in Eq.(3.21) can further be diagonalised in color space using an unitary matrix of the form.

$$U^+ = \begin{pmatrix} (E_3 + E)/N_1 & (E_1 - iE_2)/N_1 \\ (E_3 + E)/N_2 & (E_1 - iE_2)/N_2 \end{pmatrix} \tag{3.22}$$

with $E = \sqrt{E_1^2 + E_2^2 + E_3^2}$; $N_1^2 = 2E^2 + 2E_3E$, $N_2^2 = 2E^2 - 2E_3E$.

After diagonalising Eq(3.21) in color space and using the identity $Det\hat{O} = e^{tr ln\hat{O}}$ and the integral representation $ln\hat{O} = \int_o^\infty \frac{ds}{s} e^{-s\hat{O}}$ one arrives at

$$\begin{aligned} -S_{eff} &= lnZ = -4tr\left[\sum_{n=-\infty}^{\infty}\int_0^\infty \frac{ds}{s}\left[cosh(gEs)e^{-s\left[(\omega_n + \bar{A}_o)^2 + p_j^2 + m^2\right]}\right.\right. \\ &- \left.\left.e^{-s\left[\omega_n^2 + p_j^2 + m^2\right]}\right]\right] \end{aligned} \tag{3.23}$$

with $\bar{A}_o = -Ez$. The trace is now defined only over coordinate space. We note that since A_μ is an external field, no Legendre transformation is required to go from the

connected vacuum functional to the effective action .
After doing some lengthy algebra one arrives at the expression for free energy density

$$
\begin{aligned}
-\mathcal{F} &= -\frac{1}{4\pi^2}\left[\Sigma_{n=-\infty}^{\infty}(-1)^n\left[e^{in\beta gEz}\int_o^\infty \frac{ds}{s^2}(gE)coth(gEs)e^{-sm^2-n^2\frac{\beta^2 gE}{4}coth(gEs)}\right.\right.\\
&- \left.\left.\int_0^\infty \frac{ds}{s^3}e^{-sm^2-\frac{n^2\beta^2}{4s}}\right]\right]
\end{aligned}
\tag{3.24}
$$

Expanding coth z in the asymptotic form $1+1/z$, we get after separating the $n=0$ term from the other $n \neq 0$ terms

$$
\begin{aligned}
\mathcal{F} &= \frac{1}{4\pi^2}\int_o^\infty \frac{ds}{s^3}(gEs)coth(gEs)e^{-sm^2}+\\
\frac{1}{2\pi^2} & \Sigma_{n=1}^{\infty}(-1)^n\left[\cos\left(ng\beta\bar{A}_o\right)\int_o^\infty \frac{ds}{s^3}(gEs)coth(gEs)e^{-sm^2-\frac{n^2\beta^2}{4s}}e^{-n^2\beta^2 gE/4}\right.\\
&- \left.\int_o^\infty \frac{ds}{s^3}e^{-sm^2-\frac{n^2\beta^2}{4s}}\right]
\end{aligned}
\tag{3.25}
$$

Since E in Eq.(3.25) is the Euclidean electric field we need to rotate the electric field back to Minkowski space i.e. $E \to -iE$ to obtain the expression for the thermal effective action

$$
\begin{aligned}
\mathcal{F} &= \left[\frac{1}{4\pi^2}\int_o^\infty \frac{ds}{s^3}e^{-sm^2}\left[(gEs)cot(gEs)-1\right]+\frac{1}{2\pi^2}\Sigma_{n=1}^{\infty}(-1)^n\int_o^\infty \frac{ds}{s^3}\right.\\
&\left.\left[\cosh\left(ng\beta\bar{A}_o\right)(gEs)coth(gEs)e^{-in^2\beta^2 gE/4}-1\right]e^{-sm^2-\frac{n^2\beta^2}{4s}}\right]
\end{aligned}
\tag{3.26}
$$

This is the main result of our work. We can clearly see that $n=0$ provides the vacuum contribution to the effective action and $n \neq 0$ provides the finite temperature correction to it.

The spontaneous pair production rate is given by the imaginary part of the effective action given above.

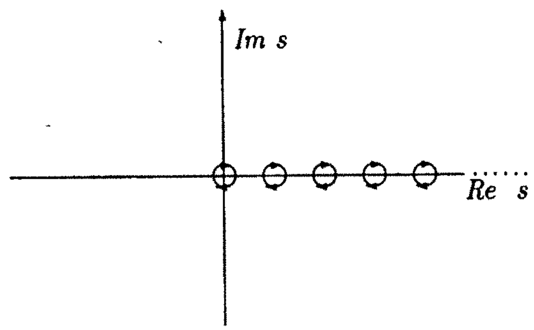

Fig-I

Contour Of Integration.

On carrying out the integration in Eq.(3.26), by choosing a contour as shown in figure-I with poles at $s = (l\pi/gE)$;we get for the imaginary part of the L_{eff}

$$Im\,[L_{eff}] = \frac{1}{4\pi^3}\sum_{n=1}^{\infty}\frac{(gE)^2}{n^2}e^{-\frac{m^2\pi n}{gE}} - \frac{1}{2\pi^2}\sum_{n=1}^{\infty}(-1)^n cosh(n\beta g A_o)\left[P.V.\int_0^{\infty}\frac{ds}{s^2}\, gE cot\, gEs \right.$$
$$\left. sin\frac{n^2\beta^2 gE}{4}e^{-sm^2-\frac{n^2\beta^2}{4s}} - \pi\sum_{l=1}^{\infty}\frac{(gE)^2}{(l\pi)^2}e^{\frac{-m^2\pi l}{gE}-\frac{n^2\beta^2 gE}{4\pi l}}\, cos\frac{n^2\beta^2 gE}{4}\right] \tag{3.27}$$

Here P.V means principal value. Although we have not evaluated the real part of the effective lagrangian, it will provide one with expressions for the thermodynamic quantities like pressure, entropy etc.

Analysis of Our Result

One can see from Eq. (3.27) that there is a sign difference between the zero temperature and the finite temperature part of the effective lagrangian. Depending on the temperature and field strength one or the other term will dominate. We have tried to evaluate the expression numerically for $z = 0$

$$
\begin{aligned}
Im\left[L_{eff}\right] = & \frac{1}{4\pi^3}\sum_{n=1}^{\infty}\frac{(gE)^2}{n^2}e^{-\frac{m^2\pi n}{gE}}\left[1+\frac{\pi}{2}\sum_{l=1}^{\infty}e^{-\frac{n^2\beta^2 gE}{4\pi l}}\ cos\frac{n^2\beta^2 gE}{4}\right] \\
& -\frac{1}{2\pi^2}\sum_{n=1}^{\infty}(-1)^n P.V.\int_0^{\infty}\frac{ds}{s^2}\, gE\ cot\ gEs\ sin\frac{n^2\beta^2 gE}{4}\ e^{-sm^2-\frac{n^2\beta^2}{4s}}
\end{aligned}
\tag{3.28}
$$

For $z = 0$ (See eq.(3.28)) we find a dramatic increment in the pair production rate at high temperature over that of vacuum, though at some intermediate temperatures the rate decreases.

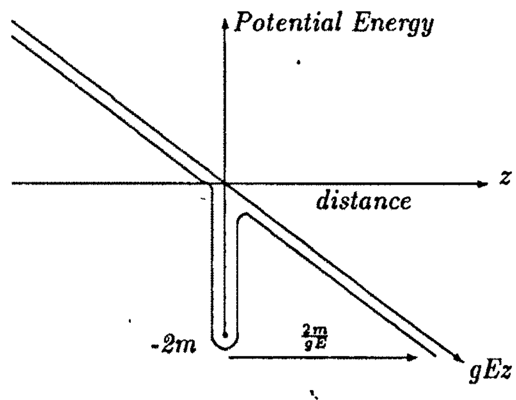

Fig-II

Potential well for quarks submitted to an external chromo-electric field gE .

It is possible to understand these phenomena, in terms of a simple potential well model, where the pair creation is viewed as tunnelling of pairs from vacuum through an energy barrier in the configuration space with maximum height $2m$ and width is $\frac{2m}{gE}$.In the presence of finite temperature the same picture still holds good. Due to thermal effects, the particles are lifted up from the bottom of the well, and as a consequence the effective barrier width, as seen by them becomes less, hence making it easier for them to tunnel out of the vacuum. This might be an explanation of the temperature corrected Schwinger expression i.e. the first term in Eq.(3.28).Moreover,

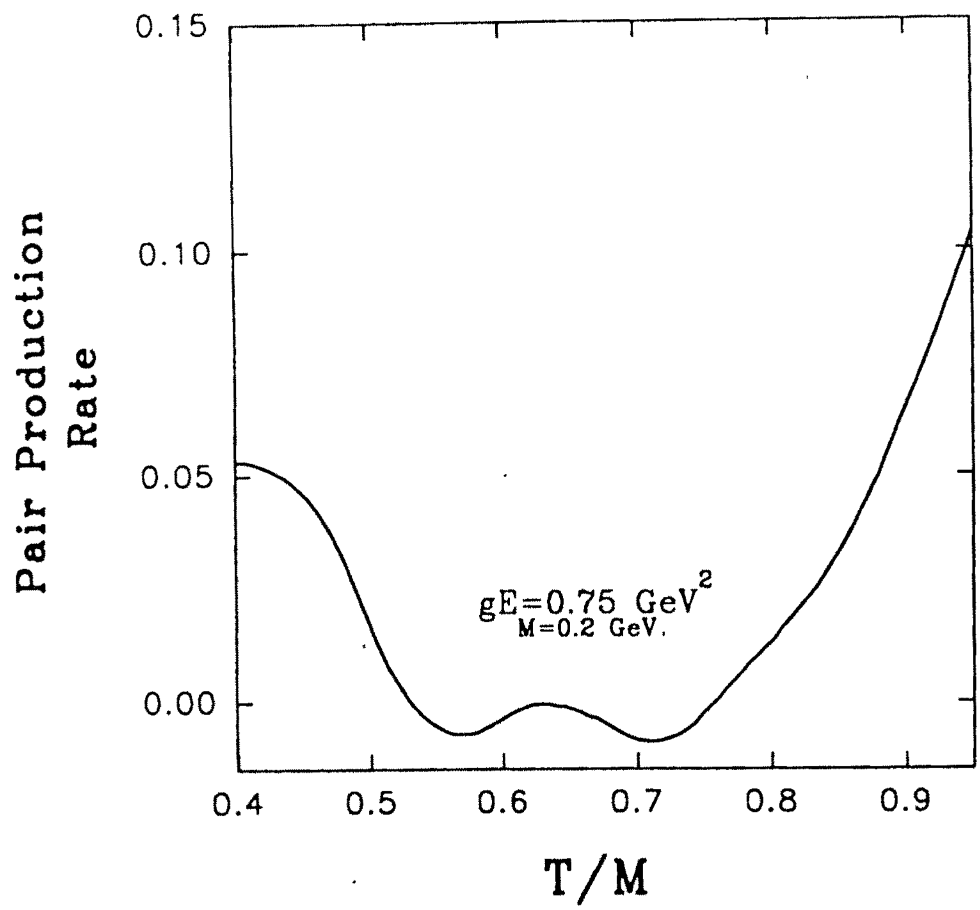

Figure 3.1: Pair Production Rate with Temperature/mass.

other than the temperature induced tunneling, at high temperature, the thermal excitations also push the particles over the barrier resulting in a significant increase in the pair production rate at high temperature. At low and intermediate temperatures, for some value of the external chromo-electric field, we find a decrease in pair production rate with respect to Schwinger's result. This effect probably reflects an increase in the width of the barrier due to thermal excitations.

From equation (3.26), we find that, at extremely high temperature the pair production rate goes as

$$Im\left[L_{eff}\right] \simeq \frac{gET^2}{6} \tag{3.29}$$

At $z \neq 0$ (eqn (3.27)), because of the presence of the cosine hyperbolic term, the pair production rate increases with increase in z [Figure].The reason behind this is that, in the presence of an electric field charged particles do not stay at rest. They move towards the source and try to cancel the electric field in the region in between. Thus, as one moves towards the source of the chromo-electric field, the field intensity increases, and hence one would expect a reasonable increment in the pair production rate as one approaches the color charged nuclear plates. In the context of heavy ion collision this would mean that if the flux tube model is correct then production rate of $q\bar{q}$ will be more as one moves away from the reaction plane. Considering the complexity of the underlying process and the successive phases that the plasma undergoes, it might be a difficult task at this stage to give a quantitative description about the signature of this phase but we believe, early signals like dilepton or direct photon might be an ideal candidate that might carry the information of this phase. From a simple minded approach to the problem, if one assumes the fluid to undergo Bjorken hydrodynamic expansion, in the following stage of its evolution, the effect of this phase may show up in the observed angular multiplicity distribution of the particles.

In summary, we have computed the pair production rate at finite temperature in the imaginary time formalism starting from the thermal partition function for a system of fermions with SU(2) color symmetry in the presence of a non-abelian external chromoelectric field.

Our results show the presence of two distinct pieces i.e. the vacuum contribution and the thermal correction to it. In the case of a $U(1)$ gauge symmetry it reduces to that of Loewe and Rojas but with a sign difference between the thermal and the vacuum contribution. It also clearly shows the spatial dependence of the temperature corrected part of the effective Lagrangian.

We have also tried to give a physical picture of the whole process in terms of particles in a simple potential well. We see that the pair production rate increases away from the plane at $z = 0$ and we expect that this effect might show up in future relativistic heavy ion collision experiments.

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