Chapter 1

Introduction

Understanding the microscopic structure of the physical universe, in general, can be thought of consisting of three steps, first, an identification of the basic particles that constitute matter; second, gaining a knowledge of what forces the particles experience; third, finding a quantitative description of the particles behaviour under the influence of the forces given the initial boundary conditions. According to the present understanding, the basic particles can be put into two catagories, namely, matter particles and gauge particles. The basic matter particles are fundamental spin 1/2 fermions called the leptons and quarks whereas the gauge particles are bosons that are exchanged between matter particles during their interactions. These fundamental particles undergo four known types of gauge interactions–gravitation (too weak to be of interest to particle physics phenomenology), electromagnetism, the strong and weak nuclear forces–and also interact with higgs boson

Historically, the study of weak interactions began with the discovery of radioactivity by Becquerel (1896) and subsequent observation that the decaying nucleus emits electrons (i e. nuclear β decay) Chadwick's observation (1914) that the electrons in β decay are emitted with a continuous spectrum of energies and subsequent calorimetric measurements of β decay (1920) seemed to suggest the violation of energy and momentum conservation laws in β decay if one assumes a two-body final state In order to save the fundamental conservation laws, W. Pauli proposed[1] a three-body final state for β decay with an extra neutral particle of near-vanishing or zero rest mass and half-interger spin (later named *neutrino* by Fermi) being emitted along with the electron and it escapes observation because of its feeble interaction with the surrounding matter

Soon thereafter, Fermi proposed[2] his theory of β-decay in close analogy to quantum electrodynamics by writing a current-current effective interaction Lagrangian density

$$\mathcal{L}_F = -(2)^{-1/2} G_F(\overline{\psi}_p \gamma_\mu \psi_n)(\overline{\psi}_e \gamma^\mu \psi_{\nu_e}) \tag{1.1}$$

The above Lagrangian density describes a 4-fermion zero-range (pointlike) interaction with a universal coupling, G_F, between the fermion pairs (e, ν_e), (p, n) $\mathcal{L}_F$, being a density, has dimension of (length)$^{-4}$, or, in energy units, dimension 4, while a fermion field has dimension 3/2 because a fermion mass term occurs in the Lagrangian in the form $m\overline{\psi}\psi$. Thus, the 4-

fermion current-current interaction is a dimension-6 operator and so G_F, the Fermi coupling constant, has units (energy)$^{-2}$. Empirically

$$G_F = 1.16639(2) \times 10^{-5} GeV^{-2} = 10^{-5} m_p{}^{-2} \tag{1.2}$$

where m_p is the proton mass. After the discovery of parity violation in weak interactions (1956), it was realised that the structure of the weak Noether current is vector-axial vector (V-A) type That culminated in the change of γ_μ into $\gamma_\mu(1-\gamma_5)$. Subsequent discovery of many other weak interaction processes and the recognition of the universality of weak interaction led to the (V-A) form[3] of the weak interaction Lagrangian given by Marshak & Sudarshan and Gell-Mann & Feynman .

$$\mathcal{L}_W = -(2)^{-1/2} \frac{G_F}{2} (J^{\mu -} J^+_\mu + J^{\mu +} J^-_\mu), \tag{1.3}$$

where the charge-raising and charge-lowering currents are given respectively by

$$\begin{aligned} J^+_\mu &= \frac{1}{2} \overline{u} \gamma_\mu (1-\gamma_5) d + \frac{1}{2} \overline{\nu} \gamma_\mu (1-\gamma_5) e + ..., \\ J^-_\mu &= \frac{1}{2} \overline{d} \gamma_\mu (1-\gamma_5) u + \frac{1}{2} \overline{e} \gamma_\mu (1-\gamma_5) \nu + ... \end{aligned} \tag{1.4}$$

Since weak interactions distinguish the handedness, we will deal with Weyl two-component spinors ψ_L and ψ_R (chiral decomposed states of 4-component Dirac spinors),

$$\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi,$$

each of which represents two physical degrees of freedom The field ψ_L annihilates a LH particle or creates a RH antiparticle, while $\psi_L^\dagger$ creates an LH particle or annihilates a RH antiparticle. For a ψ_R field the role of LH and RH are reversed. Weyl fermions having no distinct partners of opposite chirality correspond to particles that are either massless or carry no conserved quantum numbers The charged-currents can be rewritten in terms of the 2-component fields as

$$\begin{aligned} J^+_\mu &= \overline{u_L} \gamma_\mu d_L + \overline{\nu_L} \gamma_\mu e_L + ..., \\ J^-_\mu &= \overline{d_L} \gamma_\mu u_L + \overline{e_L} \gamma_\mu \nu_L + ... \end{aligned} \tag{1.5}$$

At the lowest order (tree level), $\mathcal{L}_F$ gives a very successful description of low-energy weak interaction processes But, there are quite a few problems associated with $\mathcal{L}_F$.

At low energy, the total cross-section for neutrino-electron scattering comes out to be proportional to $G_F^2 s$, where s is the Mandelstam variable defined by $s = (p_{\nu_e} + p_e)^2$ in terms of the momenta of the incoming ν and e. With increasing energy this cross-section grows without limit. On the otherhand, since ν e scattering occurs in the s-wave, the amplitudes for this process should obey the s-wave unitarity bound, viz ,

$$\sigma^s_{tot} \leq \frac{16\pi}{s}. \tag{1.6}$$

This leads to a contradiction, implying that the 4-fermion description must breakdown above a certain energy called the weak interaction cut-off Λ_{Wk} which was found to vary between 4 GeV to 300 GeV depending upon the weak interaction process under consideration.

Secondly, as one calculates higher order corrections (loops) to any lowest order weak process described by $\mathcal{L}_F$, one finds an infinite sequence of interactions of higher and higher dimension, with increasingly divergent integrals, which would require more and more arbitrary constants to render them finite. The divergence at the L-th loop goes as Λ^{2L}, Λ being the cut-off for the theory. Therefore, in a strict sense, the lowest order calculations are not reliable.

The introduction of massive vector bosons $W^{\pm}$, which play a role analogous to that of photon in QED, improved the situation. The basic interaction is then of the form

$$\mathcal{L}_W = -(2)^{-1/2} g (J^{\mu -} W_\mu^+ + J^{\mu +} W_\mu^-), \tag{1.7}$$

where the coupling constant, g, is now dimensionless. The square of $\mathcal{L}_W$ involves the propagator of the massive W boson and yields

$$\frac{g^2}{2} J^\mu \left(\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \right) J^{\nu\dagger}. \tag{1.8}$$

In order to reproduce the successful low-energy ($q^2 << M_W^2$) results of the effective Lagrangian $\mathcal{L}_F$, the coupling g must be related to G_F as

$$\frac{g^2}{2M_W^2} = \frac{4G_F}{\sqrt{2}}. \tag{1.9}$$

Although there is no dimension-6 operator in $\mathcal{L}_W$, the theory is still non-renormalizable. This is due to the fact that the longitudinally polarized W bosons are described by the polarisation vectors (ϵ_μ^L) behaving as

$$\epsilon_\mu^L \longrightarrow \frac{q_\mu}{M_W} \; as \; q \longrightarrow \infty, \tag{1.10}$$

Consequently, the W propagator approaches a constant as $q \longrightarrow \infty$ rather than decreasing as q^{-2}, and so the "canonical" dimension of the field is 2 instead of 1. Similar problem does not arise from the longitudinally polarised virtual photons in QED because the amplitudes are unchanged under the transformation

$$\epsilon_\mu \longrightarrow \epsilon_\mu + a q_\mu \tag{1.11}$$

due to gauge invariance and so the components of the photon's polarization ϵ_μ that are proportional to q_μ does not contribute to physical processes. Hence, it is quite natural to think whether or not a gauge theory can provide the remedy for non-renormalisability of $\mathcal{L}_W$.

In fact, the only renormalisable theory that accomodates the vector bosons in a fundamental way is the local gauge theory But, then the assumption that there exists a local gauge theory associated with the weak interactions led to problems; because, in contrast to the photon, the W_μ's had to be charged, parity-violating and massive, and it was not known how to construct a self-consistent renormalisable theory for such fields. This problem was solved in three stages . first, the Yang-Mills gauge theory provided a natural method of introducing charges for the vector mesons, second, the discovery of spontaneous symmetry breaking (SSB) provided a mechanism for introducing mass without violating the gauge symmetry explicitly; finally, the technical problem of proving the renormalisability was solved by using dimensional regularisation and functional integration. Thus was the birth of the electroweak[4] theory which has been successfully explaining all the physical processes involving energies upto at least $\sim$ O(100 GeV).

Despite the immense success of the SM in explaining all the low-energy phenomena, there is a hurdle of theoretical short comings of the model which strongly suggest that SM is only an important intermediate step toward the knowledge of fundamental interactions and that, at best, it is an effective theory, valid upto the scale M_X. The SM falls short of a complete theory in an aesthetic sense that the number of parameters required to describe it is nineteen i.e. six quark and three lepton masses, three mixing angles and a phase parametrising CP violation, three gauge couplings and two boson mass scales M_W and M_ϕ and θ_{QCD} parameter that describes potential strong violation of CP. Of course, the predictions of the SM have been probed to finer level of detail by precision measurements at LEP, by higher luminosity runs at the Tevatron and no discrepancies emerged between experiments and the predictions of the SM.

This thesis is based on the studies related to fermion masses and mixing within SM and beyond. As it is well-known, all the masses in the SM arise out of the spontaneous symmetry breaking of the $SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{em}$. Hence they are necessarily proportional to the order parameter v of the symmetry breakdown However, in the SM, only the masses of the gauge bosons are predicted since the constant of proportionality between their masses and v is the electric charge (and $\sin^2\theta_W$). On the other hand, both the higgs mass and those of the fermions do not have predictable values since the higgs self interaction constant λ and the Yukawa couplings h_f for each fermion are unknown parameters. In general, the process of generating mass for the fermions is non-diagonal in flavour. Consequently, some off-diagonal mass terms will ensue. This, in turn, forces a mixing between the physical states and the flavour states since the physical basis for the weak charged current interactions of quarks is not flavour diagonal. Then one wonders what fixes the observed weak mixing angles and how these angles are related to the spectrum of masses and the quark mixings and its consequences regarding the top quark mass and the quark mass matrices.

In the first half of this thesis, we study the quark masses within the framework of the SM given the CKM quark mixing matrix [5] to be symmetric. The phenomenological bounds on top quark mass and the elements of the mass matrices were studied in this ansatz for quark mixing. We found out that none of the moduli of the off-diagonal elements of the possible forms of quark mass matrices M_u and M_d that lead to the symmetric CKM matrix, are consistent with zero for these ansatze, which means that such forms for mass matrices are difficult to obtain from any symmetry. The phenomenological consistency of a particular scheme for mass matrices (based on rank one matrices) that claims to be obtaining a symmetric quark mixing was checked. And it was shown[31] that out of the three interesting solutions of the symmetric CKM matrix discussed in this scheme one is inconsistent with experiments, whereas another one requires a very heavy top quark mass ($m_t \approx 255 GeV$) to be consistent.

In the second half, we deal with the physics of massive neutrinos and its consequences. In the SM, neutrinos are massless because one excludes RH neutrinos and lepton number violating processes. Although, they can acquire mass in extensions of SM, the smallness of the neutrino mass compared to that of the charged leptons challenges our understanding. There exists an attractive scheme called 'see-saw mechanism' that explains naturally the smallness of the neutrino mass compared to the charged leptons. This mechanism can be embedded naturally in the left-right symmetric extension of the SM. The potential minimization of the most general higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model was done explicitly and its phenomenological consequences regarding neutrino masses and the see-saw relationship was explored.

We studied the limits on the elements of the neutrino mixing matrix consistent with neutrinoless double beta decay and the neutrino oscillation experiments as a function of the mixing probability (x) of ν_e with the 17 keV neutrino. Stringent limits on m_{ν_μ} (when $m_{\nu_\mu} \gg m_{\nu_\tau}$) and on the mixing matrix were found.

The plan of the thesis is as follows : chapter 2 consists of a review of some relevant topics in SM model and CP violation that provides the necessary background for chapter 3 in which the symmetric quark mixings and its consequences are discussed. In first half of chapter 4, we review the formalism for having massive neutrinos and their mixing and then, discuss how to accomodate the 17 KeV massive neutrino in the SM. The second half of chapter 4 consists of a brief review of the relevant part of the Left-Right symmetric extensions of SM followed by the potential minimization of the most general higgs representations in the context of generalised ($g_L \neq g_R$) left-right symmetric model and subsequent discussions of its phenomenological consequences regarding neutrino masses and the see-saw relationship. We state our conclusions in chapter 5.