Chapter 3: Analytical Design of Circular and Square Coil

This chapter deals with the detail design and analysis of circular and square magnetic coils for wireless charging of Electric Vehicles (EVs). Details mathematical analysis is carried out to investigate the performance of circular and square coil. Comparison between the circular and square coil topology has been carried out. Various parameters of the coil have been considered for comparison.

3.1 Basic of Electromagnetism:

The Maxwell equations are used to determine the fundamentals of electromagnetic in both the time independent as well as time dependent instances. The 4 Maxwell equations are listed below.

$$\oint \vec{E}.\overrightarrow{dl} = -\frac{\overrightarrow{\partial B}}{\partial t} \quad 3\text{-}1$$

$$\oint \vec{H}\,.\overrightarrow{dl} = \vec{J} + \left(-\frac{\overrightarrow{\partial D}}{\partial t}\right) \quad 3\text{-}2$$

$$\oint \vec{D}\,.\overrightarrow{ds} = \rho v \quad 3\text{-}3$$

$$\oint \vec{B}\,.\overrightarrow{ds} = 0 \quad 3\text{-}4$$

The quantities are respectively, the electric field ($\vec{E}$), magnetic intensity ($\vec{H}$), electric displacement ($\vec{D}$), and magnetic flux density ($\vec{B}$). The Maxwell equation is related with four laws: Faraday's law of electromagnetism, Ampere's law, and the third and fourth are Gauss laws for electric and magnetic fields. These are discussed below.

- Faraday's Law of electromagnetism:
  The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux density through the area enclosed by the loop.
- Ampere's Law:

This law states that current is a source of the magnetic field, thus the magnetic field is related to the current density.

- Gauss's law for electric field:
  This law states that the amount of total electric flux displacement through a given closed surface is proportional to the amount of volume charge ρv in the volume contained by that surface.
- Gauss's law for magnetic field:
  This states that all magnetic field lines that enter a particular closed surface must eventually leave the surface, thus there are no magnetic monopoles or sources of magnetic charge.

3.2 Electrical parameters:

There are many electrical parameters of a coil: coil resistance, coil self-inductance and magnetic field intensity, mutual inductance between coils, skin and proximity effects. We will discuss all in details one by one.

3.2.1 Coil resistance:

The resistance of a primary as well as the secondary coils for WPTS must be calculated since they constitute the fundamental constraint for such power that may be transmitted. In fact, the transformer's efficiency would be 100% if the resistances were zero. Joule's effect is brought about by coil resistances; therefore, this heat needs to be maintained under control. Therefore, while designing WPTS, minimising Joule's losses is crucial for both efficiency and transferable power.

$$R = \frac{\rho l}{A_{wire}} \qquad \text{3-5}$$

Here l is coil length,

A_{wire} is wire area,

ρ is resistivity of material.

3.2.2 Inductance and magnetic induction:

Even though a two-terminal coil stores electricity inside its magnetic field, it is still not an active electrical component. A field of magnetic attraction that develops around and becomes linked to a current-carrying wire is what causes inductance (L). A magnetic flux proportional to the conductor's current flows through it. When the magnetic flux changes due to a change in the current, the conductor produces a powerful charged force also known as EMF that resists the current change. Inductors actively resist changes in the current that passes through them. Inductance (L) is a unit of measurement for the amount of electromotive force produced per unit change in electrical current.

A magnetic flux density is looking to be a field formed by a current flowing through a material that conducts electricity. The magnetic permeability of the medium in which the conductor is placed brings it together into a magnetic field H.

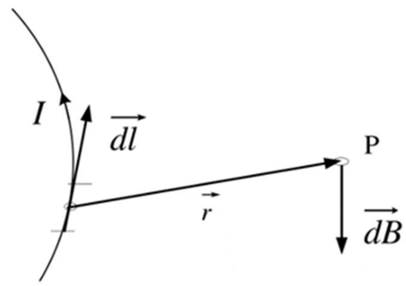

Figure 3-1Biot-Savart Law

The coupling coefficient of such coils is playing an important role in determining the performance of a WPTS. It is determined by the size, shape, and distance between the coils. In some circumstances, it may be determined analytically using the Biot-Savart rule, which yields the magnetic induction component dB created by the current I circulating in the infinitesimal section dl of such an electric circuit in distance r from P for any point P in free space.

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{I} \times \vec{r}}{r^3} \quad 3\text{-}6$$

Magnetic induction is obtained by integrating the whole circuit. Magnetic induction, for example, has the same direction as the direction of motion and a force magnitude equal to B at a point corresponding to the axis of a circular coil composed of n turns and located d from the coil's centre.

$$B = \frac{\mu_0}{4\pi} nI \frac{2\pi R^2}{(R^2 + d^2)^{3/2}} = \frac{\mu_0}{2\pi} nI \frac{A}{(R^2 + d^2)^{3/2}} = \frac{\mu_0}{2\pi} nI \frac{A}{d^3} \quad 3\text{-}7$$

Here,

R denotes the radius of a coil,

A denotes its cross-section, and

d denotes the spacing between coils.

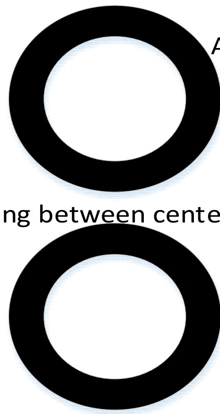

Figure 3-2 Coil spacing

Only when distance from the coil is significantly more than the coil radius, the expression of B in equation 3-7 holds. If the amount of inductive magnetism is unaltered across the cross-sectional area of the coil, self-inductance of the coil is given by [5]. The radius of the wire is denoted by r.

$$L = \mu_0 N^2 R \left[\ln\left(\frac{8R}{r}\right) - 2 \right] \qquad 3\text{-}8$$

3.2.3 Coefficient of mutual induction:

The phenomenon known as mutual induction happens when a voltage is induced in a neighbouring coil by a changing current in one coil. Figure 3-3 shows basic idea of mutual induction concept. In WPT devices, energy is wirelessly transferred from a transmitter coil to a receiver coil based on this phenomena.

The amount of mutual induction between two coils is measured by the coefficient of mutual induction, which is commonly represented by the sign M. Its definition is the relationship between the rate at which the current in one coil changes and the induced electromotive force (EMF) in the other.

The coefficient of mutual induction (M) can be found by following equation.

$$M = \frac{N_2 \emptyset_{21}}{i_1} \qquad 3\text{-}9$$

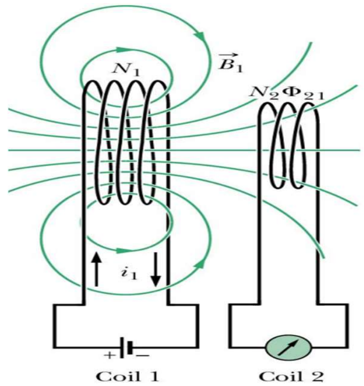

Figure 3-3 Mutual induction

3.2.4 Proximity and skin effect:

The magnetic flux will be greater in the core of a conductor than on the outside if the conductor is made up of one or more concentric circular portions. As a result, a self-induced back electromotive force would be stronger closer to the conductor's centre, resulting in a lower current density near the conductor's core relative to the conductor's surface. The skin effect occurs when the concentration at the surface increases, resulting in an increase in the conductor's effective resistance.

The level where the current density drops to 1/e, or around 0.37, of current density at the periphery, is known as the skin depth δ. Moreover, it is given by

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \qquad 3\text{-}10$$

The proximity effect, which is linked to the magnetic fields between two conductors that are adjacent to one another, also causes an increase in effective resistance. If both conductor halves are conducting current in the same direction, the magnetic flux between them is larger than the magnetic flux between them. As a result, the current distribution isn't really uniform over the cross-section, with the distant half carrying a larger share. The portions that are close together will carry a higher current density if such currents are flowing in opposing directions.

3.3 Basics of Coil Design:

Complete block diagram is shown in the Figure 3-4 below. It includes rectifier, filter, high frequency inverter, coupling coils, rectifier and battery. Other details are included in chapter:5.

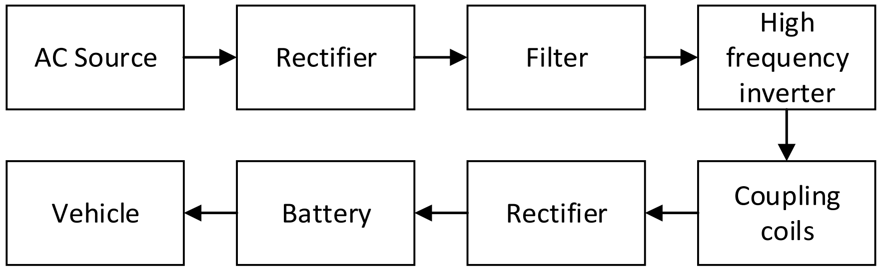

Figure 3-4 Block diagram

A wireless power transfer system is comprised of a coupling device having two or more than two coils, the configuration of which is critical in the construction of a WPTS. This chapter describes the technique for designing the coils of a wireless power transfer system (WPTS) designed to recharge the battery packs of an electric vehicle. The square and spiral connection architectures were studied. The ANSYS simulation has been used to determine all inductive characteristics as dependent variables of coil distance, turn quantity, turn proximity (for spirals only), and axial misalignment.

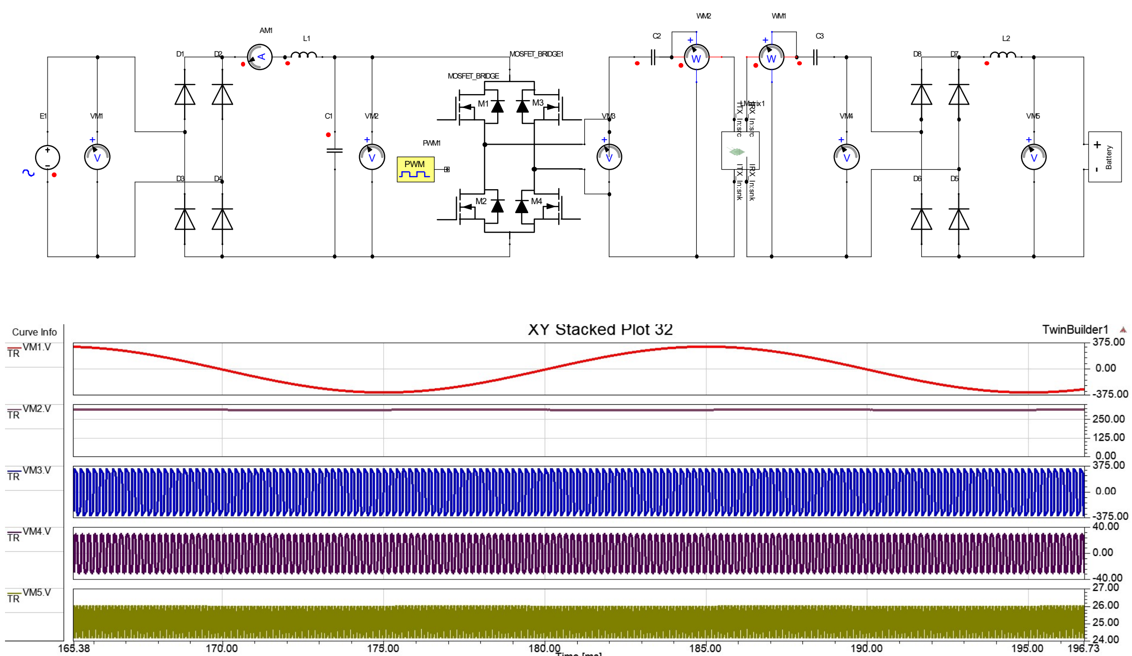

Figure 3-5 ANSYS Twinbuilder circuit and relevant waveforms

In order to construct a WPT system having resonant topology of charging the rechargeable battery of the electric vehicle used as the research case, coil-coupling architectures are examined. This data, which is provided here are used to establish the WPT requirements shown in Table 3-1.

Parameter	Symbol	Value
Nominal mains voltage	V_N	230 Volts
Nominal output	P_N	960 Watts
Resonance frequency	f	53.1 kHz
Radius of coil	r	16 cm
Misalignment between coils	d	0-20 cm

Table 3-1 WPT specifications.

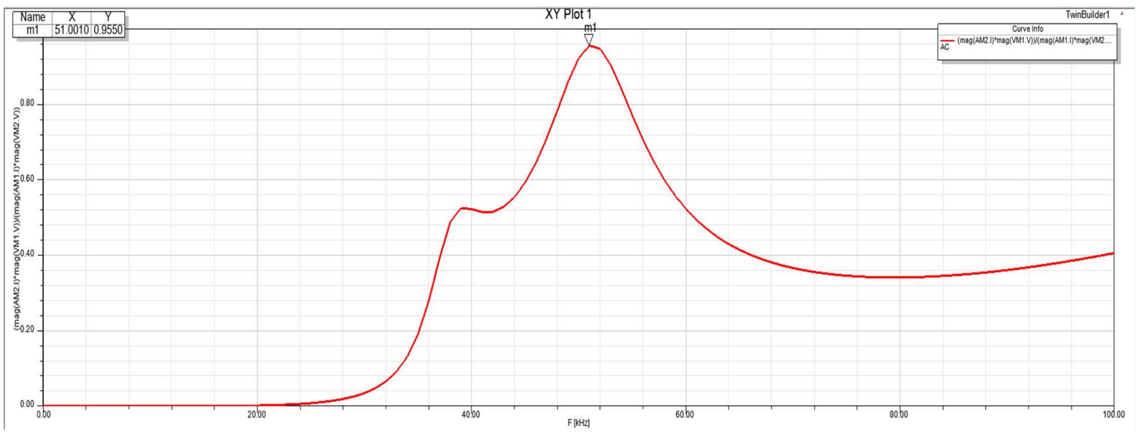

Figure 3-6 Frequency vs efficiency graph

To charge the electric vehicle at 230 V from a single-phase home outlet yields the nominal supply voltage. Maximum output power is determined by the battery pack's having maximum voltage (48 V) and charging current (20 A). The city-car dimensions for the receiver coil radius as well as coil distance are followed. However, the requirement for the operating frequency stems from a recent SAE International J2954 Task Force resolution for WPT to establish the nominal frequency of operation at 53.1 kHz.

The WPTS transmitting coil is fed by an oscillating high-frequency (HF) inverter, which is powered by a rectifying device connected to the utility grid. A cascaded DC-DC converter that takes electricity from the receiving coil changes the rectified voltage to fit the demands of the battery.

$$R_L = \frac{8}{\pi^2} R_0 \qquad 3\text{-}11$$

Here R_0 is outer radius.

In order to maintain excellent performance in general, the HF inverter's output voltage amplitude should not be significantly lower than that corresponding to the altered grid voltage. In addition to the other half of the WPTS, the rectified voltage now at the DC-DC converter's input should not be significantly greater than the battery voltage.

The standard for Q_R may be simply converted into the specification for L_R using the comparable R_L and working frequency. While we don't yet know the value of M, we may compute the normalized load current ($\bar{I}_{RL}$) from the supplied values of Q_T and Q_R using the terms in brackets in formula (3-12).

$$\bar{I}_{RL} = \frac{\bar{V}_s}{j\omega_r M}\left(\frac{k^2 Q_T Q_R}{1 + k^2 Q_T Q_R}\right) = \frac{\bar{V}_s}{j\omega_r M}\eta_r \quad 3\text{-}12$$

The lowest and utmost values of $\bar{V}_s$ ought to fall below or equal to 230 V in accordance with the aforementioned specifications, taking into account fluctuations with in grid voltage and voltage drops throughout the circuitry. Hence, 48 V is taken into consideration as the initial value of V_{DC}, and 5.8 to 580 m Ohms are the lowest and greatest levels of battery resistance. These preliminary readings allow for the evaluation of the rectified peak values of currents.

$$I_{R,pk} = \frac{4}{\pi}\frac{V_{DC}}{R_L} \quad 3\text{-}13$$

Using above equation, the value comes to be 0.1 A≤ $I_{R,pk}$≤20 A.

$$V_{S,pk} \leq \omega M I_{R,pk} = \frac{4}{\pi}\frac{V_{DC}}{R_L}\omega M \quad 3\text{-}14$$

In the resonance situation, its nominal resonant frequency, V_{DC}, $V_{S,pk}$, and minimum value of R_L may be taken into consideration. In addition, the value of mutual inductance can be derived for such minimum distance of 0.20 cm, and it should not be greater than 35 H. The transmitter maximum current may be estimated using this restriction is given by equation no. 3-15, 3-16 and 3-17.

$$I_{T,pk} = \frac{4}{\pi}\frac{V_{DC}}{\omega M} \quad 3\text{-}15$$

$$\omega^2 C_T = \frac{1}{L_T} \quad 3\text{-}16$$

$$\omega^2 C_R = \frac{1}{L_R} \quad 3\text{-}17$$

The self-inductance (L) of a coil is a measure of the ability of the coil to generate an electromotive force (EMF) in itself when the current through it changes. It depends on various factors such as the number of turns in the coil, the geometry of the coil, the material around the coil, and the presence of a core material.

The formula to calculate the self-inductance of a coil is given by:

$$L = \frac{\mu N^2 A}{l} \qquad \text{3-18}$$

Value of this current is found around 20 A.

Parameter	**Symbol**	**Value**
Minimum battery voltage	V_{Bm}	36 V
Maximum battery voltage	V_{BM}	48 V
Charging current of battery	I_B	20 A
Radius of coil	r	16 cm
Misalignment	D	0-20 cm
Resonance frequency	f	53.1 kHz
Efficiency of coupling	η	>0.95
Minimum supply voltage	V_{Sm}	100 V
HF rectifier output voltage	V_{DC}	56 V
Minimum equivalent R_L	R_{Lm}	5 Ohms
Maximum load current	I_{RLM}	20 A

Table 3-2 Analytical values

The self-inductance (L) of the transmitting and receiving coil is calculated to be about 120 H using standard electrical engineering equation and data on mutual inductance as in 3-18. According to equations 3-16 and 3-17, capacitance (C_T and C_R) equals 29.71 nF when the capacitive and inductive voltages are equal in the resonance state. The results of the above-designed approach are presented numerically in Table 3-2.

3.4 Types of coils used for EV charging:

In WPT, two distinct kinds of coil structures usually are used. They are helix as well as spiral coils, and from this point forward it is assumed that the dimensions, geometries, and number of turns of the transmitter and reception coils are equivalent.

3.4.1 Helix coil:

The helix configuration of coil is sketched in the left section of Figure 3-1(the wire radius is not scaled according to the coil dimension). The turns are arranged in an elongated shape with a base of b and a height of h, and a mean separation of R_m from the coil axis. Each coil's self-inductance can be expressed as [37,39].

Here N is the quantity of turns. By repeatedly using and adjusting its characteristics, it was found that a spiral coil with 21 turns that is arranged in a helix pattern meets the requirements for becoming self-inducting and allows for a workable wire radius of 2.5 mm.

$$L_{Helix} = \frac{0.33(R_m N)^2}{6R_m + 9h + 10b} \quad \text{3-19}$$

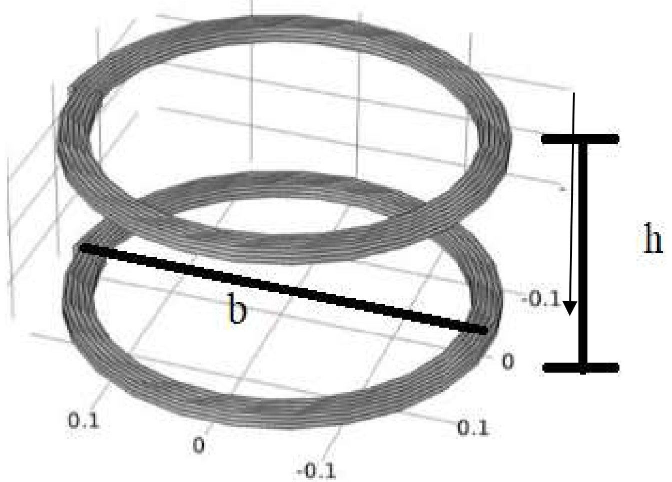

Figure 3-7 Helix coil

Figure 3-7 depicts how two aligned spiral coils are arranged. A spiral self-inductance of coils can be expressed as [6].

$$L_{spiral} = C_1 \mu_0 N^2 R_{avg} \left[\ln\left(\frac{C_2}{\emptyset}\right) + C_3\emptyset + C_4\emptyset^2 \right] \quad \text{3-20}$$

R_{avg} is average of the coil radius and $\emptyset$ is fill factor.

$$R_{avg} = \frac{(R_0 + R_i)}{2} \quad 3\text{-}21$$

$$\emptyset = \frac{R_0 - R_i}{R_0 + R_i} \quad 3\text{-}22$$

Here, R_0 and R_i are outer and inner coil radius, and coefficient C_i, where i=1,2,3… depends upon circular layout.

3.4.2 Circular coil:

Plane planner spiral coils with a high coupling coefficient and a high-quality factor are utilized for wireless power transmission. As per Figure 3-8, D_{max} is maximum outer diameter of the spiral coil, D_{min} is the lowest inner diameter of the spiral coil, N is number of spiral coils turns, S is the distance between coil turns, and W is conductor diameter.

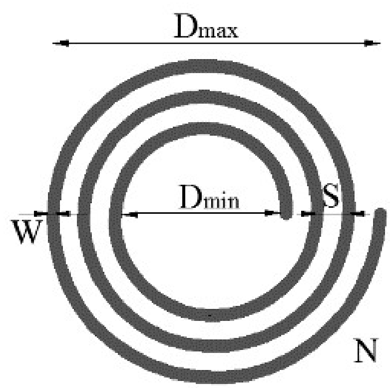

Figure 3-8 Circular coil (theoretical)

For mathematical analysis point of views, following are mainly dimensions that are to be consider from the geometry of WPT spiral coils.

$$D_{max} = D_{min} + (2N - 1)(S + W) + 2W \quad 3\text{-}23$$

$$B = \frac{D_{max} - D_{min}}{D_{max} + D_{min}} \quad 3\text{-}24$$

$$D_{avg} = \frac{D_{max} + D_{min}}{2} \quad 3\text{-}25$$

The distance between two turns should be calculated by considering following conditions.

$$S < S_{1max} = \frac{D_{lim} - 2W}{2N - 1} - W \quad 3\text{-}26$$

$$S < S_{2max} = \frac{2D_{lim} - (2W + 4r_{avg})}{2N - 1} - W \quad \text{3-27}$$

Where, S_{1ma} is allowable maximum coil internal spacing.

$$L = \mu_0 N^2 r_{avg} \left(ln \frac{2.46}{\beta} + 0.2\beta^2 \right) \quad \text{3-28}$$

$$R = R_0 + R_a \quad \text{3-29}$$

$$R_a = 320\pi^4 N^2 \left(\frac{\pi r_{avg}{}^2}{\lambda^2} \right)^2 \quad \text{3-30}$$

$$R_0 = \left(\frac{\mu_0 \omega}{2\sigma} \right)^{\frac{1}{2}} \frac{N r_{avg}}{a} \quad \text{3-31}$$

$$Q = \frac{\omega L}{R} \quad \text{3-32}$$

Here,

L – Spiral coil Inductance (Henrys)

R – Equivalent resistance (Ohms)

R_0 – Ohmic resistance (Ohms)

R_a – Equivalent radiation resistance (Ohms)

Q – Quality factor,

σ– Conductivity of the wire,

β– Filling rate,

a– Radius of wire and

ω– Angular frequency

3.4.3 ANSYS simulation model:

Designing a circular coil having 21 turns in ANSYS Electronics Desktop entails building a three-dimensional (3D) model for the coil utilizing the potent of program electromagnetic simulation capabilities. To construct a circular coil containing 21 turns using ANSYS Electronics Desktop, follow these simple steps:

- Launch the ANSYS Electronics Desktop application on your PC to begin using it.

- Geometry creation: Start by making the circular coil's 3D geometry. The software's geometry tools can be used to create a circle that represents the coil's cross-section. Include all necessary dimensions, including height, inner and outer radii, and any other pertinent information.
- Create the Coil Turns: Using the appropriate tool in ANSYS Electronics Desktop, you may extrude the 2D shape along the chosen axis to produce the 3D coil with a variety of turns once you have the 2D cross-section of the coil. Make sure that the turns are appropriately spaced and oriented, and specify the number of turns (in this case, 21 turns).
- Material Assignment: Give the coil the proper material characteristics. You can select several materials for the coil to correctly replicate its behaviour and electromagnetic properties depending on your particular application.
- Get the simulation ready: Define the simulation configuration. This entails defining the simulation parameters, the frequency of interest, and the type of analysis (such as transient, frequency domain, or steady-state).

  Apply the proper boundary conditions to the model in order to appropriately depict how the coil interacts with its surroundings. Open boundaries, boundaries of perfect electric conductors (PECs), and other specific circumstances might be among these situations.
- Make a mesh for the coil model by meshing. For usage in numerical simulation, the geometry is discredited by the mesh into smaller components. Make sure the mesh is fine enough to accurately capture the features of the coil.
- Solve the Model: Run the simulation to find the solutions to the electromagnetic field equations. The electromagnetic fields will be calculated throughout the coil's construction, including the magnetic flux density, magnetic field strength, and other pertinent values.

  Post-Processing: Examine and display the outcomes of the simulation. You may observe the coil's electromagnetic behaviour, encompassing field distributions, inductance, and other pertinent parameters, using the post-processing tools provided by ANSYS Electronics Desktop.

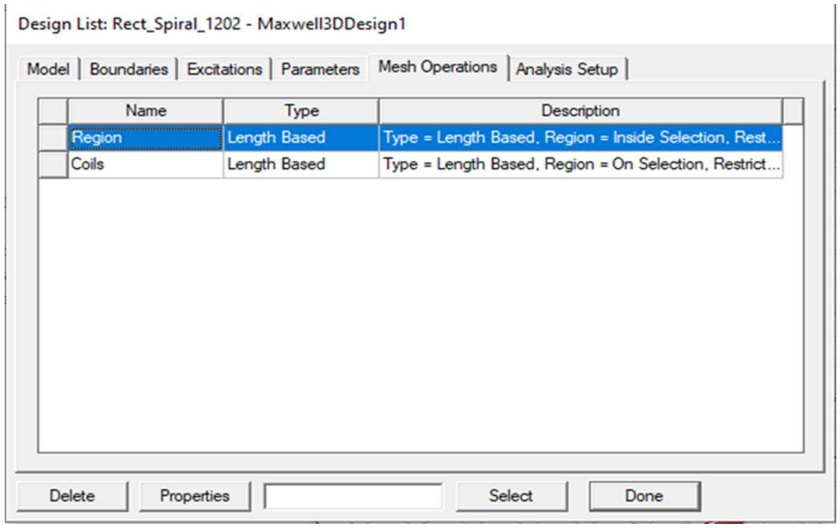

Figure 3-9 Design list parameters

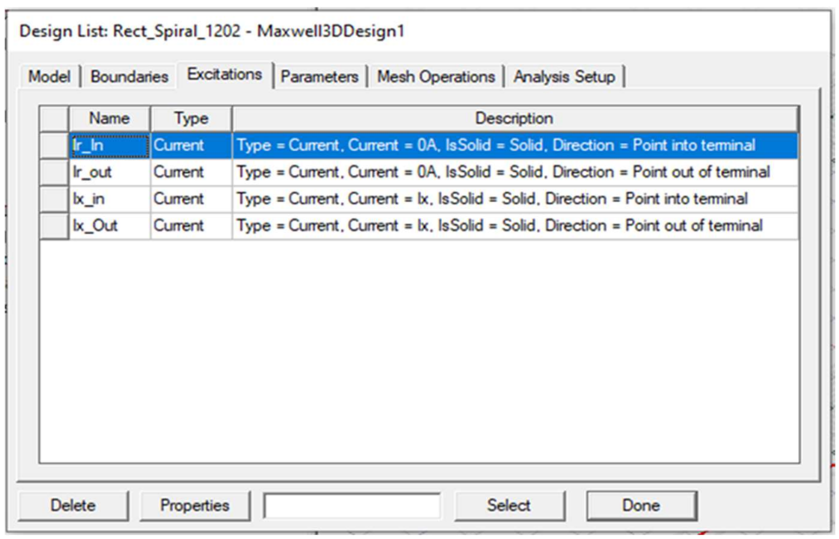

Figure 3-10 Design list: Region & Coils

Using ANSYS Electronics Desktop, you can design as well as simulate circular coils with 21 turns while learning important details about its electromagnetic behaviour and effectiveness. The outcomes of the simulation can be used to improve the coil's design for particular uses such wireless power transfer, heating by induction, or investigations into electromagnetic compatibility. Figure 3-11 shows circular coil model in ANSYS top view and Figure 3-12 shows ANSYS 3D model in isometric view.

When two coil are coaxially coupled, its mutual inductance is calculated by

$$M = \frac{\mu_0 \pi N_1 N_2 {r_{1avg}}^2 {r_{2avg}}^2}{2\left(D^2 + {r_{1avg}}^2\right)^{1.5}} \qquad 3\text{-}33$$

The system working parameters are given in the following in Table 3-3.

V	**f (kHz)**	**N**	**ZDist (mm)**	**P (W)**	**η (%)**
230	51	21	15	3350	95.5

Table 3-3 System working parameters

The equation for calculating power product is as follows:

$$P_\eta = P.\eta = \frac{V^2\omega^4 M^4 {R_L}^2}{Z_2(Z_1 Z_2 + \omega^2 M^2)^3} \qquad 3\text{-}34$$

Here, Z_1 *and* Z_2 are impedance of transmitting and receiving coil respectively.

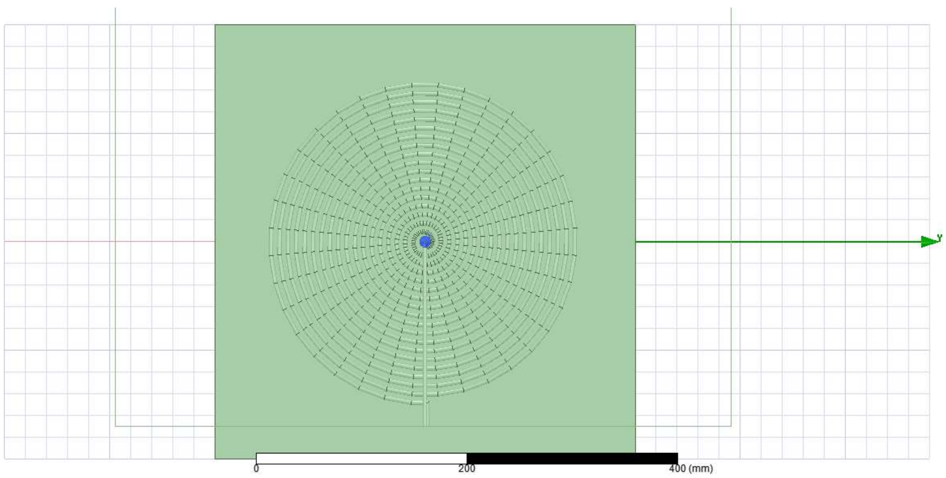

Figure 3-11 Circular coil ANSYS top view

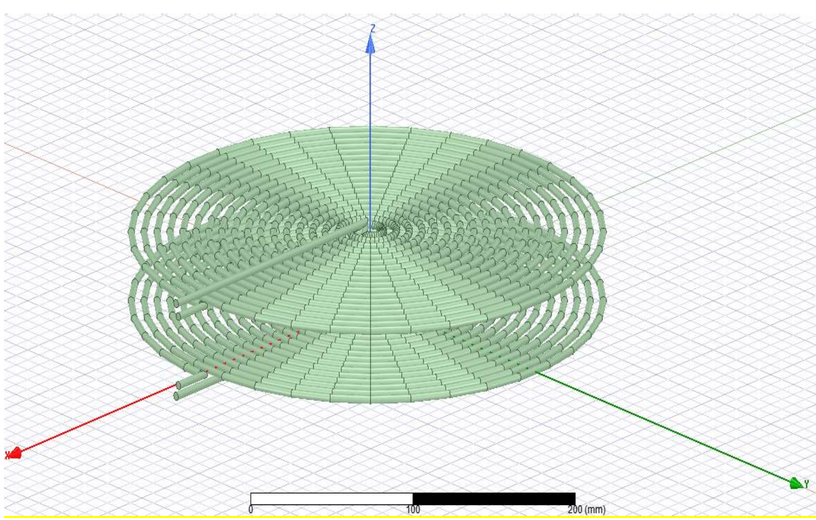

Figure 3-12 ANSYS isometric view

By considering the effect of inter-turn voltage and insulation level of the wire for air breakdown, numbers of turns of the coil are determined using wire diameter and overall structure.

3.4.4 Square coil:

With an equal distance across the magnetic coupling coils with the same set of parameters, it was demonstrated that the square coil had a higher mutual inductance as the other coil forms. The internal characteristics of the square coil were chosen as the research objectives within the condition of the identical area of the windings in order to further investigate the elements that affect the mutual inductance between the primary coil with secondary coil. The coupling coefficient is examined at various air gaps in order to investigate the impact of internal factors on the coupling coefficient.

As shown in Figure 3-13, the fundamental dimensions associated with such a spiral are N, w, s, d_i, and d_o, which correspond to the quantity of turns, cross-sectional width, distance between successive turns, inner side length, and outer side length within the spiral, respectively.

The equation for the outer most length (d_o) can be found by following equation,

$$d_0 = d_i + 2[wN + s(N - 1)] \qquad \text{3-35}$$

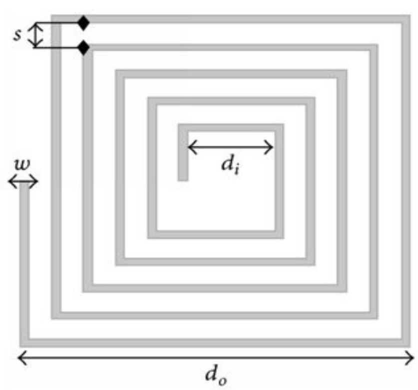

Figure 3-13 Square coil dimensions

And in a square coil, the self-inductance of the coils can be found by

$$L = 2.34\mu_0 \times \frac{N^2 . d_{avg}}{1 + 2.75 . \emptyset'} \qquad \text{3-36}$$

Here,

$$d_{avg} = \frac{d_0 + d_i}{2}$$

$$\emptyset' = \left(\frac{d_0 - d_i}{d_0 + d_i}\right)'$$

As we are using same coil for both the primary and secondary winding, the mutual inductance can be calculated by

$$M = \frac{2\mu_0 N^2}{\pi}\left[dL_n\left(\frac{d_{avg} + b}{d_{avg} + x} \times \frac{b}{h}\right) + (x - 2b + h)\right] \quad \text{3-37}$$

Where,

$$\mu_0 = 4\pi \times 10^{-7}\, {}^{H}\!/_{m}$$
$$b = \left({d_{avg}}^2 + h^2\right)^{1/2}$$
$$x = \left(2{d_{avg}}^2 + h^2\right)^{1/2}$$

And h is the distance between the coil.

By using ANSYS simulation model design procedure, one can again design 3D model in ANSYS Electronics Desktop. The resultant design ANSYS model of square planner coil is shown in Figure 3-14.

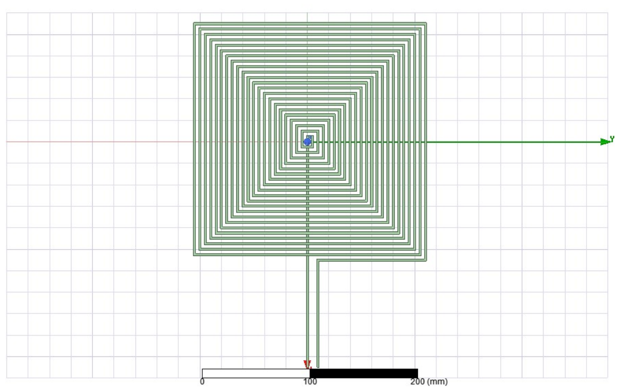

Figure 3-14 ANSYS model of planner square coil

The square construction of a coil is proposed in this section of research. This shape is highly influenced by the perimeter and rather moderately by the loop area and wire radius. As a result, the inductance of complex forms is sometimes well approximated by just a simpler shape with a comparable perimeter and/or area. There is no closed-form formula for calculating the inductance of such a generic wire polygon. Grover et al. developed a number of examples for polygons of s side lengths and R wire radius:

With the following formula, we can simply determine inductance.

$$L = \frac{2\mu_0 s}{\pi}\left[ln\left(\frac{s}{R}\right) - 0.52401\right] \qquad 3\text{-}38$$

The coil-coupling of a WPT system for charging an electric vehicle has been the subject of this chapter. The design of the coil system is initially discussed, followed by an examination of how various coil configurations affect the performance of coil with regard to inductive parameters. Both the coil topologies (circular and square) are analytically described and mathematical models are presented in this chapter 3D modelling procedure and models are formed using ANSYS Electronics Desktop.