

Synopsis

In recent times, the predictions of the Standard model (SM) of particle physics have been probed to finer level of detail by precession measurements at LEP, by higher luminosity runs at Tevatron. So far no discrepancies emerged between experiments and the predictions of the SM. Despite this spectacular success of the SM in explaining all the low-energy phenomena, there is a hurdle of theoretical short comings of the model which strongly suggest that SM is only an important intermediate step toward the knowledge of fundamental interactions and that, at best, it is an effective theory, valid upto the scale M_X . The SM falls short of a complete theory in an aesthetic sense that the number of parameters required to describe it is nineteen, six quark and three lepton masses, three quark mixing angles and a phase parametrising CP violation, three gauge couplings and two boson mass scales M_W and M_ϕ and θ_{QCD} parameter that describes potential strong violation of CP

Understanding the fermion mass hierarchy and the origin of the quark mixing is one of the outstanding problems of present day particle physics. This thesis is based on the studies related to fermion masses and mixing within the SM and beyond. The motivation for this study is to understand the possible extensions of the SM that are allowed phenomenologically and to study their predictions.

In the first part, we study the quark masses within the framework of the SM given the CKM quark mixing matrix [5] to be symmetric [21, 23, 34]. Present experimental limits [17] on the various elements of the CKM matrix indicate that it is symmetric or approximately symmetric. The elements $|V_{12}|$ and $|V_{21}|$ are quoted to be same modulo the errors and both of them lie between 0.217 and 0.223. Although only very weak bounds for $|V_{13}|$ element is known through the bound $.05 \leq q(= \frac{|V_{13}|}{|V_{23}|}) \leq .13$ at present, the decay constant $f_{B_d} \approx 220$ MeV could lead to $|V_{13}| = |V_{31}|$, in the case of $m_t > 100\text{GeV}$ Harris and Rosner[36] showed the possibility of $f_{B_d} \approx 220$ MeV by analyzing the $B_d^0 - \overline{B_d^0}$ mixing and the ϵ_K parameter of K meson system, and this was also suggested in the framework of lattice calculation [37]. It should be noted that for three generations, the assumption that V has symmetric moduli implies a single constraint on the matrix V because the unitarity requirement alone yields

$$A \equiv |V_{12}|^2 - |V_{21}|^2 = |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2$$

for three generations and experimentally the asymmetry parameter A is, in general, small i.e. $A < 10^{-4}$. Thus all the presently available data is consistent with having symmetric moduli for CKM matrix. It has been shown [21] that if V has symmetric modulus, then it is always possible to choose the phases of the quark fields so that V is also symmetric

We have shown [22] that if the CKM matrix is symmetric then the top quark mass has to be heavier than 180 GeV, to be consistent with the experimental results of ϵ_K , the parameter describing the indirect CP violation in the interactions changing strangeness by two units ($\Delta S = 2$), and the measurement on $B_d - \bar{B}_d$ mixing parameter x_d (which gives the time-integrated probability of a \bar{B}_d appearing in a B_d beam) for the Bag constant $B_K = 1, 2/3$; if the Bag constant $B_K = 1/3$ then $m_t > 275$ GeV. The parameters q and δ (CP violating phase) are constrained to be in the range

$$.113 \leq q \leq .130 \quad 8.0^\circ \leq \delta \leq 31.1^\circ$$

for the symmetric CKM matrix over the allowed range of the top quark mass $80 \text{ GeV} \leq m_t \leq 270 \text{ GeV}$. To get a comparative idea it should be noted that accurate measurements, especially at LEP of the properties of Z^0 , together with the collider and ν data yield an indirect value for m_t [8] :

$$m_t = 164^{+16}_{-17} \text{ } ^{+18}_{-21} \text{ GeV.}$$

Given the fact that CKM matrix is symmetric, one is urged to ask the important question of how to derive the symmetric quark mixing starting from the Yukawa couplings in a natural way. In this regard we tried to find the constraints [29] on the quark mass matrices for the symmetric CKM matrix. Since mass ratios of the up-quark sector and that of the down-quark one are quite different from each other, it is very difficult to get the symmetric CKM matrix being consistent with experiments in the framework of the 'calculable' quark mass matrix, where 'calculable' means that the CKM matrix is given in terms of the quark mass ratios, namely, the number of mass matrix parameters is less than ten observable quantities i.e. quark masses and CKM matrix elements. The symmetry constraint was written as an equation involving the parameters of the mass matrices using flavour projection operators in a basis where M_u is diagonal. The numerical ranges for the mod elements of M_d were given in this basis. This procedure was repeated in the basis where M_d is diagonal. Then, the necessary condition for having a symmetric V in terms of the matrices U and D was derived. A particularly interesting basis was chosen where $U = D^* P$; P being a phase matrix and the ranges for the mod elements of M_u, M_d in that basis was found out using a convenient parametrisation for V . It was noticed that none of the off-diagonal elements of M_u and M_d is consistent with zero for a symmetric V , which means such forms for mass matrices cannot be obtained from any symmetry. But, in principle there exists infinite number of other bases related to each other by similarity transformations. So it is apparent that the numbers provided for the allowed ranges of the mod elements of mass matrices are not basis independent. Finally the symmetry constraint was presented in a basis-independent form.

Recently, there was an attempt made by Tanimoto[35] to obtain an approximately symmetric CKM matrix starting from mass matrices of the type $M_{U,D} = \kappa_{U,D} M_0 + X_{U,D}$ where $\kappa_{U,D}$ are numerical constants; M_0 is a real 3×3 , rank-one matrix and the matrices X_U and X_D are correction terms that have to be added to M_0 to obtain the non-zero masses of the light two generation quarks since the rank-one mass matrix M_0 has only one non-vanishing eigenvalue. The phenomenological validity of this scheme was checked and we have shown[31] that out of the three interesting solutions of the symmetric CKM matrix discussed in this scheme one

is inconsistent with experiments, whereas another one requires a very heavy top quark mass ($m_t \approx 255 \text{ GeV}$) to be consistent.

The second part of the thesis deals with the physics of massive neutrinos. The question of whether or not the neutrinos have non-zero masses concerns one of the most important issues in both particle physics and astrophysics. In the minimal SM the neutrinos are predicted to be massless due to the absence of right-handed neutrinos as well as the lepton number violating processes. However, they can acquire mass in extensions of SM involving either new $SU(2)$ singlet neutral fermions (to generate Dirac mass) or new Higgs representations (to generate Majorana mass). There exists an attractive scheme called 'see-saw mechanism' that explains naturally the smallness of the neutrino mass compared to the charged leptons. This mechanism can be embedded[54] naturally in the left-right symmetric extension of the SM. We have made an analysis[53] of the spontaneous symmetry breaking for the Higgs sector taking various Higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model, including the Higgs choice that predicts the correct low energy ratio of $\frac{m_b}{m_\tau}$. The minimisation of these potentials was carried out explicitly and its phenomenological consequences regarding neutrino masses had been studied. A modification in the see-saw relationship between the vacuum expectation values v_L and v_R was found out when left-right parity is broken spontaneously. The mass spectrum for different Higgs choices consistent with potential minimisation had been used to study the evolution of the couplings g_L and g_R according to the Renormalisation Group equation.

Although none of the observational claims[45] of a 17 keV neutrino (with 1 % mixing with the electron neutrino), which was first observed by Simpson[44] in 1985, seems definitive at this time, the renewed possibility that a heavy mass eigenstate is mixed with the electron neutrino is sufficiently surprising to warrant an examination of its consequences. We studied the constraints[43] put by the existence of a 17 keV mass eigenstate on the neutrino mixing matrix from the various oscillation data, neutrinoless double beta decay and the limit on the ν_e, ν_μ and ν_τ masses. In the limit when all the three eigenvalues are nondegenerate and the mass differences are larger than 100 eV^2 , we identify ν_τ with the 17 keV neutrino and vary the mixing probability between 3% and 0.3%. We find a very narrow allowed region for the various mixing angles. The allowed values of m_{ν_μ} lie between 145 keV and 205 keV for 1% mixing and between 135 keV and 240 keV for 3% mixing (our result differs from the earlier similar works with 3% mixing, where some approximations were made). The $\nu_e \rightarrow \nu_\mu$ oscillation probability is found to lie between .001 and .002 for consistency. We then considered the allowed amount of the symmetry breaking when ν_μ and ν_τ form a pseudo-Dirac particle. We found the mass difference to be less than $9 \times 10^{-8} \text{ eV}$, which puts stringent limits on the 'symmetry breaking effect on the neutrino mass matrix

To summarise, the phenomenological bounds on top quark mass and the elements of the mass matrices were studied given the ansatz for quark mixing is symmetric. We found that none of the moduli of the off-diagonal elements of the possible forms of quark mass matrices M_u and M_d that lead to the symmetric CKM matrix, are consistent with zero for these ansätze, which means that such forms for mass matrices are difficult to obtain from any symmetry. The phenomenological consistency of a particular scheme for mass matrices that claims to

be obtaining a symmetric quark mixing was checked. The potential minimisation of the most general Higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model was done explicitly and its phenomenological consequences regarding neutrino masses and the see-saw relationship was explored. We studied the limits on the elements of the neutrino mixing matrix consistent with neutrinoless double beta decay and the neutrino oscillation experiments as a function of the mixing probability (x) of ν_e with the $17keV$ neutrino. Stringent limits on m_{ν_μ} (when $m_{\nu_\mu} \gg m_{\nu_\tau}$) and on the mixing matrix were found