PART - I

CHAPTER - II

ESTIMATION OF THE FRACTION DEFECTIVE IN CURTAILED SAMPLING PLANS BY THE METHOD OF MOMENTS

2.1 In this Chapter we introduce Curtailed Sampling Plans, giving the scope for curtailment of inspection, the statements of the plans considered, the definition of random variables associated with these plans, etc. Two situations associated with reporting of the inspection results are described and the estimates of the fraction defective by the method of moments are obtained under the two Situations in these Plans.

2.2 The usual single sampling plan by attributes is defined by three numbers (i) the lot size N, (ii) the sample size n, (iii) the acceptance number c and the decision rule - accept the lot if the number of defectives in the sample is equal to or less than the acceptance number c, otherwise reject it. In this plan the number of articles to be inspected, for deciding whether a lot is to be accepted or rejected, is fixed. One may however know, at a certain inspection stage, before all the units of the sample are inspected, whether a lot is going to be rejected or accepted. This may happen at the end of a certain number of inspections, which is less than the fixed size of the sample, irrespective of the result of inspection of the remaining units, and we may curtail the inspection earlier. The curtailing can be done at the rejection stage or at the acceptance stage. Let us consider a case of curtailing at the rejection stage for n=30, c=4. Suppose the fifth defective appears at the tenth inspection, where the articles are inspected one by one. Then, irrespective of the results thereafter, it is certain that the lot is to be rejected and we can curtail inspection at this stage. As our second example, we consider the case of curtailing at the acceptance stage for the same values of n and c as above. Suppose there is no defective article found among the first twentysix articles inspected. Then one can decide to accept the lot without inspecting the remaining articles and we can curtail inspection at this stage.

The use of curtailed sampling is not always desirable. In the case of 100% inspection of the rejected lots, the question of curtailing at the rejection stage does not arise. It may be desirable to use curtailed sampling when it is necessary to know only whether a lot is to be accepted or rejected or when the inspection is destructive or expensive.

Thus, we can have curtailment in the inspection at the rejection stage or at both the rejection and acceptance stages. The former situation is accounted in Plan 2 and the latter in

Plan 3. The usual Single Sampling Plan is defined as Plan 1. We summarize all the above information in the following statements of the plans.

2.2.1 Plan 1 : Inspect a random sample of n units from the lot. Accept the lot if there are fewer than k defectives. Reject the lot if there are k or more defectives.

Plan 2: Inspect randomly selected units of the lot one at a time until either k defectives have been observed or until n units have been inspected. Reject the lot if k defectives are observed. Accept the lot if n units are inspected, provided that the number of defectives observed in them is less than k.

Plan 3: Inspect randomly selected units of the lot one at a time until either k defectives have been observed or n-k+1 nondefectives have been observed. Accept the lot if there are n-k+1 nondefectives. Reject the lot if there are k defectives.

In all these plans k and n are predetermined numbers. In general, k will be much less than n. k is known as the rejection number and is related to the acceptance number c by the relation c=k-1. The minimum value of k is 1 in Plans 1 and 2, and 2 in Plan 3. Plan 3 reduces to Plan 2 for k=1.

Patil [48] has shown that the determination of k and n will be the same for all the three plans, for given producer's and consumer's risks, since the probability of acceptance of a lot is the same for all the three plans.

2.3 We define the discrete random variables x,y,z, and i in the following way:

x = number of defectives in an inspected articles.

y = number of articles inspected when the kth defective is found.

z = number of articles inspected when the (n-k+1)th nondefective is found.

i = number of defectives found when sampling is curtailed by the finding of the (n-k+1)th nondefective.

We note that $i=z-(n-k+1)$ ...(2.1)

Further, we define a discrete random variable s which takes the values 0,1,..,k-1 when a lot is accepted and the values k,k+1,...,n when a lot is rejected. In Plan 1, s is associated only with x. In Plan 2, s is associated with x for s=0,1,...,k-1 and is associated with y for s=k, k+1,...,n. In Plan 3, s is associated with i for s=0,1,...,k-1 and is associated with y for s=k, k+1,...,n.

We have assumed that the fraction defective (p) remains constant over the entire production run. Further, we have assumed that the lot size is large (preferably $N > 10n$)

implying that the probability of any inspected article of the lot to be defective is p. This applies to the Type B situation of Dodge and Romig [19(b)] .

Then the probability distributions of the random variable s in the respective plans are as follows:

Plan 1:

$$f_1(s)=\begin{cases}\binom{n}{x} p^x q^{n-x}, & s=x=0,1,..,k-1\\[2ex] \binom{n}{x} p^x q^{n-x}, & s=x=k,k+1,..,n.\end{cases} \qquad ...(2.2)$$

Plan 2:

$$f_2(s)=\begin{cases}\binom{n}{x} p^x q^{n-x}, & s=x=0,1,.,k-1\\[2ex] \binom{y-1}{k-1} p^k q^{y-k}, & s=y=k,k+1,..,n.\end{cases} \qquad ...(2.3)$$

Plan 3:

$$f_3(s)=\begin{cases}\binom{n-k+i}{n-k} q^{n-k+1} p^i, & s=i=0,1,..,k-1\\[2ex] \binom{y-1}{k-1} p^k q^{y-k}, & s=y=k,k+1,..,n ,\end{cases} \qquad ...(2.4)$$

where q=1-p and $0 \leq p \leq 1$.

The first part of each of the $f_j(s)$ for j=1,2,3, represents the situation associated with the acceptance of a lot and the second part represents the situation associated with the rejection of a lot. Thus $\sum_{s=0}^{k-1} f_j(s)$ for j=1,2,3 gives the probability of acceptance of a lot and $\sum_{s=k}^{n} f_j(s)$

for j=1,2,3 gives the probability of rejection of a lot. Furthermore, since a lot will either be accepted or rejected it follows that

$$\sum_{s=0}^{k-1} f_j(s) + \sum_{s=k}^{n} f_j(s) = 1, \quad j=1,2,3 \qquad \ldots(2.5)$$

Then (2.5) satisfies the necessary condition for $f_j(s)$ to be a probability function. Alternatively, (2.5) is evident for j=1, since it represents the sum of a binomial probability function and for j=2,3, equation (2.5) is true due to the following identity between the binomial and inverse (negative) binomial distributions.

$$\sum_{x=k}^{n} \binom{n}{x} p^x q^{n-x} = \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k} \qquad \ldots(2.6)$$

This equality was proved by Patil [47] and Morris [44].

2.4 We now consider the Situations A and B connected with the mode of reporting of the results of sampling inspection.

2.4.1 Situation A: This Situation takes into consideration the complete information of the results of sampling inspection that one would have on hand legitimately. In Plan 1, the complete information of the inspection is the number of defectives in n inspected articles irrespective of the fact whether a lot is accepted or rejected. In Plan 2, it is (i) the number of defectives in n inspected articles when a lot is accepted and (ii) the number of articles inspected when the inspection is curtailed by finding the kth defective. In Plan 3, it is (i) the number of defectives found when the inspection is curtailed by finding the (n-k+1)th nondefective, and (ii) the number of articles inspected when the inspection is curtailed by finding the kth defective. A lot is accepted if (i) happens and is rejected when (ii) happens.

2.4.2 Situation B: This Situation takes place when censored information of Type I is reported. Four cases arising from four different modes of reporting the results of sampling inspection associated with only curtailed sampling plans are considered:

Case I: The inspector acting under Plan 2 reports whether a lot is accepted or rejected as also the number of defectives observed.

Case II: The inspector acting under Plan 2 reports whether a lot is accepted or rejected as also the number of articles inspected.

Case III: This case is similar to Case I except that the inspection is carried out according to Plan 3.

Case IV: The inspector acting under Plan 3 reports whether a lot is accepted or rejected as also the number of nondefectives observed.

Although Cases I and III look similar, the maximum likelihood estimates of the fraction defective are not similar, as will be seen later in Chapter III. On the other hand, Cases II and IV yield similar maximum likelihood estimates, although there is no apparent similarity. In the method of moments, the estimates of fraction defective are different for all the Cases.

2.5 We shall now consider the probability functions of the reported observational character in both the Situations A and B.

2.5.1 Probability functions under Situation A are the same as those given earlier, namely, (2.2), (2.3), and (2.4).

2.5.2 In Situation B, the observational character reported in each case is conveniently represented by a discrete random variable as follows:

t = number of defectives observed in the sample when inspection is carried out according to Plan 2,

u = number of articles inspected when inspection is carried out according to Plan 2,

v = number of defectives observed in the sample when inspection is carried out according to Plan 3.

w = number of nondefectives observed in the sample when inspection is carried out according to Plan 3.

t, u, v, and w appear respectively in Cases I, II, III and IV.

Recalling the physical meaning of k and n, the predetermined constants of the sampling plans, we determine for the Situation B the various values attained by t,u,v, and w. t assumes the values 0,1,..,k-1 when a lot is accepted and k when a lot is rejected. u assumes the value n when a lot is accepted and the values k, k+1, ...,n when a lot is rejected. v assumes the values 0,1,2,..,k-1 when a lot is accepted and k when a lot is rejected. w attains the value n-k+1 when a lot is accepted and the values 0,1,...,n-k when a lot is rejected. The probability functions of t,u,v, and w are as follows:

Case I :

$$g_1(t)=\begin{cases} \binom{n}{t} p^t q^{n-t}, & t=0,1,..,k-1 \\ \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k}, & t=k \end{cases} \qquad \ldots(2.7)$$

Case II :

$$g_2(u)=\begin{cases} \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}, & u=n \\ \binom{u-1}{k-1} p^k q^{u-k}, & u=k,k+1,..,n. \end{cases} \qquad \ldots(2.8)$$

Case III:

$$g_3(v)=\begin{cases} \binom{n-k+v}{n-k} q^{n-k+1} p^v, & v=0,1,...,k-1 \\ \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{v-k}, & v=k \end{cases} \qquad \ldots(2.9)$$

Case IV :

$$g_4(w)=\begin{cases} \sum_{i=0}^{k-1} \binom{n-k+i}{n-k} q^{n-k+1}\, p^{i}, & w=n-k+1 \\ \binom{w+k-1}{k-1} p^{k} q^{w}, & w=0,1,\ldots n-k \end{cases} \qquad \ldots(2.10)$$

The first part of each of the probability functions represents the situation associated with the acceptance of a lot and the second part represents the situation associated with the rejection of a lot. This may clarify the ambiguity regarding the repetition of u=n in $g_2(u)$. One assigns the probability given by the first part of $g_2(u)$ when it is reported that u=n and the lot is accepted under Plan 2. Furthermore, one assigns the probability given by the second part of $g_2(u)$ when it is reported that u=n and the lot is rejected under Plan 2.

2.6 In both the Situations, Situations A and B, our estimation process is based on the inspection of several lots (any number of lots will do). We assume that T lots have undergone the inspection under one of the plans and the information of the inspection supplied belongs to either Situation A or B. Each accepted or rejected lot will give one observation associated with x, y, or i in Situation A and with t,u,v or w in Situation B. In Situation A under Plan 3, the inspector may record either i or z, since these quantities are related by linear expression (2.1).

2.6.1 In Situation A, let the observed frequencies associated with x,y,z or i be as follows:

$a_{x,j}$ = number of accepted lots under Plan j in which the number of defectives in the sample was x(j=1,2).

$r_{x,1}$ = number of rejected lots under Plan 1 in which the number of defectives in the sample was x.

$r_{y,j}$ = number of rejected lots under Plan j in which the number of articles in the sample was y(j=2,3).

$a_{z,3}$ = number of accepted lots under Plan 3 in which the number of articles in the sample was z.

$a_{i,3}$ = number of accepted lots under Plan 3 in which the number of defectives in the sample was i.

Then the total number of accepted lots, $T_{a,j}$(j=1,2,3), and the total number of rejected lots $T_{r,j}$(j=1,2,3) will be in the respective plans :

Plan 1

$$T_{a,1} = \sum_{x=0}^{k-1} a_{x,1} \qquad T_{r,1} = \sum_{x=k}^{n} r_{x,1}$$

Plan 2

$$T_{a,2} = \sum_{x=0}^{k-1} a_{x,2} \qquad T_{r,2} = \sum_{y=k}^{n} r_{y,2}$$

Plan 3

$$T_{a,3} = \begin{cases} \sum_{i=0}^{k-1} a_{i,3} & \text{when the random variable i is recorded} \\ \sum_{z=n-k+1}^{n} a_{z,3} & \text{when the random variable z is recorded.} \end{cases} \qquad T_{r,3} = \sum_{y=k}^{n} r_{y,3}$$

Note that $T = T_{a,j} + T_{r,j}$ for j=1,2,3.

Further, in the notation of the classical theory of estimation based on fixed sample sizes, we have T observations on the random variable s from one of the populations $f_j(s)$ (j=1,2,3). For example, in Plan 2 s takes the value x $a_{x,2}$ times and it takes the value y $r_{y,2}$ times, making a total of T s's.

2.6.2 In Situation B, let the observed frequencies associated with t,u,v and w, variables be as follows:

$a_{t,2}$ = number of accepted lots under Plan 2 where the number of defectives in the sample is t(t=0,1,...,k-1),

$r_{u,2}$ = number of rejected lots under Plan 2 where the number of articles inspected in the sample is u(u=k,k+1,..,n),

$a_{v,3}$ = number of accepted lots under Plan 3 where the number of defectives in the sample is v(v=0,1,...,k-1),

$r_{w,3}$ = number of rejected lots under Plan 3 where the number of nondefectives in the sample is w(w=0,1,..,n-k).

Furthermore, out of the T lots, $T_{a,j}$ and $T_{r,j}$ the number of accepted and rejected lots under Plan j (j=2,3,) have the following relations;

$$T = T_{a,2} + T_{r,2} = \sum_{t=0}^{k-1} a_{t,2} + T_{r,2}, \qquad \text{for Case I:}$$

$$T = T_{a,2} + T_{r,2} = T_{a,2} + \sum_{u=k}^{n} r_{u,2}, \qquad \text{for Case II:}$$

$$T = T_{a,3} + T_{r,3} = \sum_{v=0}^{k-1} a_{v,3} + T_{r,3}\,, \qquad \text{for Case III;}$$

and $$T = T_{a,3} + T_{r,3} = T_{a,3} + \sum_{w=0}^{n-k} r_{w,3}\,, \qquad \text{for Case IV.}$$

It may be then noted that the observed frequencies for $t=k$, $v=k$, and $w=n-k+1$ are respectively $T_{r,2}$, $T_{r,3}$ and $T_{a,3}$ and for $u=n$ under the acceptance of a lot, $T_{a,2}$:

2.7 Estimation by Method of Moments:

2.7.1 Situation A: The first moment of the random variable s in each of the plans is as follows:

Plan 1:

$$E(s) = \sum_{s=0}^{k-1} s\, f_1(s) + \sum_{s=k}^{n} s\, f_1(s)$$

$$= \sum_{x=0}^{k-1} x \binom{n}{x} p^x q^{n-x} + \sum_{x=k}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= np \qquad \ldots(2.11)$$

Plan 2:

$$E(s) = \sum_{s=0}^{k-1} s\, f_2(s) + \sum_{s=k}^{n} s\, f_2(s)$$

$$= \sum_{x=0}^{k-1} x \binom{n}{x} p^x q^{n-x} + \sum_{y=k}^{n} y \binom{y-1}{k-1} p^k q^{y-k}$$

$$= np\, B(p, n-1, k-2) + (k/p)\left[1 - B(p, n+1, k)\right] \qquad \ldots(2.12)$$

Plan 3:

$$E(s) = \sum_{s=0}^{k-1} s\, f_3(s) + \sum_{s=k}^{n} s\, f_3(s)$$

$$= \sum_{i=0}^{k-1} i \binom{n-k+i}{n-k} q^{n-k+1} p^{i} + \sum_{y=k}^{n} y \binom{y-1}{k-1} p^{k} q^{y-k}$$

$$= (n-k+1)\, B(p,n+1,k-1)/q$$

$$+(k/p)\, [1-B(p,n+1,k)] - (n-k+1) B(p,n,k-1)$$

$$=J_4/p - (n-k+1)\, B(p,n,k-1) \qquad \ldots(2.13)$$

where $J_4 = (n-k+1)(p/q)\, B(p,n+1,k-1)$

$$+ k\,[1-B(p,n+1,k)] \qquad \ldots(2.14)$$

and $$B(p,n,k)= \sum_{x=0}^{k} \binom{n}{x} p^{x} q^{n-x} \qquad \ldots(2.15)$$

Furthermore, recalling the definitions of the observed frequencies $a_{x,1}$, $r_{x,1}$, $a_{x,2}$, $r_{y,2}$, $a_{i,3}$ and $r_{y,3}$ associated with the random variables x,y, and i, the observed arithmetic mean $(\bar{s})$ based on T observations in the respective plans is as follows:

Plan 1:

$$\bar{s} = \left[\sum_{x=0}^{k-1} x a_{x,1} + \sum_{x=k}^{n} x r_{x,1} \right] / T \qquad \ldots(2.16)$$

Plan 2:

$$\bar{s} = \left[\sum_{x=0}^{k-1} x a_{x,2} + \sum_{y=k}^{n} y r_{y,2} \right] / T \qquad \ldots(2.17)$$

Plan 3:

$$\bar{s} = \left[\sum_{i=0}^{k-1} ia_{i,3} + \sum_{y=k}^{n} yr_{y,3} \right] / T \qquad \ldots(2.18)$$

Then equating the r.h.s. of equations (2.16), (2.17), and (2.18) respectively to r.h.s of equations (2.11),(2.12), and (2.13) we have the following equations to estimate the fraction defective (p) :

Plan 1:

$$p = \left[\sum_{x=0}^{k-1} xa_{x,1} + \sum_{x=k}^{n} xr_{x,1} \right] / nT \qquad \ldots(2.19)$$

Plan 2:

$$p = \frac{np^2\ TB(p,n-1,k-2) + kT\ [1-B(p,n+1,k)]}{\sum_{x=0}^{k-1} xa_{x,2} + \sum_{y=k}^{n} yr_{y,2}} \qquad \ldots(2.20)$$

Plan 3:

$$p = \frac{TJ_4 - Tp(n-k+1)\ B(p,n,k-1)}{\sum_{i=0}^{k-1} ia_{i,3} + \sum_{y=k}^{n} yr_{y,3}} \qquad \ldots(2.21)$$

It is revealed from equations (2.20) and (2.21) that one needs iteration to estimate p by the method of moments under Curtailed Sampling Plans. It is stated in section 2.6 that one can have choice of reporting either i or z when a lot is accepted under Plan 3. It is then revealed from (2.21) that this equation is to be used when i is reported. We give now the estimating equation in case z is reported. In this case the probability function will be of the following form:

$$f_4(s) = \begin{cases} \binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)}, & s=z=n-k+1, n-k+2, \ldots, n \\ \binom{y-1}{k-1} p^k q^{y-k}, & s=y=k, k+1, \ldots, n \end{cases} \qquad \ldots(2.22)$$

The probability function of s is given by the first part of $f_4(s)$ when it is known that the lot is accepted and is given by the second part when it is known that the lot is rejected. This removes the ambiguity regarding the overlapping of the ranges of s. Overcoming of similar ambiguity in $g_2(u)$ was discussed in section 2.5.2. Thus in this case the first moment of s is

$$E(s) = \sum_{z=n-k+1}^{n} z \binom{z-1}{n-k} q^{n-k+1} p^{z-(n-k+1)} + \sum_{y=k}^{n} y \binom{y-1}{k-1} p^k q^{y-k}$$

$$= (n-k+1) B(p, n+1, k-1)/q + (k/p)\,[1 - B(p, n+1, k)]$$

$$= J_4/p \qquad \ldots(2.23)$$

Further the observed arithmetic mean of s in this case is

$$\bar{s} = \frac{\sum_{z=n-k+1}^{n} z a_{z,3} + \sum_{y=k}^{n} y r_{y,3}}{T} \qquad \ldots(2.24)$$

Therefore equating the r.h.s. of (2.23) and (2.24) we get the following estimating equation:

$$p = T J_4 / \left[\sum_{z=n-k+1}^{n} z a_{z,3} + \sum_{y=k}^{n} y r_{y,3} \right] \qquad \ldots(2.25)$$

It may be noted that (2.21) and (2.25) are not basically the same.

Significance of this observation will be evident from the fact that the estimating equations by the method of maximum likelihood under such circumstances are similar, as shown in Chapter III.

2.7.2 Situation B :

Recalling the definitions of the random variables t, u, v, and w and their respective probability functions g_1, g_2, g_3 and g_4, we get the first moment of these random variables as follows:

Case I :

$$E(t) = \sum_{t=0}^{k} t\ g_1(t)$$

$$= \sum_{t=0}^{k-1} t \binom{n}{t} p^t q^{n-t} + k \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k}$$

$$= np\ B(p,n-1,k-2) + k\ [1-B(p,n,k-1)] \qquad \ldots(2.26)$$

Case II:

$$E(u) = \sum_{u=k}^{n} u\ g_2(u)$$

$$= n \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x} + \sum_{u=k}^{n} u \binom{u-1}{k-1} p^k q^{u-k}$$

$$= n\ B(p,n,k-1) + (k/p)\ [1-B(p,n+1,k)] \qquad \ldots(2.27)$$

Case III :

$$E(v) = \sum_{v=0}^{k} vg_3(v)$$

$$= \sum_{v=0}^{k-1} v \binom{n-k+v}{n-k} q^{n-k+1} p^v + k \sum_{y=k}^{n} \binom{y-1}{k-1} p^k q^{y-k}$$

$$=(n-k+1)\, B(p,n+1,k-1)/q - (n-k+1)\, B(p,n,k-1)$$

$$+ k\, [1-B(p,n,k-1)] \qquad ...(2.28)$$

Case IV:

$$E(w) = (n-k+1)\, g_4\, (n-k+1) + \sum_{w=0}^{n-k} wg_4(w)$$

$$= (n-k+1) \sum_{i=0}^{k-1} \binom{n-k+i}{n-k} q^{n-k+1} p^i$$

$$+ \sum_{w=0}^{n-k} w \binom{w+k-1}{k-1} p^k q^w$$

$$=(n-k+1)B(p,n,k-1) + (k/p)\, [1-B(p,n+1,k)]$$

$$-k\, [1-B(p,n,k-1)] \qquad ...(2.29)$$

Furthermore, recalling the definitions of the observed frequencies $a_{t,2}$, $T_{r,2}$, $r_{u,2}$, $T_{a,2}$, $a_{v,3}$, $T_{r,3}$, $r_{w,3}$ and $T_{a,3}$ associated with the random variables t, u, v and w, the observed arithmatic means $\bar{t}, \bar{u}, \bar{v}$ and $\bar{w}$ based on T observations in the respective cases are as follows:

Case I :

$$\bar{t} = \left[\sum_{t=0}^{k-1} ta_{t,2} + k\,T_{r,2}\right]/T \qquad \ldots(2.30)$$

Case II:

$$\bar{u} = \left[nT_{a,2} + \sum_{u=k}^{n} ur_{u,2}\right]/T \qquad \ldots(2.31)$$

Case III:

$$\bar{v} = \left[\sum_{v=0}^{k-1} va_{v,3} + kT_{r,3}\right]/T \qquad \ldots(2.32)$$

Case IV:

$$\bar{w} = \left[(n-k+1)\,T_{a,3} + \sum_{w=0}^{n-k} wr_{w,3}\right]/T \qquad \ldots(2.33)$$

Then equating the r.h.s. of equations (2.30), (2.31), (2.32) and (2.33) respectively to the r.h.s. of equations (2.26), (2.27),(2.28) and (2.29) we have the following estimating equations:

Case I :

$$p = \frac{\sum_{t=0}^{k-1} ta_{t,2} + \left\{kT_{r,2} - kT\,[1-B(p,n,k-1)]\right\}}{n\,T\,B(p,n-1,k-2)} \qquad \ldots(2.34)$$

Case II :

$$p = \frac{kT\,[1-B(p,n+1,k)]}{\sum_{u=k}^{n} ur_{u,2} + \left\{nT_{a,2} - nT \cdot B(p,n,k-1)\right\}} \qquad \ldots(2.35)$$

Case III :

$$q=\frac{(n-k+1)\ T\ \ B(p,n+1,k-1)}{\sum_{v=0}^{k-1} va_{v,3}+(n-k+1)TB(p,n,k-1)+\left\{kT_{r,3}-kT\ [1-B(p,n,k-1)]\right\}} \qquad \ldots(2.36)$$

Case IV :

$$p=\frac{kT\ [1-B(p,n+1,k)]}{\sum_{w=0}^{n-k} wr_{w,3}+kT\ [1-B(p,n,k-1)]\ +\left\{(n-k+1)T_{a,3}-(n-k+1)TB(p,n,k-1)\right\}} \qquad \ldots(2.37)$$

In Case III it is convenient to estimate q rather than p and then to estimate p by the relation p=1-q.

Since we have

$$E\ (T_{r,2}) = T\ [1-B(p,n,k-1)] \qquad \ldots(2.38)$$

$$E(T_{a,2}) = T\ B(p,n,k-1), \qquad \ldots(2.39)$$

$$E(T_{r,3}) = T\ [1-B(p,n,k-1)]\ , \qquad \ldots(2.40)$$

and $E(T_{a,3}) = T\ B(p,n,k-1),$ ...(2.41)

We have the following approximations:

$$T_{r,2} \doteq T[1-B(p,n,k-1)], \qquad \ldots(2.42)$$

$$T_{a,2} \doteq T\ B(p,n,k-1), \qquad \ldots(2.43)$$

$$T_{r,3} \doteq T\ [1-B(p,n,k-1)]\ , \qquad \ldots(2.44)$$

and $T_{a,3} \doteq T\ B(p,n,k-1).$ ...(2.45)

Then using the above approximations (2.42), (2.43), (2.44), and (2.45) the estimating equations (2.34), (2.35), (2.36) and (2.37) may be approximated as follows:

Case I :

$$p \doteq \frac{\sum_{t=0}^{k-1} ta_{t,2}}{nT\ B(p,n-1,k-2)} \qquad \ldots(2.46)$$

Case II:

$$p \doteq \frac{kT\left[1-B(p,n+1,k)\right]}{\sum_{u=k}^{n} ur_{u,2}} \qquad \ldots(2.47)$$

Case III:

$$q \doteq \frac{(n-k+1)\ T\ B(p,n+1,k-1)}{\sum_{v=0}^{k-1} va_{v,3} + (n-k+1)T\ B(p,n,k-1)} \qquad \ldots(2.48)$$

Case IV:

$$p \doteq \frac{kT\left[1-B(p,n+1,k)\right]}{\sum_{w=0}^{n-k} wr_{w,3} + kT\left[1-B(p,n,k-1)\right]} \qquad \ldots(2.49)$$