CHAPTER II

PREDICTION OF COLLEGE ACHIEVEMENT IN MATHEMATICS

Introduction

For the prediction of college success, another important factor is mathematical ability. Like language factor this has been often found in factor analysis. (Thurstone, Burt, Vernon etc.) Even otherwise the significance of this factor, that is, mathematical ability for success in a science course can be easily understood.

As a student leaves school and joins college, many changes occur. The change of syllabus that takes place at college, may demand additional higher specific mental abilities, other than those which are already measured in SSCE Course. These specific higher abilities cannot be assumed necessarily a linear function of measured abilities, and if the underlying function is curvilinear, the correlations of measures of college performance with measured abilities underscore the true relationship. Hence the problem-

Problem

The object of this study is to find answer to the problem:

Whether linear function is adequate for predicting the achievement in Mathematics at college and whether the prediction can be improved by using non-linear polynomial regressions?

Method

The marks in SSCE Mathematics and PScE Mathematics were used in this analysis. The prediction of PScE Mathematics (which is the criterion variable) from SSCE Mathematics is dealt with in this study. The bivariate distribution (Appendix 2) was prepared first and then the analysis was done by least square method. The statistical computations necessary for the analysis an shown below step by step.

Step 1. CALCULATION OF DATA REQUIRED FOR FITTING POLYNOMIAL OF THE THIRD DEGREE

1	2	3	4	5	6	7	8	9	10	11	12
x	Totals y Arrays	xy	x^2y	x^3y	f_x	$f_x x$	$f_x x^2$	$f_x x^3$	$f_x x^4$	$f_x x^5$	$f_x x^6$
-6	-52	312	-1872	11232	8	-48	288	-1728	10368	-62208	373248
-5	- 9	45	- 225	1125	3	-15	75	- 375	1875	-9375	46875
-4	-81	324	-1296	5184	15	-60	240	- 960	3840	-15360	61440
-3	-103	309	- 927	2781	28	-84	252	- 756	2268	-6804	20412
-2	-108	216	- 432	864	32	-64	128	- 256	512	-1024	2048
-1	-89	89	- 89	89	32	-32	-32	- 32	32	- 32	32
0	-72	0	0	0	35	0	0	0	0	0	0
1	-69	-69	-69	-69	39	39	39	39	39	39	39
2	-21	-42	-84	-168	25	50	100	200	400	800	1600
3	14	42	126	378	28	84	252	756	2268	6804	20412
4	31	124	496	1984	16	64	256	1024	4096	16384	65536
5	37	185	925	4625	9	45	225	1125	5625	28125	140625
6	37	222	1332	7992	8	48	288	1728	10368	62208	373248
$\sum$	-485	1757	-2115	36017	278	27	2175	765	41691	19557	1105515

Step 2. SOLUTION OF SIMULTANEOUS EQUATIONS FOR FITTING A POLYNOMIAL

	1 N	2 x^2	3 x^2	4 x^3	5 y
1	278	27	2175	765	-485
x		2175	765	41691	1757
x^2			41691	19557	-2115
x^3				1105515	36017
	278	27	2175	765	-485
	1.0	0.097122302	7.823741007	2.751798561	-1.744604317
		2172.3777	553.7590	41616.7014	1804.1043
		1.0	0.254909172	19.157212579	0.830474507
			24533.2051	2963.3592	1219.6317
			1.0	0.120789729	0.049713508
				305791.9352	2642.6937
				1.0	0.008642130

Step 3. FITTING A CUBIC EQUATION

∴ d=0.008642130

c+d (0.120789729)=0.049713508

c=0.048669627

b+c (0.254909172)+d(19.157212579) = 0.8304745073

b=0.652509051

a+b (0.097122302)+c(7.823741007)+d(2.751798561)= -1.744604317

a = -2.2125337456

∴ The fitted cubic equation is-

$$y= -2.212537456+0.652509051x+0.048669627x^2 + 0.008642130x^3$$

Where $y = \frac{Y - 32}{5}$ and $x = \frac{X - 67}{5}$

Step 4. FITTING A QUADRATIC EQUATION

$c = 0.049713508$

$b + c\,(0.254909172) = 0.830474507$

$b = 0.817802078$

$a + b\,(0.097122303) + c(7.823741007) = -1.744604317$

$a = -2.212976749$

$\therefore$ The fitted quadratic equation is-

$$y = -2.212976749 + 0.817802078x + 0.049713508x^2$$

Where, as before, $y = \dfrac{Y - 52}{5}$ and $x = \dfrac{X - 67}{5}$

Transferred to original units, this becomes

$$Y = 30.775166307 - 0.514519990X + 0.009942702X^2$$

Step 5, FITTING A LINEAR EQUATION

$b = 0.830474507$

$a + b\,(0.97122302) = -1.744604317$

$a = -1.825261913$

$\therefore$ The fitted linear equation is-

$$y = -1.825261913 + 0.830474507x$$

In original units, this becomes

$$Y = 0.830474507X - 12.768101534$$

Step 6. TESTING THE SIGNIFICANCE OF THE INCREASE IN THE SUMS OF SQUARES FOR REGRESSION DUE TO EACH ADDITIONAL DEGREE OF FITTING

S.S. due to linear regression $= (2172.3777)(0.830474507)^2$
$= 1498.26$

S.S. due to quadratic regression $= 1498.26 + (24533.2051)(0.049713508)^2$
$= 1498.26 + 60.63$
$= 1558.89$

S.S. due to cubic regression $= 1558.89 + (305781.9352)(0.008642130)^2$
$= 1558.89 + 22.84$
$= 1581.73$

Results:

We now set up an analysis of variance for testing the significance of the increase in the sums of squares for regression due to each additional degree of fitting.

Table 2.1 Testing the Significance of Each Additional Degree in Fitting of the Polynomial Regression of PScE Mathematics Marks on SSCE Mathematics Marks

Source	SS	DF	MS
Linear regression	1498.26	1	1498.26
Excess due to quadratic	60.63	1	60.63
Excess due to cubic	22.84	1	22.84
Residual	2399.14	274	8.76
Total	3980.87	277	

Evidently, there is significant linear regression and also the fitting of quadratic makes a worthwhile improvement, but the cubic is not of significant value.

The results obtained on ungrouped data are presented below:

The fitted linear and quadratic equations are:

$$Z_2 = 0.833036038X_2 - 12.906798564$$

$$Z_2 = 0.009870741X_2^2 - 0.501753067X_2 + 30.268571800$$

The significance of the linear and quadraties regressions is tested by following analysis of variance (given in Table 2.2):

Table 2.2 Testing the Significance of Linear and Quadratic Regressions of PScE Mathematics Marks on SSCE Mathematics Marks (Un-grouped)

ANALYSIS OF VARIANCE

Source	SS	DF	MS	F
Linear regression	38097.33	1		
Quadratic regression	39722.38	2		
Difference	1625.05	1	1625.05	7.38
Residual	60594.24	275	220.34	
Total	100316.62	277		

Analysis on Normalized Standard Scores

As done in the previous chapter, we now examine the effect of normalization of raw marks on the results. The same procedure of converting raw units to normalized standard scores was used by percentile rank method. The marks in SSCE mathematics were converted to normalized standard scores as shown below:

Table 2.3 SSCE Mathematics Marks Converted to Normalized Standard Scores

Raw marks in SSCE mathematics intervals	f	Percentile rank	Normalized standard scores
95 - 99	8	98.56	2.19
90 - 94	9	95.50	1.70
85 - 89	16	91.01	1.34
80 - 84	28	83.09	0.96
75 - 79	25	73.56	0.63
70 - 74	39	62.05	0.31
65 - 69	35	48.74	-0.03
60 - 64	32	36.69	-0.34
55 - 59	32	25.18	-0.67
50 - 54	28	14.39	-1.07
45 - 49	15	6.65	-1.50
40 - 44	3	3.42	-1.82
35 - 39	8	1.44	-2.19

The analysis for fitting of curves over the above normalized standard scores was done as before, and the results were obtained as shown in the following analysis of variance table :

Table 2.4 Testing the Significance of Each Additional Degree in Fitting of the Polynomial Regression of PScE Mathematics Marks on Normalized SSCE Mathematics Marks

Source	SS	DF	MS
Due to linear regression	1505.28	1	1505.28
Excess due to quadratic	60.29	1	60.29
Excess due to cubic	8.47	1	8.47
Residual	2459.96	274	8.98
Total	3980.87	277	

Here again we find that the linear regression and the addition of quadratic make significant contribution, but the cubic does not come out significant.

Next we examine the results on a standardized aptitude test in mathematics. The test designated as Numerical Ability Test formed a part of the composite scholastic aptitude test standerdized by Examination reform and research unit., M.S. University of Baroda. It consisted of 36 items designed to measure mathematical reasoning abilities in Algebra, Geometry and Arithmetic. The test was administered to the university students admitted to the PScE course in 1964. These students appeared for

PSc examination in 1965. The Numerical Ability Test scores and PScE Mathematics marks were analysed to study the relationship between the two variables.

The following analysis of variance table shows the results obtained:

Table 2.5 Testing the Significance of Each Additional Degree in Fitting of the Polynomial Regression of PScE Mathematics Marks on the Standardized Mathematics Test Scores

Source	SS	DF	MS
Linear regression	895.93	1	895.93
Excess due to quadratic	100.01	1	100.01
Excess due to cubic	3.43	1	3.43
Residual	4215.53	348	12.11
Total	5214.90	351	

In case of aptitude scores also, we find that there is significant linear regression and the fitting of quadratic also makes a significant contribution, but the cubic is not of significant value.

The results obtained on ungrouped data are presented below:

The fitted linear and quadratic equations are:

$$P_m = 1.543838351T_n + 26.045436767$$

$$P_m = 0.075243906T_n^2 - 1.203967885T_n + 48.960509046$$

The significance of linear and quadratic regression is tested by following analysis of variance(given in Table 2.6):

Table 2.6 Testing the Significance of Linear and Quadratic Regressions of PScE Mathematics Marks on Standardized Mathematics Scores (Un-grouped)

ANALYSIS OF VARIANCE

Source	SS	DF	MS	F
Quadratic regression	26556.03	2		
Linear regression	24095.25	1		
Difference	2460.78	1	2460.78	8.29
Residual	103579.06	349	296.79	
Total	130135.09	351		

The implications of these results, that is, curvilinear relationship of the test scores with the criterion of college Mathematics should be noted at this stage.

It may be pointed out that the each item in the standardization has its discriminating index which is usually based on the assumed linear relationship, while the the final test score has curvilinear relationship with the criterion. Also in Gayen's work on 'Management of Achievement in Mathematics', discriminating curves for some items are shown which are curvilinear in shape. These findings suggest that there is a need for making some modification in item statistics such as discriminating index of items, which may take into account curvilinear relationship.

In this regard, Gayen's approach of discriminating curves and the discriminating power of items is noteworthy. It gives the complete exposition of the underlying relationship and the discriminating power of the items at all levels of ability scale and hence be made use of in such investigations.

Conclusions:

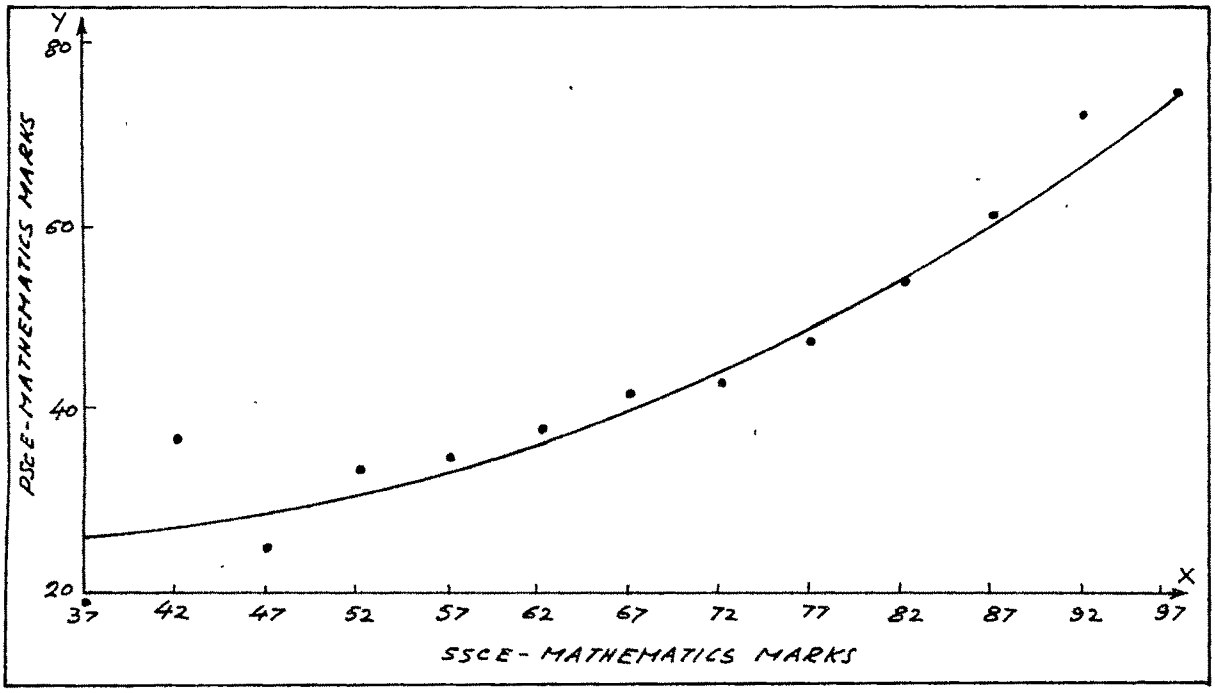
From the above study, the following conclusions are drawn:

1. The empirical mathematical functions of educational measurements - achievement and aptitude measures for predicting college achievement in mathematics are found to be significantly linear.

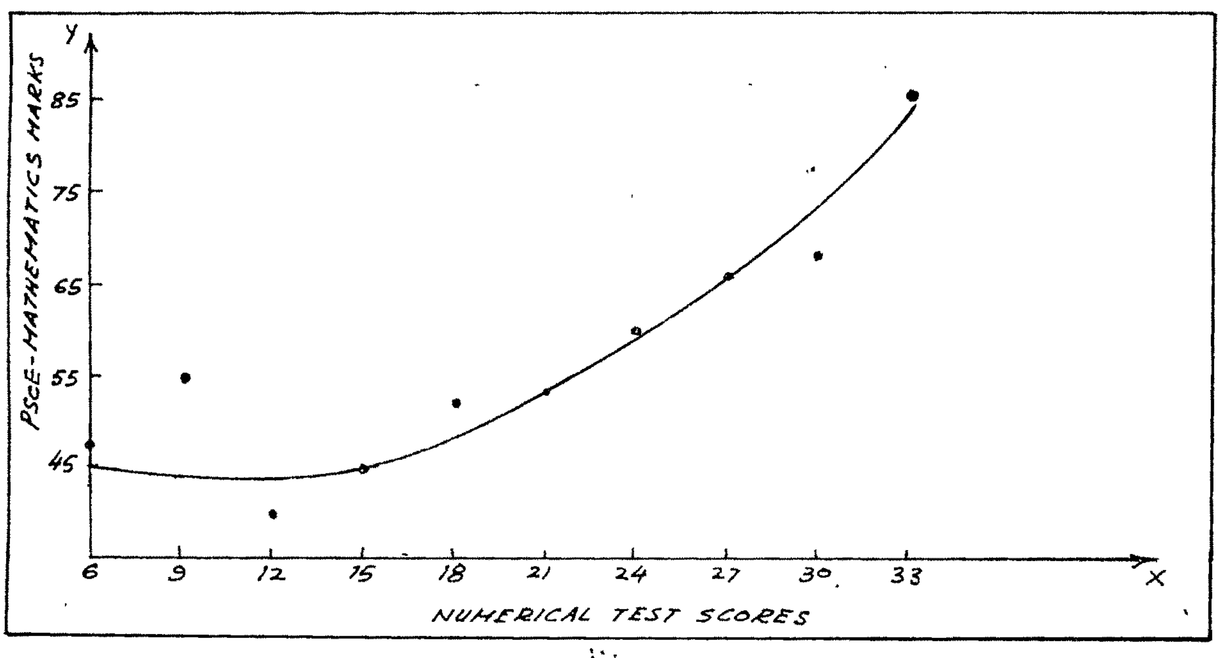
2. Addition of quadratic does make a significant contribution, while cubic does not.

3. Both the educational measures show similar curvilinear trends of prediction functions.(Figh 2)

4. The findings on normalized standard scores are the same as those obtained in case of raw units.



(i)



(ii)

FIG. 2

GRAPHS SHOWING THE RELATIONSHIP OF THE CRITERION VARIABLE PSCE MATHEMATICS WITH (i) SSCE MATHEMATICS AND (ii) NUMERICAL TEST SCORES