CHAPTER 1

INTRODUCTION

Chapter -1

INTRODUCTION

1. Introduction

The main Endeavour of any scientific method is to help human being towards its betterment, to this end and statistical studies have been continuously playing an important role.

Today the science of statistics is an indispensible part of any and every sphere of human activity and is extensively applied in framing polices and formulating decision in a large number of diversified fields Covering Natural, Economic, Physical, Social sciences and Life Sciences. According to Prof. P.C Mahalnobis, "statistics is essentially an applied science. Its only justification lies in the help it can give in solving a problem."

The formulation and growth of the theory of probability during 18^{th} and 19^{th} centuries brought about a sharp and important changes in the basic premises of scientific thinking. Scientific investigators during this period began to realize a close resemblance between the laws of uncertainties governing the outcome of games of chance and the laws of variations observed by them in apparently uncontrolled phenomena in their fields of study. This led astronomers, physicists, geneticists, engineers, agriculturists etc. to believe that a stochastic or probabilistic model (or approach) could possibly explain the variability of observations in fields of scientific inquiry, where such variations were unavoidable.

It became, however, apparent even as early as in nineteenth century that no matter how strongly one believed in the deterministic model, it was not possible to use them beyond limits. A stochastic model was clearly needed as a realistic basis for explaining natural phenomenon characterized by inherent variability. Neyman in Journal of American Statistical Association (1960) reaffirmed that 'currently in the period of dynamic indeterminism in science, there is hardly a serious piece of research, which, if treated realistically does not involve operations on stochastic processes'. It was the growing complexity of physical sciences and later in biological and social sciences that inadequacy of deterministic models was realized and led to the gradual replacement of such models by stochastic models.

1.1 STATISTICS AS A SCIENCE OF INDUCTIVE INFERENCE

As we have already remarked earlier statistics is concerned with collection of data and with their analysis and interpretation. The methods by which data are to be collected has given rise to different techniques and this itself has given a branch or area in statistics called sampling. Next comes the question as to what the data tell us. This answer depends not only on the data but also on the background knowledge of the situation or phenomenon; the latter is formalized in the assumptions under which the analysis enters. The process of inference involved in statistics is of an inductive nature – inferring from particular to the general or from sample to the population. It is here that the effectiveness of statistic lies which has evolved and is in a continuous process of evolving the scientific methodology based on the theory of probability to meet the challenging needs of such inferences. Thus the development in probability theory and statistical inference are to go hand in hand. Statistics today has become an indispensible tool in planning of experiments for any scientific inquiry and in drawing valid inferences on the basis of data that could be quantified. Instead of going into how the data are to be obtained, we would assume for our purpose that they are rather given and describe in brief some principal lines of approach of statistical analysis.

CONDITIONAL INFERENCE

One of the fundamental problems in statistics is that of specification of an appropriate model to represent the phenomenon under study and to make analysis. It is indeed obvious that the validity of statistical inference depends on the appropriateness of the model. In most applications the model is parametric and if it can be determined in advance from theoretical considerations, and statistical inferences can be drawn using classical theory. This is the view of late Prof. R. A. Fisher according to whom there is 1: 1 correspondence between the model and its analysis.

There are situations when we come across data that are collected from operational studies or a researcher feels that it is extremely unlikely that any particular specification will represent exactly the phenomenon under investigation. In the former, data are not taken from well designed experiments or surveys having a specific underlying frame work. In such cases data analysis cannot confine itself to a prescribed model and hence cannot be unique. We have to examine and discuss more or less the adequacy of any proposed framework before we build statistical theories on it. The main difficulty faced by the statistician in analyzing data collected from operational studies is that he has first to evolve a model from the data, test its adequacy on it or a similar data and then to make final inferences. Thus the inferences drawn are always conditional. The decision to use conditional or unconditional inference has to be made by the experimenter (researcher) before the experiment and may be based on his prior knowledge obtained from his own experience and / or of other workers in that field. If the decision is to use unconditional inference, then available inference procedures (Classical or Bayesian) may be used. However, if the decision is to use conditional inference;

then the research worker has to base his inferences on the specification evolved through the data and then to go for final inference.

Examples and need of such inference procedures are abound. They occur in econometrics, regression analysis, ANOVA models, outliers, and other branches of statistics. In all these cases, uncertainties exist and one has to resolve them before making final inferences; hence they were given the name testi-testing, testi-mating and testi-predicting by Bancroft (1975). For testimating a new name 'testimator' has been proposed by Sclove, Morris and Radhakrishnan (1972). In all such cases, where we use conditional inference it is important that the effect of preliminary test(s) on subsequent inference should always be taken into account. This aspect was often neglected by applied statisticians.

Suppose we are interested in the estimation of θ in f(x ; θ) when a random sample of size n say $(x_1, x_2 ...., x_n)$ is available, f is completely known say for θ and in addition either a guess of θ say θ_0 or an interval (θ_1, θ_2) both known, is given in which θ is assumed to lie. This priori information is sometimes available from past experience or similar studies and we are interested in estimators of θ which behave nicely in the neighbourhood of θ_0 (or in an interval). However, we do not assume any distribution of θ but wish to utilize the information about θ.

CLASSICAL INFERENCE

In this type of inference the data are assumed to be repeated values on random variables which, we postulated to follow a joint probability distribution p belonging to some known class P. Frequently, the distributions are indexed by a parameter θ (say) taking values in a set Ω, so that $P = \{P_\theta / \theta \in \Omega\}$. The aim of statistical analysis is to specify a plausible value of θ in terms of a statistic $t = t(x_1, ..., x_n)$ where t is supposed to be measurable. But there is no unique method for the specification of t, though various methods for choosing t have been proposed in the literature. This is the problem of point estimation of θ as enunciated by Fisher about 1920s. If instead of giving a single value of 't' as, an estimate of θ, we determine a set of values for which we can plausibly assert that it does or does not contain θ , we call this estimation by confidence sets or hypothesis testing. It was fist formulated by Jerzy Neyman in his 1937 paper and later developed by Wolfwitz, Stein, Hodges, Guttman and others. It was remarked by them that in some sense estimation by confidence sets or methods may be more meaningful.

In contrast to point estimation in which we try to find out a plausible value of the parameter on the basis of the information provided by the sample observations, in statistical hypothesis testing we are to choose between two possible actions regarding the hypothesized value(s) of the parameter. i.e., to decide that the distribution is a particular member of a family which is known except for the parameters. In the context of testing of hypothesis these two actions are called acceptance or rejection of the hypothesis.

1.2 BAYESIAN INFERENCE

In Bayesian approach the parameter is assumed to be random variable with an a priori density function, this distribution expresses the state of knowledge or ignorance about θ before the sample data are analyzed. Given the probability model, the prior distribution and the data set $(x_1, x_2 \ldots\ldots, x_n)$, Bayes theorem is used to calculate the posterior probability density function $P(\theta / D)$ of θ, where D denotes the prior and sample information and on the basis of posterior distribution inferences about θ are drawn. Thus the Bayesian method of reasoning seems rather deductive.

Bayesian inference is an especially important consideration in those areas of application where the sample data may be either expensive or difficult to obtain, such as reliability and life testing experiments.

PRIOR DISTRIBUTION :

The prior distribution g(θ) on the parameter space Ω is specified before data became available and is modified using the data to determine a posterior distribution, which is the conditional distribution of θ given the observations say $x_1,\ldots\ldots,x_n$. The other difficulties in Bayesian analysis are :(i) There is no convincing definition of optimality, (ii) The optimal procedure depend heavily oon the assumed nature of probability model.

Some other concepts used in Bayesian analysis stem from decision theory such as risk, Bayes risk, Minimax and Bayes rules etc. Since the Bayes risk of a decision rule depends on the choice of the prior distribution and is a real number, it is possible to order. The optimal choice then would be the one which minimizes the Bayes risk. How does one select a known density g(θ) to express uncertainty about θ, is a problem which remains open and controversial? In many practical situations the statistician will possess some subjective apriori information concerning possible values of θ. This information may often be summarized and made objective by the choice of a suitable prior distribution on the parameter space. It is perhaps the most difficult task in Bayesian analysis. Although a few guidelines have been given regarding the choice of a prior distribution; yet none seems satisfactory. Summarizing about the Bayes rules we may say that we are interested in them because of (i) they are admissible and (ii) form complete class.

In Bayesian set-up the experimenter expresses his belief about the parameter by prior distribution and his misjudgment by a loss function.

Bayesian methods are now becoming widely accepted as a way to solve applied statistical problems in industries and government. Research groups in various disciplines like econometrics, education, law, archaeology, engineering, medical and life sciences are using Bayesian inferential methods to obtain optimum solutions to their problems.

DIFFERENCE BETWEEN CLASSICAL AND BAYESIAN INFERENCE

In simple language, the main difference between Bayesians and classical statistics is that the Bayesians treat the state of nature (e.g., the value of a parameter) as a random variable, whereas the classical way of looking at is that it's a fixed but unknown value, and that putting a probability distribution on it does not make sense.

Bayesian methods provide alternatives that allow one to combine prior information about a population parameter with information contained in a sample to guide the statistical inference process.

The classical estimation method originally proposed by Hamilton involves a two step procedure in which model parameters are estimated first (usually by maximum likelihood estimation), and inference on hidden states is subsequently drawn holding these parameter estimates fixed.

Advances in computational capacity have more recently spurred a number of papers employing alternative, Bayesian estimation methods based on Monte-Carlo techniques. In contrast to classical methods these methods permit simultaneous inference on both the model parameters and hidden states.

1.3 VARIOUS TYPES OF LOSS FUNCTIONS

Any decision-making situation consists a non-empty set Θ of possible states of nature, sometimes referred to as the parameter space and a non-empty set A of actions available to decision maker. Under these two situations, nature chooses a point θ in Θ and the decision maker without being informed of the choice of nature, chooses an action d in A. As a consequence, there may incur some loss which will depend on d and θ. Thus, loss is a function of θ and d defined the product space Θ x A say L(θ,d). The function L(.,.) is known as the loss function.

In point estimation problems, the action space consists of the set of all possible values of θ. Thus, it may be the whole parameter space or a subset of it. To ease the problem a sampling experiment is often conducted to collect the data. The data is considered to be an observation of the random variable x which is assumed to have a probability distribution f(x/θ), when the true state of nature is θ. The decision maker chooses an estimate/ class of estimates $\hat{\theta}$ as the value of the function of the random variable x say T(x) for the given observed value x i.e. $\hat{\theta} = T(x)$. The function T(.) is called the estimator and its value T(x) when x is observed is the estimate for θ. Naturally, the loss L(θ,d) now reduces to L(θ,T(x)) which is a random variable and depends on the sample outcome.

The basic problem of decision theory is : Given a loss function L(θ,d), a decision d and the risk R(θ,d) which criterion should one choose for adopting d? The ideal solution would be to choose a d for which R(θ,d) is minimum for all θ. Unfortunately, this is not possible. The decision theory as formulated and developed by Wald in a series of paper beginning in 1939 was an attempt to unify the statistical theories of estimation and testing of hypothesis which become especial cases now.

For point estimation a number by loss functions are available in the literature. These can be broadly classified into two groups, viz. symmetric and asymmetric. More generally we may have the idea of General Entropy Loss functions which includes Asymmetric Loss Function(ASL). Both types of loss functions have extensively been used in estimation problems. Among various symmetric loss functions (Berger (1985), Martz & Waller (1982)), the quadratic loss function or the squared error loss function (SELF) is very popular and widely used in Bayesian analysis. The main reason behind its popularity is that, it was used in estimation problems when unbiased estimators of parameter θ were being considered. A second reason is its relationship with classical least square theory. Finally the use of 'SELF' makes the calculation relatively straight forward and simple (mean of the posterior distribution). A number of situations may arise in practice where 'SELF' may be appropriately used, especially when under estimation and over estimation are of equal importance.

Inspite of above mentioned justifications for 'SELF' there may be practical situations when the real loss function may not be symmetric i.e. overestimation and underestimation are not equally penalized. Situations may exist when the overestimation may lead to more serious consequences than the underestimation or vice-versa.

For example suppose that a producer produces some electronic device, wants to estimate the failure rate of his products. If his estimate is larger than the real value, he will have to incur additional resources to improve the technology to increase the reliability of his products. On the other hand if he underestimates the real value, he may lose customers and his market share may decrease because the real reliability of his products will now be less than the value he offers. In extreme cases, underestimating the failure rate may even cause the ruin. Hence, underestimation of the failure rate will lead to worse consequences than overestimation. Similarly, overestimation (space shuttle challenger case **Ref**: Basu and Ebrahimi (1991)) may lead to worse consequences than underestimation. Due to these reasons and others, Berger (1985) points out that justification for 'SELF' has a little merit. In order to bring the statistical model nearer to practical situations, the use of asymmetric loss functions and General Entropy Loss Functions (GELF) is suggested. Varian (1975) in his applied study to real estate assessment introduced an Asymmetric Loss Function called LINEX (Linear Exponential), which rises approximately exponentially on one side of zero and approximately linearly on the other side of zero. This loss function was extensively used by Zellner (1986) in estimation of scalar parameter and prediction of a scalar random variable in Gaussian (normal) model. Use of LINEX function has been justified by Lindley (1968), Zellner and Geisel (1968), Canfield (1970), Smith (1980), Schabe (1986), Basu and Ebrahimi (1991), Pandey and Rai (1992), Srivastava & Rao (1992), Srivastava (1996), Srivastava and Kapasi (1999), Srivastava and Tank (2001), Srivastava and Tanna (2001) and others.

1.4 ASYMMETRIC LOSS FUNCTIONS

Various loss functions have been considered under the category of Asymmetric loss functions and some of them are described as below.

LINEX LOSS FUNCTION

The Linex loss function is an Asymmetric Loss Function, which was introduced by Klebanov (1972) and used by Varian (1975) in the context of real estate assessment. Zellner (1986) used it for estimation of a scalar parameter and prediction of a scalar random variable. Both Zellner (1986) and Varian (1975) have discussed its behaviour and various applications. The linex loss function is defined as $L(\theta, a) = \exp(a(\hat{\theta} - \theta)) - a(\hat{\theta} - \theta) - 1$, $a \neq 0$

For small values of $|a|$,

$$L(\theta, a) \cong \frac{a^2}{2}(\hat{\theta} - \theta)^2$$

Thus, Linex is almost symmetric and not too different from a Squared Error Loss Function (SELF) and, therefore, Bayes estimates and predictions, based on linex loss, are quite near to those obtained from SELF.

MODIFIED LINEX LOSS FUNCTION

According to Basu and Ebrahimi (1991), when the parameter θ is a scale parameter, we may take $\Delta = (\hat{\theta}/\theta) - 1$, where $\hat{\theta}$ is an estimate of θ. They define *modified linex loss* function as

$$L(\Delta) = b[\, e^{a\Delta} - a\Delta - 1],\ b > 0,\ a \neq 0 \quad \text{Where } \Delta = \left(\frac{\hat{\theta}}{\theta} - 1\right) \qquad \text{\_\_\_\_\_\_(1.4.1)}$$

The sign and magnitude of 'a' represents the direction and degree of asymmetry respectively. The positive value of 'a' is used when overestimation is more serious than under estimation, while a negative value of 'a' is used in reverse situations. $L(\Delta)$ rises exponentially when $\Delta < 0$ and almost linearly when $\Delta > 0$. The loss function defined by (1.4.1) is known as the LINEX loss function. 'b' is the factor of proportionality.

GENERAL ENTROPY LOSS FUNCTION

Calabria and Pulcini (1996) defined *generalized entropy loss function* as

$$L(\theta,\hat{\theta})=b\left[\left(\frac{\hat{\theta}}{\theta}\right)^{p}-p\ln\left(\frac{\hat{\theta}}{\theta}\right)-1\right],\qquad p\neq 0, b>0 \tag{1.4.2}$$

as a valid alternative to the modified linex loss.

This loss is a generalization of the entropy loss used by several authors (for example, Dey and Liu, 1992; Dey et al., 1987) where the shape parameter 'p' is equal to unity (1). The more general version of (1.4.2) allows different shapes of the loss function to be considered when $p > 0$, a positive error $(\hat{\theta}>\theta)$ causes more serious error than a negative error and when $p < 0$, a negative error $(\hat{\theta}<\theta)$ causes more serious error than the positive error).

In particular, for $p = 1$, we have entropy loss function given by

$$L(\theta,\hat{\theta}) = b\left(\frac{\hat{\theta}}{\theta} - \ln\frac{\hat{\theta}}{\theta} - 1\right)$$

However, if $\left|\frac{\hat{\theta}-\theta}{\theta}\right| \cong 0$, we have $L(\theta,\hat{\theta}) \cong \frac{1}{2}\left(\frac{\hat{\theta}}{\theta}-1\right)^{2}$ · which resembles SELF.

1.5 BAYESIAN POINT ESTIMATION

In Bayesian estimation, statistical inference is made when we are given a model, a distribution of parameters and a loss function associated with the decision, we make for the parameter under this setup and experimenter expresses his belief about the real situation via a prior distribution and the misjudgment by loss function. Before collecting the sample data, the experimenter specifies a prior distribution say $g(\theta)$ which reflects his knowledge or ignorance about the parameter on the basis of the sample data. The experimenter specifies the loss function say $L(x/\theta)$. The prior information $g(\theta)$ with sample information $L(x/\theta)$ is then combined by Bayes theorem to get the posterior distribution $P(\theta/x)$ as:

$$P(\theta/x) = \frac{L(x/\theta)g(\theta)}{\int L(x/\theta)g(\theta)d\theta} \qquad \text{\_\_\_\_\_\_(1.5.1)}$$

Where integration is taken over the whole parameter space. This posterior distribution P(θ/x) is thus, an inferential statement in the Bayesian view point. Consider that we wish to obtain a point estimate for θ under some specified loss function $L(\theta, \hat{\theta})$ where $\hat{\theta}$ is the estimate of θ. In Bayesian approach an estimate $\hat{\theta}$ is selected such that it minimizes the posterior risk, which is the average loss for the specified prior distribution P(θ/x). Under different loss functions different Bayes estimaters may be obtained for the same prior distribution.

ESTIMATION UNDER SQUARED ERROR LOSS FUNCTION

A loss function which is often used for point estimation problem is the Squared Error Loss Function.

$$L(\theta, \hat{\theta}) = \Delta^2 \qquad \text{\_\_\_\_\_\_(1.5.2)}$$

Where $\Delta = (\hat{\theta} - \theta)$ and may be considered as error due to estimation.

The Bayes estimator under the loss (1.5.2) is the value which minimizes.

$$E[L(\theta, \hat{\theta})/x] = \int (\hat{\theta} - \theta)^2 \, P(\theta/\underline{x}) \, d\theta \qquad \text{\_\_\_\_\_\_(1.5.3)}$$

Obviously, $\hat{\theta} = \hat{\theta}_s = E(\theta/\underline{x}) = \int \theta \, P(\theta/\underline{x}) \, d\theta$ ______(1.5.4)

Minimizes (1.5.4) and thus posterior mean is the Bayes estimator.

ESTIMATION UNDER LINEX LOSS FUNCTION

The LINEX loss function suggested by Varian (1975) is

$$L(\theta, \hat{\theta}) = b\, e^{a\Delta} - c\Delta - b \;\; ; \;\; a, c \neq 0, b > 0 \qquad \text{\_\_\_\_\_\_(1.5.5)}$$

Since, the loss function should be such that it has a minimum value viz. zero at $\theta = \hat{\theta}$ we must have ab = c

Therefore (1.5.5) reduces to

$$L(\theta, \hat{\theta}) = b\,[e^{a\Delta} - a\Delta - 1] \;\; ; \;\; a \neq 0, b > 0 \qquad \text{\_\_\_\_(1.5.6)}$$

LINEX loss has two constants, a and b which give the freedom to tailor the loss according to our needs by choosing them appropriately. The function for various choices has been shown graphically by Zellner (1986). Thus, LINEX loss could be used in situation where loss function is asymmetric.

While estimating θ by $\hat{\theta}$, and denoting E_{POST} as the posterior expectation we have:

$$L(\Delta) = b\left[e^{a(\hat{\theta}-\theta)} - a(\hat{\theta}-\theta) - 1\right] \;\; ; \;\; where \;\; \Delta = (\hat{\theta}-\theta)$$

$$E_{POST}\; L(\Delta) = b\left[E_{POST}\; e^{a(\hat{\theta}-\theta)} - a\,E_{POST}(\hat{\theta}-\theta) - 1\right]$$

$$\frac{dE_{POST}}{d\theta} = b\left[E_{POST}\; e^{a(\hat{\theta}-\theta)} - a\,E_{POST}(1) - 0\right] = 0$$

$$\Longrightarrow E_{POST}\; e^{a(\hat{\theta}-\theta)} = 1$$

i.e. $\hat{\theta} = -\frac{1}{a}\, log\, E_{POST}\; (e^{-a\theta})$

Provided $E_{POST}\; (e^{-a\theta})$ exist and is finite.

Thus, we see that Bayes estimator which is the mean of posterior probability distribution function under 'SELF', is proportional to the Moment Generating Function of posterior probability distribution function under LINEX loss.

Basu and Ebrahimi (1991) modified the loss function for estimating a scale parameter i.e. they defined Δ as:

$\Delta = \left(\frac{\hat{\theta}}{\theta} - 1\right)$ then

$$L(\Delta) = b\left[e^{a\left(\frac{\hat{\theta}}{\theta}-1\right)} - a\left(\frac{\hat{\theta}}{\theta} - 1\right) - 1\right]$$

$$therefore\ \frac{d\,E_{POST}\,L(\Delta)}{d\theta} = 0$$

$\Rightarrow E_{POST}\left(\frac{1}{\theta} e^{a\left(\frac{\hat{\theta}_B}{\theta}-1\right)}\right) = e^a E_{POST}\left(\frac{1}{\theta}\right)$, solving this we get $\hat{\theta}_B$, the estimator under L(Δ).

ESTIMATION UNDER GENERAL ENTROPY LOSS FUNCTION

A suitable alternative to modified LINEX loss is the General Entropy Loss (GEL) proposed by Calabria and Pulcini (1996) given by:

$$L_E(\hat{\theta}, \theta) \propto \left\{\left(\hat{\theta}/\theta\right)^p - p\ \ln\left(\hat{\theta}/\theta\right) - 1\right\}$$

Whose minimum occurs at $\hat{\theta} = \theta$.

If we are considering prior distributions, then the Bayes estimate of θ under GELF is in a closed form and is given by $\hat{\theta} = [E(\theta^{-p}\,|x)]^{-1/p}$ provided that $E_\theta(\theta^{-p})$ exists and is finite.

- When p = 1, the Bayes estimate (4.1.1.2) coincides with the Bayes estimate under the weighted squared error loss function $(\hat{\theta} - \theta)^2/\theta$, used by Varde (1969) for deriving Bayes estimate of R(t).
- When p = -1, the Bayes estimate (4.1.1.2) coincides with the Bayes estimate under the squared error loss function.

The Bayes estimator of θ under entropy loss function is obtained by putting p = 1 in $\hat{\theta}=[E(\theta^{-p}|x)]^{-1/p}$ which is the posterior harmonic mean.

For the negative values of p, i.e., p = - u (say), the form of the generalized entropy loss function reduces to $L(\theta,\hat{\theta})=\left(\frac{\theta}{\hat{\theta}}\right)^{u}-u\ln\frac{\theta}{\hat{\theta}}-1$.

In particular for u = 1, $L(\theta,\hat{\theta})=\frac{\theta}{\hat{\theta}}-\ln\frac{\theta}{\hat{\theta}}-1$. In this case the Bayes estimator works out to be posterior arithmetic mean.

PROBLEM OF TESTIMATION

Any given real life situation can be modeled via some probability distribution having some known mathematical form except for the constants (parameters) involved in it. Almost every parameter has its own physical interpretation in terms of real life situation.

The efforts are to estimate these parameter(s) in the best possible manner so as to provide 'best' estimator(s). Sometimes we might have 'additional' information about the parameter of interest which could be utilized to hopefully improve the estimator. Such type of informations are common in Bio-statistics and health statistics. For example we might know due to past studies that the hemoglobin level of school going girls is θ_0 and we wish to use this information for the estimation of the hemoglobin level for some population of school going girls. We might take this information as such and use it in while proposing an estimator for θ (say the hemoglobin level of entire population under study) or this available information might be tested (verified) using a test of significance and the given information is incorporated on the basis of outcome of this test.

TESTIMATION PROCEDURE:

Suppose that we have the guess information: in the form of a point θ_0 or an interval (θ_1, θ_2) and the sample information $(x_1, x_2 \ldots\ldots, x_n)$ then obtain: (i) The 'best' estimator of θ using $(x_1, x_2 \ldots\ldots, x_n)$ by Maximum Likelihood Estimator or some other suitable method of estimation (ii) Test $H_0 : \theta = \theta_0$ against a suitable alternative (one tailed or two tailed, mostly two tailed), if H_0 is accepted utilize this information, otherwise ignore it. Thus, we combine testing procedure with the estimation procedure and in the literature such procedure has been termed as 'TESTIMATION'.

1.6 REVIEW OF LITERATURE

In this section a review of the literature related to the problems under study in the area of inferences based on Asymmetric Loss Function and those utilizing guess information has been made.

Bancroft (1944) was the first statistician to consider the impact of preliminary test of significance on subsequent problem of estimation.

Thompson (1968) was the first to introduce the idea of shrinkage technique using point as well as interval guess. Canfield (1970) introduced the idea of Asymmetric Loss Functions.

Several authors have proposed estimator(s), weighted estimator(s), shrinkage testimators for the scale parameter of single parameter Exponential distribution. Pandey and Srivastava (1987) among others, proposed some improved shrinkage testimators, where the arbitrariness in the choice of shrinkage factor was removed.

Earlier studies were confined to the use of symmetric loss functions (mostly 'SELF') but later on in several studies the superiority of Asymmetric Loss Functions was established, like Zellner (1986), Basu and Ebrahimi (1991), Calabria and Pulcini (1994, 1996) among others..

Srivastava and kapasi (1999) have proposed Conditional – Guess testimator(s) for the mean life in single parameter & two parameter exponential distribution. Srivastava and Tank (2003) have proposed sometimes pool estimator for Exponential distribution. Properties of these estimators have been studied under asymmetric loss function.

Pandey and Srivastava (1987), Pandey and Singh (2007) have proposed shrinkage testimator(s) for the variance of Normal distribution and have studied the properties of these using 'SELF' and asymmetric loss function (ASL).

Katti (1962), Shah(1975), Arnold and Al-Bayatti (1970), Waiker et al. (1989) have proposed double stage shrinkage testimator of the mean for an Exponential distribution and the variance of Normal distribution.

Srivastava and Tanna (2007 & 2012) have proposed double stage shrinkage testimator for the mean life of an Exponential distribution under 'General Entropy Loss Function' and under asymmetric loss function.

Pandey and Singh (1984) considered estimating shape parameter of Weibull distribution by shrinkage towards an interval. Pandey, Srivastava and Malik (1989) studied some shrinkage testimators for the shape parameter of Weibull distribution.

1.7 AN OUTLINE OF PROBLEMS UNDER INVESTIGATION

In the present thesis, an attempt has been made to study the properties of various parameters of Exponential distribution, Normal distribution and Weibull distribution using various asymmetric loss functions and we have proposed some improved estimator(s) for various parameter(s) for different probability distributions in terms of reduced risk(s).

CHAPTER – I

Chapter - I is introductory, and it covers the basic idea of Classical and Bayesian Inference procedures. It also provides a brief review of literature. In the same chapter Bayesian estimation procedures under various loss functions have been discussed.

CHAPTER – II

Chapter - II deals with the problems of **one sample** shrinkage testimators of Exponential Distribution and Normal Distribution under Asymmetric Loss Function. The Exponential distribution has a variety of statistical applications in life testing and reliability and other fields. Normal distribution occupies a very important place in Statistical studies. Vaious testimators for different parameters of both the distributions have been proposed and their risk properties have been studied.

Several authors have proposed estimator(s), weighted estimator(s), shrinkage testimators for the scale parameter of single parameter Exponential distribution. Pandey and Srivastava (1987) among others, proposed some improved shrinkage testimators, where the arbitrariness in the choice of shrinkage factor was removed.

Srivastava and kapasi (1999) proposed Conditional – Guess testimator(s) for the same distribution. Srivastava and Tank (2003) proposed sometimes pool estimator for Exponential distribution under asymmetric loss function.

In this chapter, we have proposed single stage shrinkage testimator(s) for the scale parameter of Exponential distribution for several choices of the shrinkage factors and the properties of these have been studied using asymmetric loss functions.

Pandey and Srivastava (1987), Pandey and Singh (2007) have proposed shrinkage testimator(s) for the variance of Normal distribution and have studied the properties of these using 'SELF' and asymmetric loss function (ASL). In this chapter, we have proposed several estimators for the variance of Normal distribution for different choices of shrinkage factors, and the properties of these have been studied using asymmetric loss function. It has been found that the proposed testimators dominate the usual estimator(s) in terms of reduced risk.

Further the use of asymmetric loss function facilitates to provide better control over the 'risk' of the proposed testimators by choosing the degree of asymmetry and level of significance carefully. Recommendations regarding these two have been attempted.

CHAPTER - III

Chapter - III deals with the problems of **double stage** shrinkage testimators of Exponential Distribution and Normal Distribution under Asymmetric Loss Function.

The first stage sample is used to test $H_0 : \theta = \theta_0$ and if H_0 is not rejected, it is suggested to use the prior knowledge being supported by a test, in estimating θ.

However, if H_0 is rejected, then take $n_2 = (n - n_1)$ additional observations $x_{21}, x_{22}, \ldots . x_{2n_2}$ and use the pooled estimator i.e. we do not use the prior knowledge and obtain a second sample to make up for the loss of the prior knowledge and estimate θ using both the samples.

Such techniques were presented by Katti (1962), Shah (1975), Arnold and Al-Bayatti (1970), Waiker et al. (1989). We have proposed 'Double stage shrinkage testimators' for the scale parameter of an Exponential distribution and the variance of Normal distribution. Properties of these proposed testimator(s) have been studied under asymmetric loss function and attempts have been made regarding the use of such procedures.

It has been observed that General Entopy Loss Function has appeared as a valid alternative to Modified LINEX loss function, so it is of interest to study the risk properties of various testimators using General Entropy Loss Function (GELF).

In particular not many attempts have been made to study shrinkage testimators under GELF with this motivation the next chapters of the present work have been devoted to such study.

CHAPTER - IV

Chapter – IV has been devoted to the study of risk properties of single stage shrinkage testimators for various parameters of interest in Exponential and Normal distribution under 'General Entropy Loss Function'. The risk properties of these have been studied and recommendations regarding the degrees of asymmetry and level of significance have been made.

CHAPTER - V

In the Chapter – V, we have extended the work done by Srivastava and Tanna (2007 & 2012) they have proposed double stage shrinkage testimator for the mean life of an Exponential distribution under 'General Entropy Loss Function'. Some new estimators have been proposed by removing the arbitrariness in the choice of shrinkage factors and 'Double stage shrinkage testimators' have been proposed for Exponential and Normal distributions for their mean life and variance respectively. Properties of these testimator(s) have been studied using 'General Entropy Loss Function' and recommendations for sample sizes, level(s) of significance and degrees of asymmetry have been made.

CHAPTER – VI

Pandey and Singh (1984) considered estimating shape parameter of Weibull distribution by shrinkage towards an interval. Pandey, Srivastava and Malik (1989) studied some shrinkage testimators for the shape parameter of Weibull distribution.

In this chapter, we have proposed some improved shrinkage estimators for the shape parameter of the Weibull distribution when it is known apriori that β (shape parameter) lies in the interval (β_1, β_2). We have studied the properties of this estimator using asymmetric loss function and it has been found that it is preferable to the other estimators, in terms of having smaller risk.