2
FOURIER AND WAVELET TRANSFORMS

2.1. Time and Frequency representation of signals

2.1.1 Introduction

The most common representation of signals and waveforms is in the time domain. However, most signal analysis techniques work only in the frequency domain. The concept of the frequency domain representation of a signal is quite difficult to understand when one is first introduced to it. The following section attempts to explain the frequency domain representation of signals.

2.1.2 Time and Frequency domain

The frequency domain is simply another way of representing a signal. For example, consider a simple sinusoid as shown in Figure 2.1.

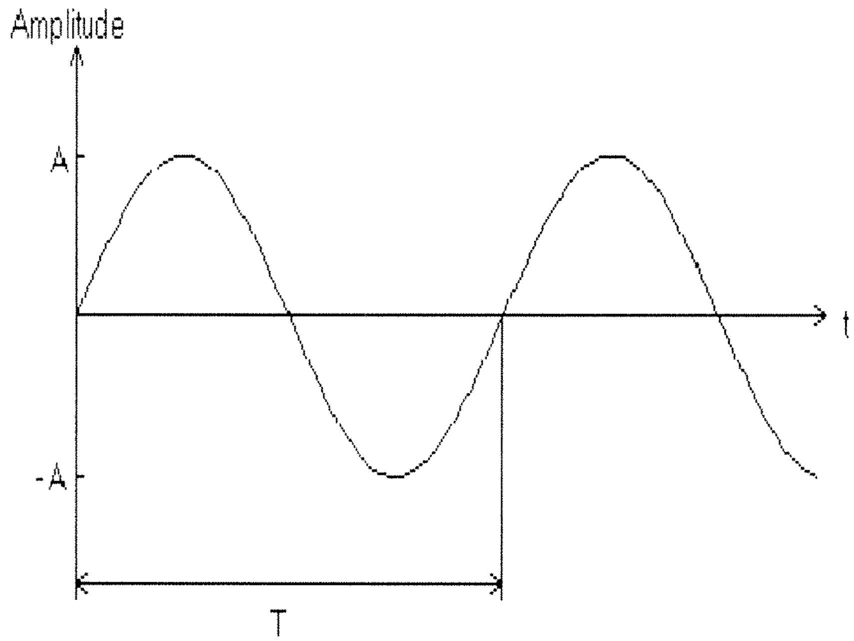

Fig.2.1 A simple sinusoidal function

The time – amplitude axes on which the sinusoid is shown define the time plane. If an extra axis is added to represent frequency, then the sinusoid would be as illustrated in Figure 2.2.

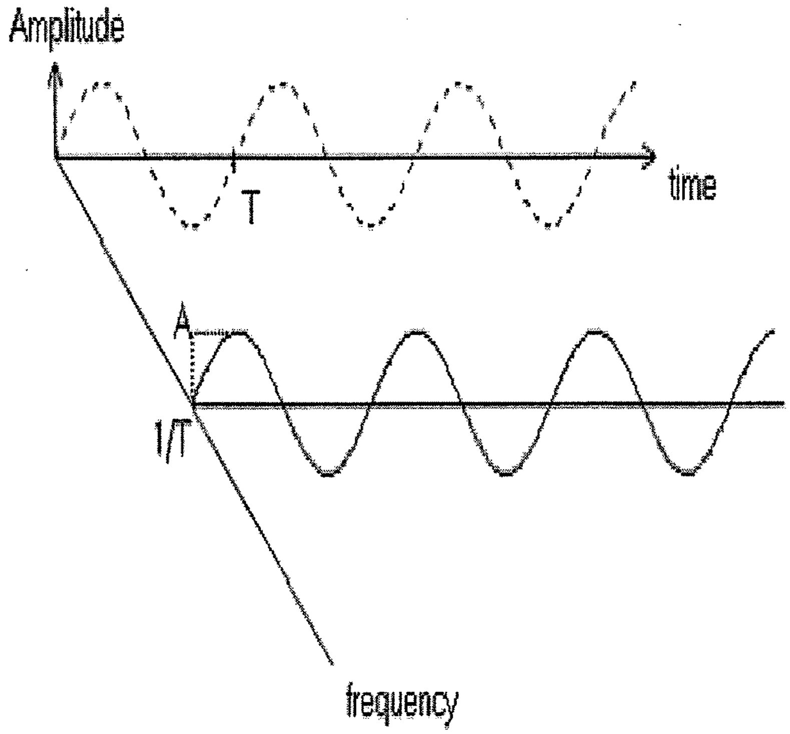

Fig.2.2 Time – Frequency representation of a sinusoidal function

The frequency – amplitude axes define the frequency plane in a manner similar to the way the time plane is defined by the time – amplitude axes. This frequency plane represents the frequency spectrum of the original signal with respect to time. The frequency plane is orthogonal to the time plane, and intersects with it on a line which is the amplitude axis. Note that the time signal can be considered to be the projection if the sinusoid onto the time plane (time – amplitude axes). The actual sinusoid can be considered to be as existing some distance along the frequency axis away from the time plane. This distance along the frequency axis is the frequency of the sinusoid, equal to the inverse of the period of the sinusoid.

The waveform also has a projection onto the frequency plane. If you imagine yourself standing on the frequency axis, looking toward the sinusoid, you would see the sinusoid as simply a line. This line will have a height equal to the amplitude of the sinusoid. So, the projection of the sinusoid onto the frequency plane is simply a line equal to the amplitude of the sinusoid.

These two projections mean that the sinusoid appears as a sinusoid in the time plane (time – amplitude axes), and as a line in the frequency plane (frequency – amplitude axes) going up from the frequency of the sinusoid to a height equal to the amplitude of the sinusoid.

It should be noted very carefully that all the information about the sinusoid (frequency, amplitude and phase) is represented in the time plane projection, but all phase information is lost in the projection onto the frequency plane. If the full signal is to be reconstructed from the frequency representation then an additional graph called the phase diagram is needed. The phase diagram is simply a graph of the phase versus frequency, similar to the amplitude versus frequency graph obtained from the frequency plane.

2.2 Discrete Fourier Transform (DFT) and Fast Fourier Transform (FFT) [7-23]

2.2.1 Introduction

The *Fourier Transform* provides the means of transforming a signal defined in the time domain into one defined in the frequency domain. When a function is evaluated by numerical procedures, it is always necessary to sample it in some fashion. This means that in order to fully evaluate a Fourier transform with *digital operations*, it is necessary that the time and frequency functions be sampled in some form or another. Thus the digital or *Discrete Fourier Transform* (DFT) is of primary interest

2.2.2 The Fourier Transform

The Fourier transform is used to transform a continuous time signal into the frequency domain. It describes the continuous spectrum of a non-periodic time signal. The Fourier transform $X(f)$ of a continuous time function $x(t)$ can be expressed as :

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-i2\pi ft}\,dt \qquad (2.1)$$

The Inverse transform is:

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{i2\pi ft} df \quad \text{.......} \quad (2.2)$$

2.2.3 The Discrete Fourier Transform (DFT)

This is used in the case where both the time and the frequency variables are discrete (which they are if digital computers are being used to perform the analysis). Let *x(nT)* represent the discrete time signal, and let *X(mF)* represent the discrete frequency transform function. The Discrete Fourier Transform (DFT) is given by

$$X(mF) = \sum_{n} x(nT) e^{-inm2\pi FT} \quad \text{.......} \quad (2.3)$$

Where

$$x(nT) = \frac{1}{N} \sum_{m} X(mF) e^{inm2\pi FT} \quad \text{.......} \quad (2.4)$$

2.2.4 The Fast Fourier Transform (FFT)

The fast Fourier transform (FFT) is simply a class of special algorithms which implement the discrete Fourier transform with considerable savings in computational time. It must be pointed out that the FFT is not a different transform from the DFT, but rather just a means of computing the DFT with a considerable reduction in the number of calculations required.

Since this section is intended as an introduction to the Fourier transform, a rigorous development of the underlying theory of the FFT will not be attempted here. While it is possible to develop FFT algorithms that work with any number of points, maximum efficiency of computation is obtained by constraining the number of time points to be an integer power of two, e.g. 1024 or 2048.

2.2.5 Applications of FFT

Once the waveform has been acquired and digitized, it can be Fast-Fourier-Transformed to the frequency domain. The FFT results can be either real and imaginary, or magnitude and phase, functions of frequency. The choice of output format belongs to the user. Since the FFT generates the frequency spectrum for a time domain waveform, some fairly simple applications, e.g., harmonic analysis, distortion analysis, vibration analysis, and modulation measurements, might suggest themselves immediately. Another important area is that of frequency response estimation. A linear, time-invariant system can be stimulated with an impulse function. Its output, the impulse response, can then be acquired and fast-Fourier-transformed to the frequency domain. The FFT of the impulse response, referred to as the frequency response function, completely characterizes the system. Once a system's frequency response function is known, one can predict how that system will react to any waveform. This is done by Convolution. An important aspect of the FFT is that convolution can easily be performed through frequency-domain multiplication. Let's say you know a system's impulse response, given by $h(t)$, and an input waveform given by $x(t)$. The output, say $y(t)$, caused by $x(t)$, can be computed in the classical manner by the convolution integral. But this is tedious and slow. An easier and faster approach is to FFT $x(t)$ and $h(t)$ to the frequency domain. Then the product of their frequency domain functions can be formed, giving $Y(f) = X(f) H(f)$. Forming this product corresponds to time domain convolution, and the convolution result can be obtained by Inverse-Fourier-Transforming (IFT) the $Y(f)$ function back to the time domain.

Correlation is another useful operation that the FFT makes easier. Mathematically, correlation looks and is performed in a manner similar to convolution. The difference is that one of the frequency domain functions is conjugated before the frequency domain product is formed.

Although the operations of convolution and correlation may look similar, their applications are not. Correlation is a sort of searching or looking for similarities between two waveforms. When two waveforms have absolutely no similarity, like uncorrelated noise, their correlation function is zero. On the other hand, correlation two waveforms that are exactly alike produce a perfect correlation function.

This property of finding similarities makes correlation a useful tool for detecting signals that are hidden or masked by other signals.

Another useful property of correlation is its ability to indicate delay. This is particularly useful in measuring things like path delay, path diversity, and echo return times.

2.3 Introduction to Wavelet Transforms

The wavelet transform is a mathematical tool that cuts up data, functions or operators into different frequency components, and then studies each component with a resolution matched to its scale. For example, in signal analysis, the wavelet transform allows us to view a time history in terms of its frequency components, which means it maps a one-dimensional signal of time, $f(t)$, into a two dimensional signal function of time and frequency [7-23]. The wavelet transform represents the signal as a sum of wavelets at different locations (positions) and scales (frequency bands). The wavelet coefficients essentially quantify the strength of the contribution of the wavelets at these locations and scales.

Next sections of this chapter are devoted to representing a general introduction to the wavelet transform and its applications in power system areas. Wavelet types, conditions, efficiency, basic mathematics and applications to power systems are illustrated.

2.3.1 Wavelet and Multi-resolution

A wavelet is a small wave, which has its energy concentrated in time to give a tool for analyzing transient, non-stationary, or time-varying phenomena. A wavelet still has oscillating wave-like characteristics but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. Different wavelets are shown in Figure 2.3.

The wavelet transform can be accomplished in three different ways namely as: the Continuous Wavelet Transform (CWT), the Wavelet Series (WS) and Discrete Wavelet Transform (DWT). The DWT maps a sequence of numbers into a sequence of numbers much the same way the Discrete Fourier transform (DFT) does.

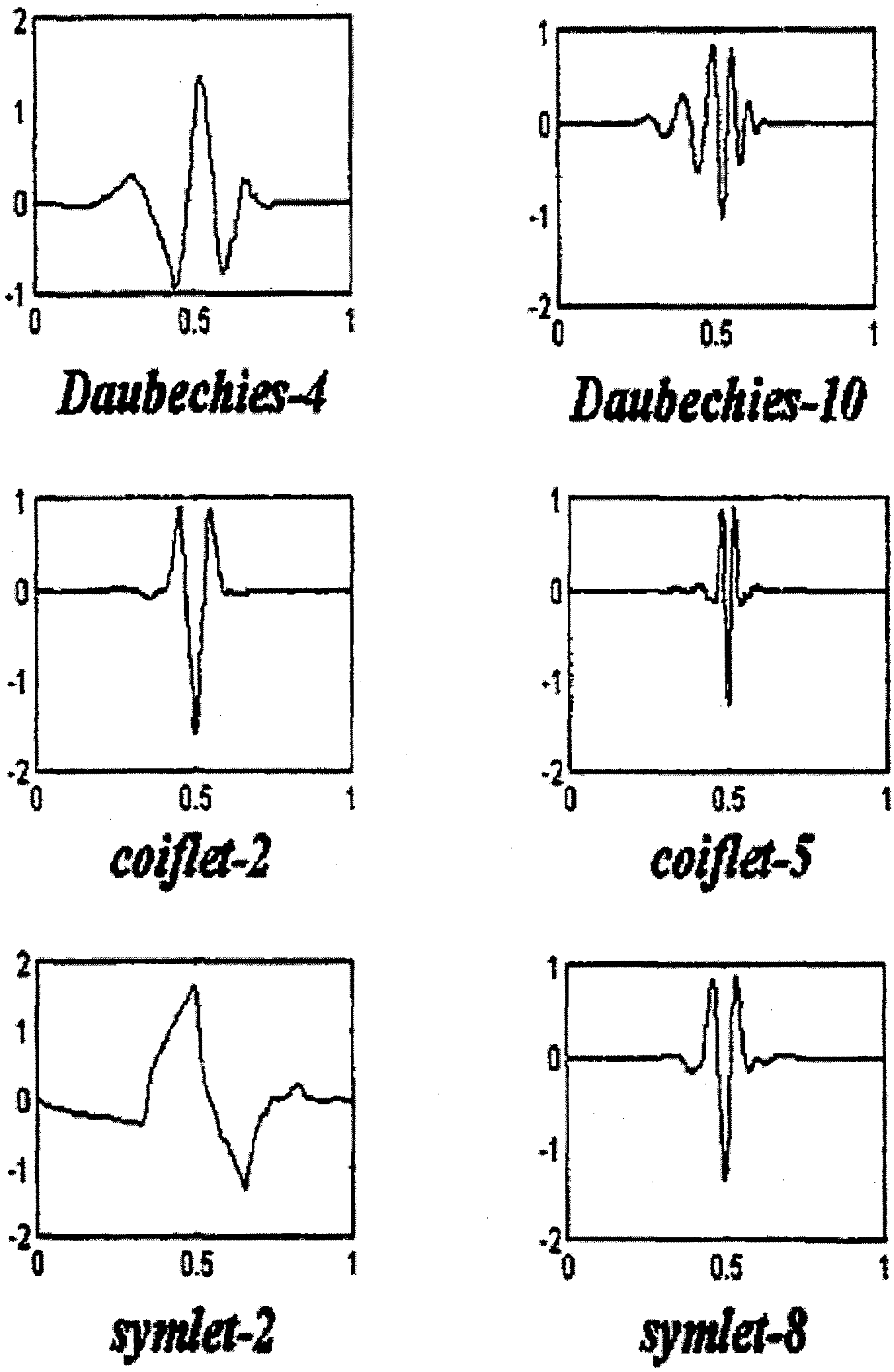

Fig.2.3 Examples of Different Wavelet Functions

The discrete wavelet transform (DWT) is sufficient for most practical applications in power systems and for reconstruction of the signal. It provides enough information, and offers an enormous reduction in the computation time. It is considerably easier to implement when compared to the continuous wavelet transform. The discrete wavelet coefficients measure the similarity between the signal and the scaled and translated versions of a scaled wavelet, $\Psi_{a,b}$.

On the other hand, Multi-Resolution Analysis (MRA) is used to analyze a signal at different frequencies with different resolutions. The goal of MRA is to develop representations of a complicated signal $f(t)$ in terms of several simpler ones and study them separately. This goal will help in achieving two important properties. The first is the localization property in time of any transient phenomena, and the second is the presence of specific frequencies at different resolution levels.

The DWT uses selected wavelets as digital Filters with different cutoff frequencies to analyze a signal at different scales. In MRA, the signal is passed through a series of discrete filters "selected mother wavelet" to analyze and localize the high and the low frequencies that embedded in the signal.

2.3.2 Wavelet Properties

Wavelets have three main properties:

- They are building blocks to decompose and reconstruct signals. This means complicated signals can be decomposed and represented as simple building blocks in terms of the selected wavelets.
- The wavelet expansion gives a Time-frequency localization of the signal. This means most of the energy of the signal is well represented by a few expansion coefficients chat are localized in the time and frequency domains.
- The calculation of the wavelet coefficients from the signal can be done efficiently. This means that by using orthogonal wavelets, the distorted signal coefficients in the wavelet domain are simply given as the inner product of the signal with the wavelet function, which greatly simplifies the transform algorithm [24-30]

2.3.3 Wavelet Efficiency

Wavelet transforms have been proven to be very efficient in signal analysis. Due to the abovementioned properties of the wavelet transform, the following advantages can be gained:

- Wavelet expansion coefficients represent a component that is itself local and are easier to interpret. Therefore, the location of these coefficients can be used to detect and localize any sudden change in the signal due to fault or abnormal system condition. Furthermore, the energy of these coefficients will assist in extracting features that can classify such transient event in terms of its magnitude, frequency components and duration.
- MRA that decomposes a signal at different resolution levels will allow a separation of components that overlap in both time and frequency. This property will be useful in detecting and classifying multiple transient events that may take place in the same monitored window.
- The wavelet transform coefficients represent the energy of the transient signal. These coefficients will be used to measure the magnitude of the transient signal and quantify the transient events.
- The rapidly drop off in the size of the coefficients, with increasing translation and scaling factors, will assist in representing the transient event by using only small number of coefficients. This will help in designing an automated recognition system that has the ability to store a large number of transient events using a small number of coefficients.
- MRA and DWT calculations are efficiently performed by digital computers. Discrete wavelet transform (DWT) computation relies on convolution and decimation or interpolation. These operations depend on addition and multiplication. Furthermore, the number of mathematical operations for DWT is in the order of (N) which is lower than that for the Fast Fourier Transform (FFT) algorithm which needs (N log(N))operations. This computational speed feature of the DWT will help in implementing the automated recognition system on-line and for real time applications.

2.4 Wavelet and Multi-resolution analysis Mathematical Representation

2.4.1 General Introduction

The Wavelet transform will be proposed in this thesis as a tool to solve protection problems associated with series compensated lines. Using wavelet properties, detection, discrimination, localization and classification of any transient phenomena such as fault or abnormality within the signal can be achieved. Such analysis of any transient event will help in localizing the transient event in the time domain and clarifying the presence of any specific frequency components at different resolution levels.

To achieve our goal and to workout wavelet-based solution technique the mathematical details of the proposed tool is highlighted. The mathematical concepts of the Wavelet transforms (WT) and Multi-Resolution Analysis (MRA) are presented and discussed further in this Chapter. The Analysis and Synthesis procedures of multi-resolution analysis are discussed and applied on selected examples. The time-frequency "scale" plane and localization and partitioning of signal energy at different resolution levels are also presented in this chapter.

2.4.2 General Mathematical Preliminaries

The purpose of this section is to introduce the mathematical notations and tools that are useful to present Wavelet Transform theory. Some definitions of vector spaces and related mathematical relations are introduced.

2.4.3 Vector Spaces

The totality of vectors that can be constructed by scalar multiplication and vector addition from vectors in a given set is called a vector space. A set of vectors that is capable of generating the totality of vectors by these operations is said to span the space. If the set consist of the least number of vectors chat span the space, the set is called a Basis of the space. The number of vectors in the basis is called the dimension of the space. N-basis vectors generate an n-dimensional space. Any subset of r-basis vectors from the basis of an r-dimensional subspace.

2.4.4 Norms

The concept of the distance is generalized in the case of vectors through the use of norms. The norm of a vector x, $||x||$ is a real non negative number such that:

$\|x\| = 0$ *if and only if* $x = 0$

$\|cx\| = |c|\|x\|$ *for all scalars c vectors x*.. (2.5)

$\|x_1 + x_2\| \leq \|x_1\| + \|x_2\|$ *for all* x_1 *and* x_2...(2.6)

There exist many norms for vectors. Three of the commonly used ones are:

$$\|x_1\| = \sum |x_i|$$
$$\|x_2\| = \sqrt{\sum |x_i|^2} \quad \text{.. (2.7)}$$
$$\|x\|_\infty = \max |x_i|$$

2.4.5 Inner Product

It is a scalar "a" obtained from two vectors *f(t)* and *g(t)*, by an integral. It is denoted as:

$a = \langle f(t), g(t) \rangle = \int f(t) g(t) dt$ .... (2.8)

The length of a vector " *norm*" can be defined in terms of the inner product as:

$\|f(t)\| = \sqrt{|\langle (f(t), g(t)) \rangle|}$ .... (2.9)

2.4.6 Hilbert Spaces

It is a complete inner product space with orthogonal basis, where any signal *f (t)* Є L^2® satisfies the following condition:

$\int_{-\infty}^{\infty} |f(t)|^2 dt \langle \infty$ .... (2.10)

This means that the signal *f (t)* has finite energy.

2.4.7 Basis

A set of vectors $\Phi_k(t)$ spans a vector space F. if any element $f(t)$ in that space can be expressed as a linear combination of members of that set. This means that $f(t)$ can be written as:

$$f(t)=\sum_k a_k \phi_k(t) \quad (2.11)$$

with $k \in Z$ the set of integers and $a, t \in R$. $\Phi k(t)$ is known as the expansion set and a_k is known as the expansion coefficients. The expansion set $\Phi_k(t)$ forms a basis set or basis if the set of expansion coefficients $\{a_k\}$ are unique for any particular $f(t) \in F$. There may be more than one basis for a vector space. However, all of them have the same number of vectors, and this number is known as the dimension of the vector space.

The expansion set $\Phi_k(t)$ forms an orthogonal basis if its inner product is zero:

$$\langle \phi_k(t), \phi_l(t) \rangle = 0 \text{ For all } k \neq l \quad (2.12)$$

The expansion set $\Phi_k(t)$ forms an orthogonal basis if the inner product can be represented as:

$$\langle \phi_k(t), \phi_l(t) \rangle = \delta(k-l) = \begin{cases} 0 & k \neq l \\ 1 & k = l \end{cases} \quad (2.13)$$

This means that in addition of being orthogonal, the basis is normalized to unity norm.

$$\|\phi_k(t)\| = 1 \text{ for all } k \quad (2.14)$$

For an orthonormal basis, the set of expansion coefficients $\{a_k\}$ can be calculated using the inner product,

$$a_k = \langle \phi_k(t), f(t) \rangle \quad (2.15)$$

Therefore, having an orthonormal basis, any element in the vector space $f(t) \in F$, can be written as:

$$f(t)=\sum_k \langle \phi_k(t), f(t) \rangle \phi_k(t) \quad (2.16)$$

This expansion formulation is extremely valuable. The inner product of *f(t)* and $\Phi_k(t)$ produce the set of coefficients a_k. This set of coefficients a, can be used linearly with the basis vectors $\Phi_k(t)$ to give back the original signal *f(t)*.

2.5 Wavelet Transform and Multilevel representation

The Wavelet Transform is a tool that can cut any signal into different frequency components and then study each component at a certain resolution level. The WT depends on two sets of functions known as scaling functions and wavelet functions. In order to implement a multi-level presentation of a signal we will start by defining the scaling function and then use it to represent the wavelet function.

2.5.1 The Scaling Function

The scaling function $\Phi(t)$ is a function that belongs to the Hilbert space. The scaling set $\Phi_k(t)$ is defined as a set of integer translations of a basis scaling function $\Phi(t)$, where:

$$\phi_k(t) = \phi(t-k) \text{ forall } k \in Z; \phi_k \in L^2(R) \quad (2.17)$$

and L^2 ® is the Hilbert space, which can be represented by a set of subspaces *(Vj{j Є Z})*, where *Z* is the set of integers.

The set of scaling functions $\Phi_k(t)$ span the subspace V_0 defined as:

$$V_0 = span[\phi_k(t)] = span_k\{\phi(t-k), k \in Z\} \quad (2.18)$$

if *f(t)* is a finite energy signal *{f(t) Є* L^2*®}* , then an approximated version of *f (t) Є Vo* ,can be represented in terms of the scaling function as shown in Figure 2.4 and can be expressed according to Equation 2.19 as:

$$f(t) = \sum_k a_k \phi_k(t) \quad (2.19)$$

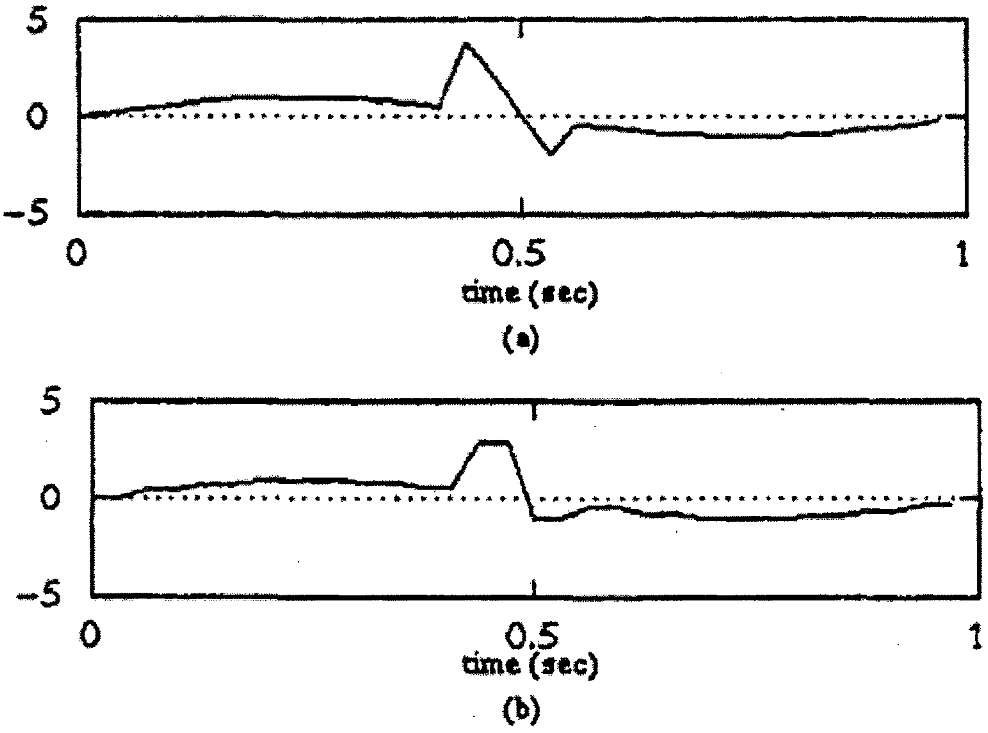

Fig.2.4 Approximation of the input signal, (a) the input signal,
(b) Approximation of the input signal using Haar scaling function

Keeping in mind the containment property, the scaling function *Φ(t)* can be expressed in terms of a weighted sum of shifted *Φ(2t)*:

$$\phi(t) = \sum h(n)\sqrt{2}\phi(2t-n) \quad n \in Z \qquad (2.20)$$

Where the coefficients *h(n)* are a sequence of real or complex number called the scaling function coefficients (or the scaling filter coefficients) and the $\sqrt{2}$ maintains the unity norm of the scaling function with the scale of two. This equation is called the multi-resolution analysis equation. It can be utilized to represent the signal at different resolution levels. This is presented in the following subsection.

2.5.2 Multilevel representation using Scaling function

In order to represent a signal *f(t)* at different resolution levels, the used scaling function *Φ(t)* must be translated and scaled. Therefore, the two dimensional family of scaling function $\Phi_{j,k}(t)$ is presented as:

$$\phi_{j,k}(t) = 2^{jl^2}\phi(2^j t - k) \quad (2.21)$$

where, j is the scaling factor and k is the translation factor. This two dimensional family can span different subspaces *(Vj{j Є Z})* as:

$$V_j = span_k\{\phi_{j,k}(t)\} = span_k\{2^{jl^2}\phi_k(2^j t - k)\} \quad (2.22)$$

for all integers k.

This means that any signal $f(t) \in L^2$® can be approximated and represented at different resolution levels *(f(t) Є Vj)*, as:

$$f(t) = \sum_k a_k 2^{jl^2}\phi(2^j t - k) \quad (2.23)$$

The multi-level representation of the signal *f(t)* is shown in Figure 2.5. The Haar scaling function is scaled and translated to represent the input signal at five resolution levels. As the scale j changes in Equation 2.23 changes, more details are added to the approximated version and a more similar version of the original signal can be achieved. These details, which exist in between each of the two approximated versions of the signal, are very important in analyzing and monitoring the original signal. These details can be extracted by using the wavelet function.

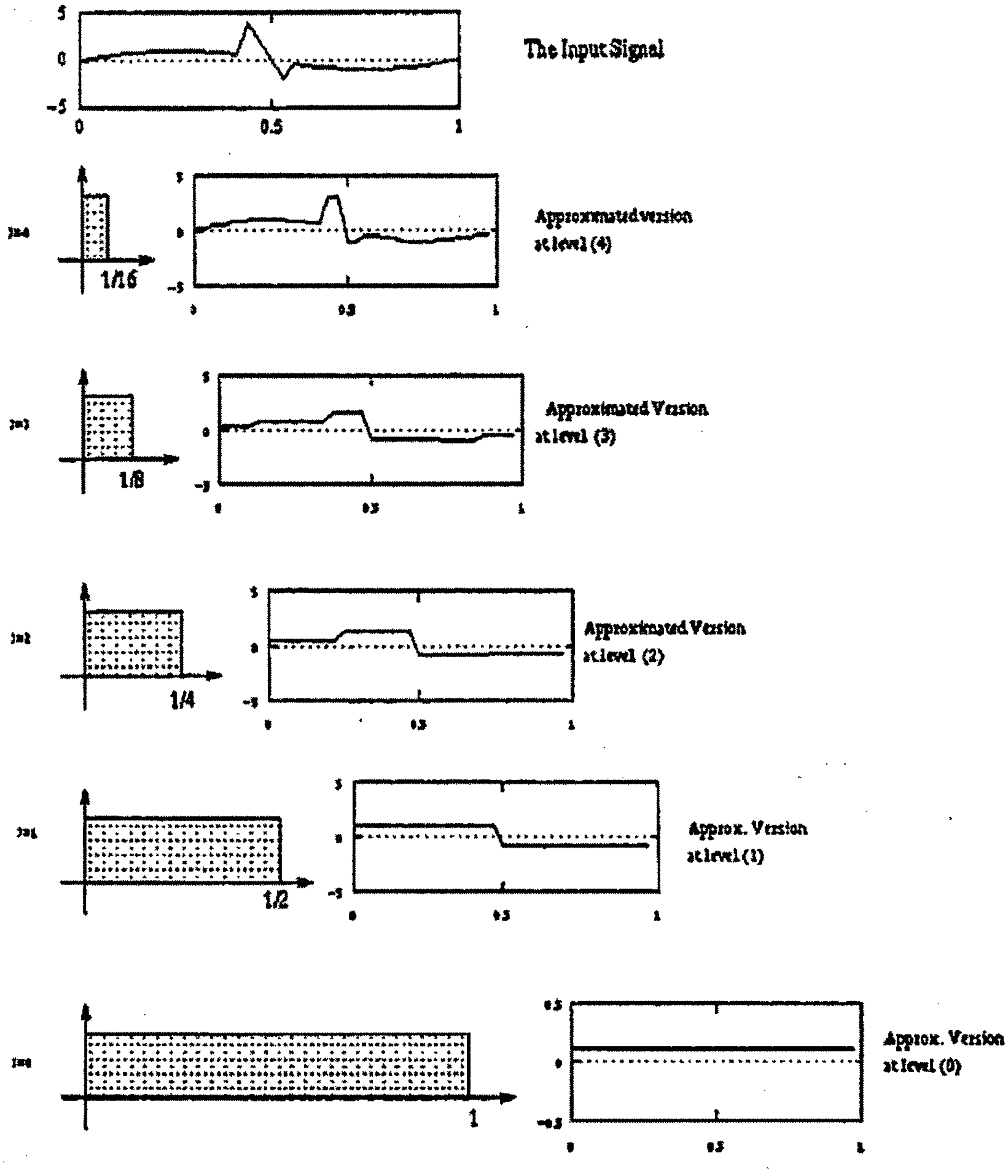

Fig.2.5 Multilevel representation of an input signal using the Haar scaling function

2.5.3 The Wavelet Function

More important features of a signal can be extracted by using a function chat spans the difference between various approximated versions obtained using the scaling function $\Phi_{j,k}(t)$. This can be achieved by using the wavelet function $\Psi_{j,k}(t)$. As indicated by the containment property, the subspace Vo, is embedded in the subspace V_1, $Vo \in V_1$. In order to move to a finer subspace V_1, from a coarser subspace Vo , one must add another subspace in between, which is known as the complement subspace *Wo*, This is illustrated clearly in Figure 2.6. Since these wavelets reside in the space spanned by the next narrower scaling function. Then they can be represented by a weighted sum of shifted scaling functions at that space. Far example $\Psi(t)$ resides in the space *Wo*,, and $Wo \in V_1$ .Therefore, $\Psi(t)$ an be represented by a weighted sum of shifted scaling function $\Phi(2t)$. This is illustrated in Figure 2.6 and mathematically can be presented by:

$$\psi(t) = \sum_{n} h_l(n)\sqrt{2}\phi(2t-n) \qquad n \in Z \quad \text{.......................................(2.24)}$$

for some set of wavelet coefficients (wavelet Filter coefficients) $h_l(n)$, where.

$$h_1(n) = (-1)^n h(1-n) \quad \text{...(2.25)}$$

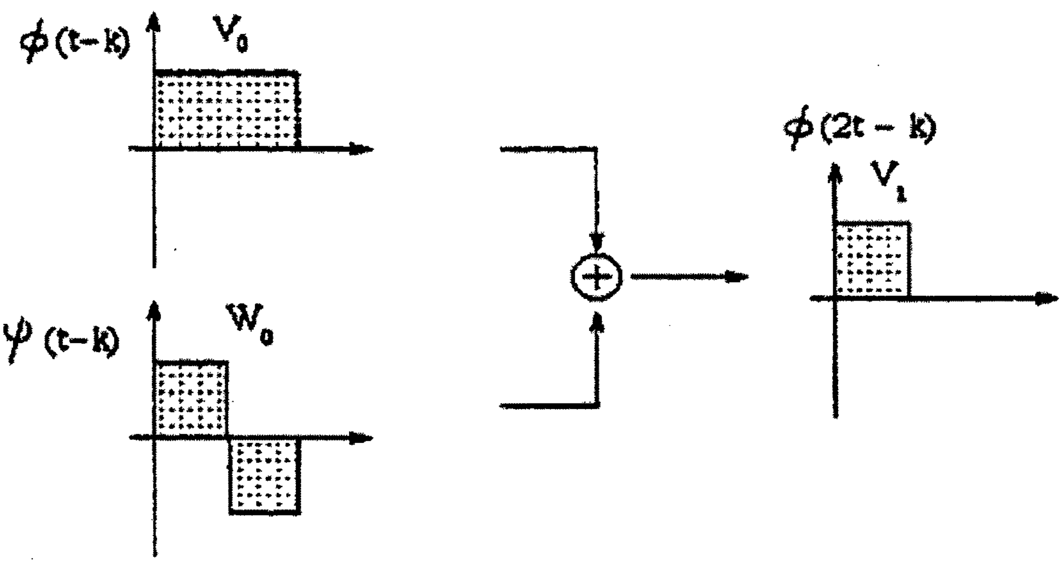

Fig.2.6 Moving to a finer space using the wavelet, $\Psi_{j,k}(t)$ and scaling function, $\Phi_{j,k}(t)$

Again, the scaled wavelet function can be utilized to extract different details that reside in between different approximated versions of the signal. This will be discussed in the following subsection.

2.5.4 Multilevel representation using the Wavelet function

As the scaling function $\Phi_{j,k}(t)$ spans Vo, and $\Phi_{1,k}(t)$ spans V_1, there are particular functions which spans Wo, and W_1. Therefore, as the scaling function $\Phi_{j,k}(t)$, spans Vj, the wavelet function $\Psi_{j,k}(t)$, spans Wj , Where Wj is the orthogonal complement of Vj, . This means that all members of Vj are orthogonal to all members of Wj , ($Vj \perp Wj$).

Therefore, the space Vj, can be represented in terms of a set of subspaces where each subspace can be spanned using the scaling and wavelet functions. This is mathematically represented as:

$$V_j = V_{j-1} \oplus W_{j-1} = V_0 \oplus W_0 \oplus W_1 \oplus W_2 \oplus ... \oplus W_{j-1} \quad (2.26)$$

Therefore, any signal $f(t) \in L^2(R)$ can be represented as a series expansion by using a combination of the scaling function and wavelets function:

$$f(t) = \sum_{k=-\infty} c_k \varphi(t-k) + \sum_{k=-\infty} \sum_{j=0} d_{j,k} \psi(2^j t - k) \quad (2.27)$$

where, c_k, are the approximated coefficients of the last approximated version and $d_{j,k}$, are the detail coefficients at different scales.

Equation 2.27 represents the signal $f(t)$ at different resolution levels in terms of one approximated version and different details that exist in between different approximated versions. The first summation gives the approximated version of the signal $f(t)$ in terms of the scaling function. The second summation gives different details that can be extracted in terms of the wavelet function at different scales. The summation of the approximated version and the different detail versions will represent the original signals $f(t)$.

Figure 2.7 shows the details of the input signal at different resolution levels by using the Haar wavelet function. It is clear form the figure that as the scale changes more resolution is achieved.

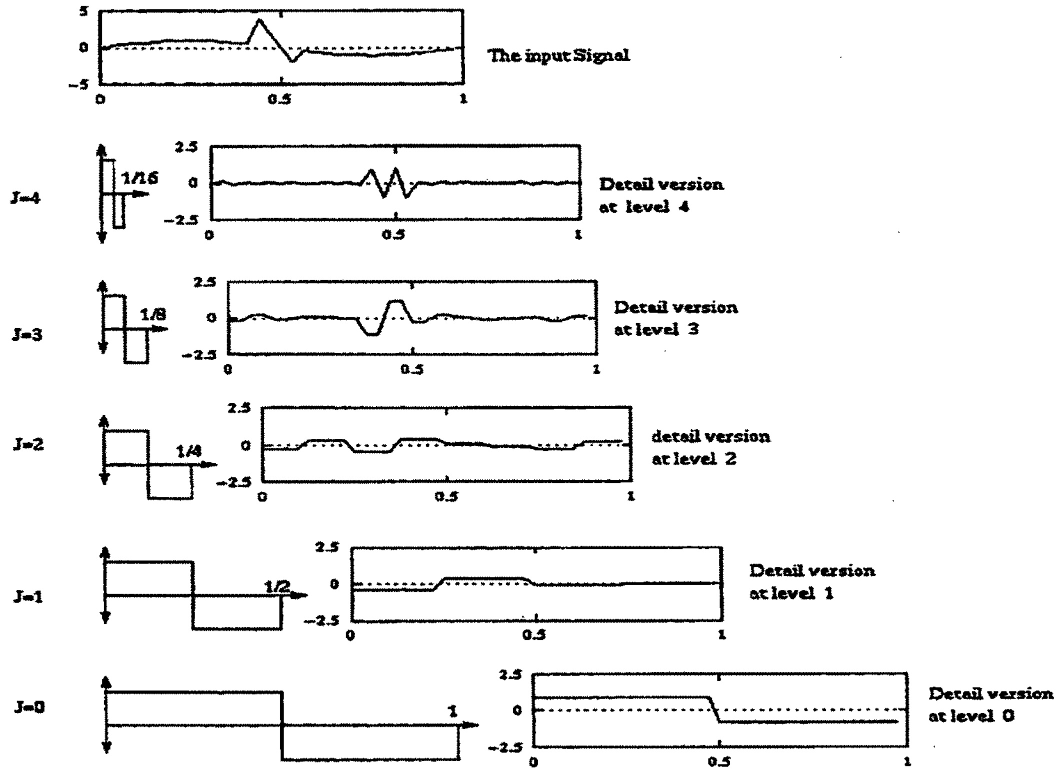

Fig.2.7 Multilevel representation of an input signal using wavelet function

2.5.5 Multi resolution analysis

Multi-resolution analysis (MRA) is used to decompose any signal and represent it at different resolution levels. The goal of multi-resolution analysis (MRA) is to develop representations of a complicated signal *f(t)* in terms of several simpler ones and study them separately. This goal will help in achieving two important properties. The first is the localization property in time of any transient phenomena. And the second is the presence of specific frequencies at different resolution levels. In multi-resolution analysis, the signal is decomposed to find a time-frequency picture of the signal and then reconstructed to get back the original signal.

In **MRA**, the first stage divides the spectrum into two equal frequency bands; the second stage subdivides the lower frequency band into quarters, and so on. In other words, the Discrete Wavelet Transform (DWT) coefficients for any signal, periodic or non-periodic, can be computed by using a multi-rate filter bank. The total number of the resolutions that can be achieved depends on the number of sampling points, which can be controlled by the sampling frequency and the window size of the data. This is best explained by Figure 2.8.

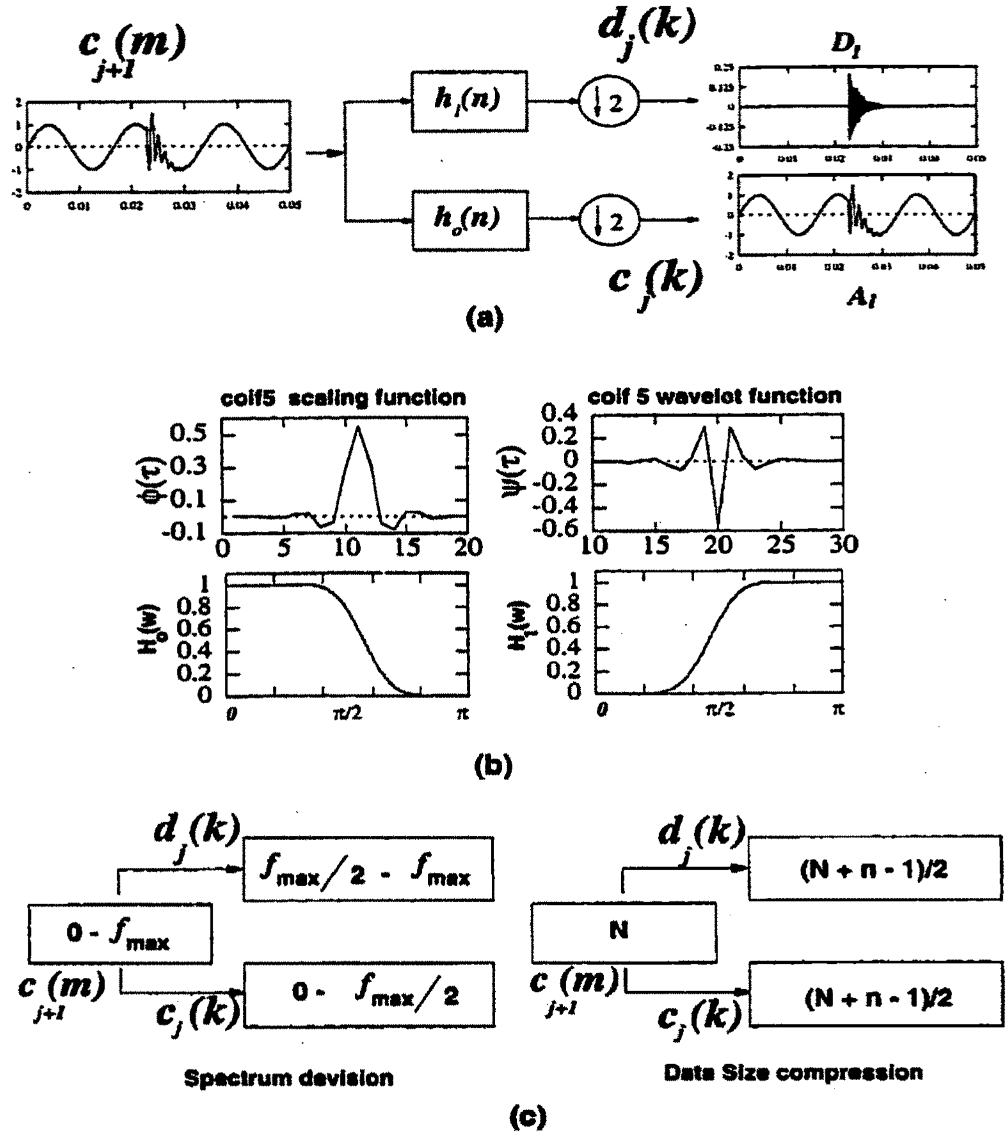

Fig.2.8 One stage MRA and Wavelet Filters –(a) decomposing into detail and approximated version,(b) Coiflt-5 scaling and wavelet function and their frequency response,(c) Spectrum division and coefficients size compression

In other words, an analysis filter bank efficiently calculates the discrete wavelet transform (DWT) using banks of digital filters and down-samplers, and the synthesis filter bank calculates the inverse discrete wavelet transform (IDWT) to reconstruct the signal from the transform. Figure 2.9 shows five-levels of multi-resolution signal decomposition using the Haar scaling and wavelet functions.

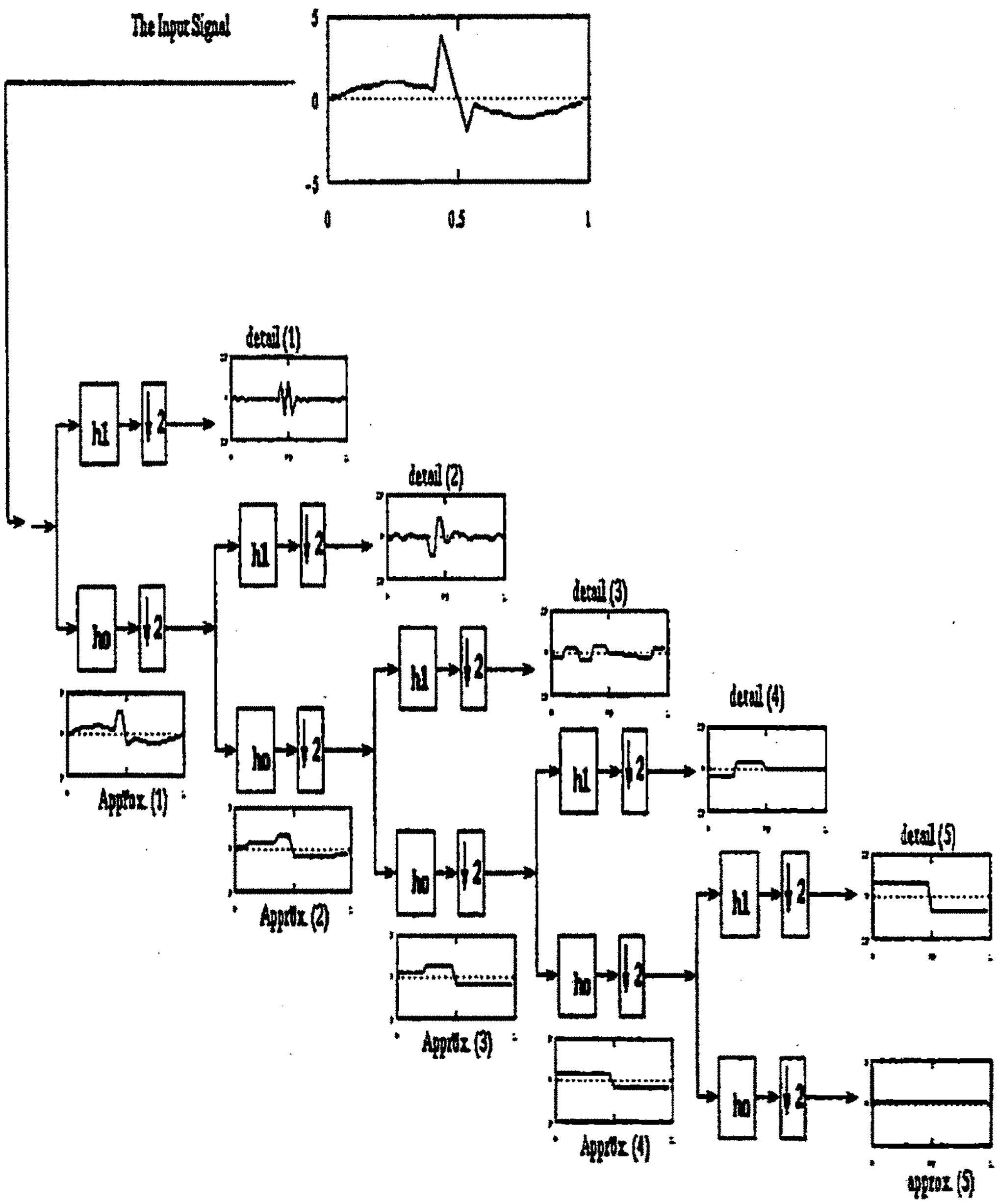

Fig.2.9 Five level multi-resolution signal decomposition

2.6 Review of Wavelet Applications in Power Systems

Several works have been developed in many areas with the aim of this tool, specially, in the last ten years have been met the potential benefits of applying WT to power systems due to, among other, the interest in analyzing and processing the voltage-current signals in order to make a real time identification of transients in a fast and accurate way.

The aim of the following subsection is to provide a descriptive overview of the wavelet transform applications in power systems to those who are novel in the study of this subject. For this purpose, the main publications carried out in this field have been analyzed and classified by areas.

In the mainstream literature, wavelets were first applied to power system in 1994 by Robertson [24] and Ribeiro [25]. From this year the number of publications in this area has increased as Fig. 2.10 shows.

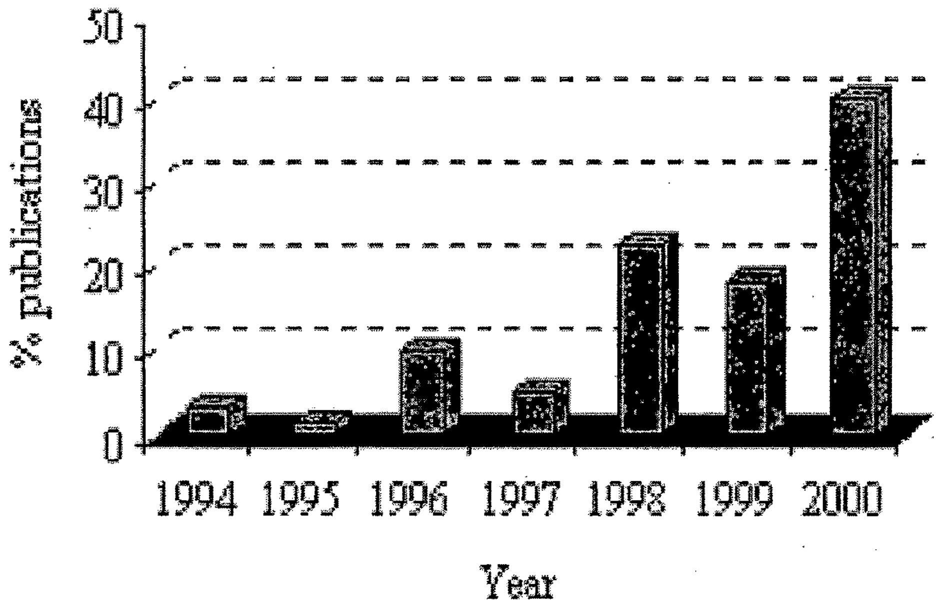

Fig.2.10- Evolution of Wavelet Publication in Power System

The main focus in the subsection has been on identification and classification methods from the analysis of measured signals, however, few works use wavelet transform as an analysis technique for the solution of voltages and currents which propagate throughout the system due, for example, a transient disturbance.

The most popular wavelet transform applications in power systems are the following:

- Power system protection
- Power quality
- Power system transients
- Partial discharges
- Load forecasting
- Power system measurement

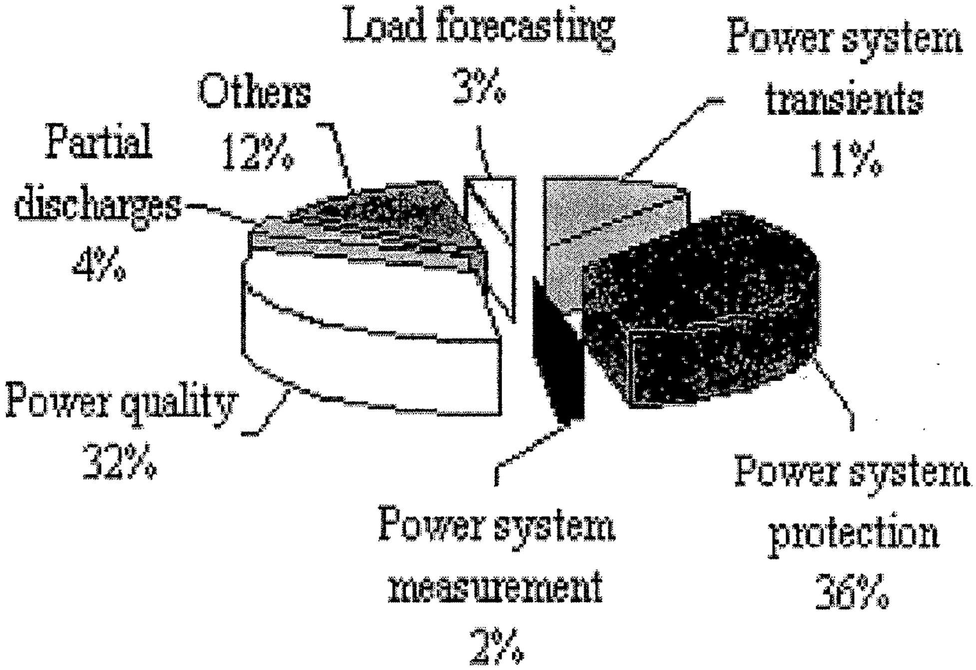

Fig.2.11- Percentage of Wavelet publications in different power system areas

Fig 2.11 shows the percentage of publications in each area; the areas in which more works have been developed are the protection and power quality field. Next section will present the applications of the wavelet Transform in the area of Power System Protections.

The potential benefits of applying wavelet transform for improving the performance of protection relays have been recognized in recent years [26-42]. In 1996, Chaari et al [26] introduce wavelets for the power distribution relaying domain to analyze transient earth faults signals in a 20 kV resonant grounded network as generated by EMTP; in the same year J. Momoh et al present an algorithm to develop a feature extractor suitable for training an Artificial Neural Network for fault diagnosis using the wavelet transform, in this case data was obtained from experimentation. At 1998

Magnago and Abur set up the development of a new investigation line in the area of fault location using wavelets, for this purpose, the fault generated traveling wave is processed by the wavelet transform to reveal their travel times between the fault and the relay locations; EMTP simulations are used to test and validate the proposed fault location. In 1999 the same authors extend the method to the identification of the faulted lateral in radial distribution systems [27] and in 2000 present an improved method for their earlier papers [28]. Similar methods for fault location can be found in [29], [30]. High impedance fault identification [31]-[32]-[33] is other application area of wavelet transform, for example, in [34] Charytoniuk presents a comparative analysis for arc fault time location, frequency and time frequency (wavelet) domain, the author demonstrates that the wavelet approach is strongly affected by the choice of a wavelet family, decomposition level, sample rate and arcing fault behavior. The application of wavelets to auto-reclosure schemes [35]-[36] is develop to accelerate trip of power transmission lines, wavelet transform is adopted to analyze the fault transients generated by the secondary arc and permanent faults and the numerical results reveal that certain wavelet components can effectively be used to detect and identify the fault relevant characteristics in transmission systems and then to distinguish between transients and permanent fault. The wavelet transform is also applied for the bars [37] , motors [38-40], generators [41]-[42] and transformer protection [43-48], in most of this cases, the spectrum of signals is analyzed with the wavelet transform to develop online detection algorithm to detect insulation degradation, inrush and to carry out the precise discrimination between internal and external faults

2.7 Summary

Wavelet Transforms have been successively applied in a wide variety of research areas. Recently, wavelet analysis techniques have been proposed extensively in the literature as a new tool to be implemented in different power engineering areas. The wavelet transform analysis is proposed as a new tool for monitoring power system related problems.

The scaling property of the selected wavelet function to be used in decomposing the signal will assure the ability of the MRA technique to detect any transient event and localize it in the time and frequency domains. Selecting Ortho-normal wavelets, multi-resolution analysis will have the ability to distribute the energy of the distorted signal in terms of the expansion coefficients of the wavelet domain. Therefore, both the expansion approximated and detail coefficients will give an indication about the energy content of the distorted signal in certain time and frequency bands. This feature can be used to classify different power system protection related problems. From the other side, the energy of the wavelet coefficients can be combined with the localization property to give a measure of the transient events like initiation of any fault. The small values of the expansion coefficients will give us an indication about the resolution levels that contains low energy of the transient signal and hence can be ignored for data compression purposes. This can reduce the large volume of transient's data to a manageable size. It will provide a higher quality of information about the transient events to be analyzed by power system engineers. Furthermore, the expansion coefficients of the highest resolution levels can be ignored for de-noising purposes. Using these properties of multi-resolution signal decomposition, an automated recognition system can be designed to detect, localize and classify different types of power system faults or transient events related problems.

Next chapters of this work will be describing some techniques using tools like Fourier Transforms and Wavelet Transforms to prevent overreach/ mal-operation of distance relays for protection of Series Compensated Transmission Lines.