Appendix B

Exact variance formula for a given member of EGOE(1+2)-s

For completeness, we reproduce here the formula for spectral variances generated by each member of EGOE(1+2)-**s**. Given a one plus two-body Hamiltonian H, the fixed-S spectral variance $\sigma^2(m,S) = \langle H^2 \rangle^{m,S} - [\langle H \rangle^{m,S}]^2$ will be a fourth order polynomial in m and $S(S+1)$ [Fr-69, No-86]. This gives

$$\sigma^2(m,S) = \sum_{p=0}^{4} a_p\, m^p + \sum_{q=0}^{2} b_q\, m^q\, S(S+1) + c_0\, [S(S+1)]^2 . \tag{B1}$$

The parameters (a_i, b_i, c_i) follow from $\sigma^2(m,S)$ for $m \leq 4$ and to determine these inputs one has to construct H matrices for m up to 4. However an elegant method, allowing $\sigma^2(m,S)$ to be expressed in terms of $(\epsilon_i, V^{s=0,1}_{ijkl})$, is to use the embedding algebra $U(N) \supset U(\Omega) \otimes SU(2)$. With respect to this algebra, as pointed out in [Ko-79, Ko-02a], $h(1)$ decomposes into a scalar $\nu = 0$ part [given by the first term in the first equation in Eq. (2.3.3)] and an irreducible one-body part with $\nu = 1$. The $\nu = 0$ and $\nu = 1$ parts transform, in Young tableaux notation [He-74], as the irreps [0] and $[21^{\Omega-2}]$ respectively of $U(\Omega)$. Similarly $V^s(2)$, $s = 0,1$ decompose into $\nu = 0,1$ and 2 parts. The scalar parts $V^{\nu=0:s=0,1}$ can be identified from Eq. (2.3.3) and they will not contribute to the variances. The effective one-body parts $V^{\nu=1:s=0,1}$, generated by $V^{s=0,1}_{ijkl}$, are defined by the induced single particle energies $\lambda_{i,j}(s)$ given ahead in Eq. (B2). The diagonal induced energies $\lambda_{i,i}(s)$ are identified for the first time in [Ko-79].

However for EGOE(1+2)-**s** it is possible to have $\lambda_{i,j}(s)$, $i \neq j$. Now the irreducible two-body part $V^{\nu=2:s=0} = V - V^{\nu=0:s=0} - V^{\nu=1:s=0}$ and similarly $V^{\nu=2:s=1}$ is defined. It should be noted that the two $\nu = 0$ parts of $V(2)$ transform as the $U(\Omega)$ irrep [0] and the two $\nu = 1$ parts of $V(2)$ transform as the irrep $[21^{\Omega-2}]$. Similarly $V^{\nu=2:s=0}$ transforms as the irrep $[42^{\Omega-2}]$ and the $V^{\nu=2:s=1}$ as the irrep $[2^2 1^{\Omega-4}]$. Using these and the group theory of $U(N) \supset U(\Omega) \otimes SU(2)$ algebra as given by Hecht and Draayer [He-74], a compact and easy to understand expression for fixed-S variances emerges, with $\mathcal{S}^2 = S(S+1)$, $m^x = \Omega - m/2$, $X(m,S) = m(m+2) - 4S(S+1)$ and $Y(m,S) = m(m-2) - 4S(S+1)$,

$$
\begin{aligned}
\sigma^2_{H=h(1)+V(2)}(m,S) = & \frac{(\Omega+2)mm^x - 2\Omega\,\mathcal{S}^2}{\Omega(\Omega-1)(\Omega+1)} \sum_i \tilde{\epsilon}_i^2 \\
& + \frac{m^x\,X(m,S)}{2\Omega(\Omega-1)(\Omega+1)} \sum_i \tilde{\epsilon}_i \lambda_{i,i}(0) \\
& + \frac{(\Omega+2)m^x\left[3Y(m,S)+16\mathcal{S}^2\right] - 8\Omega(m-1)\mathcal{S}^2}{2\Omega(\Omega-1)(\Omega+1)(\Omega-2)} \sum_i \tilde{\epsilon}_i \lambda_{i,i}(1) \\
& + \frac{\left[(m+2)m^x/2+\mathcal{S}^2\right]X(m,S)}{8\Omega(\Omega-1)(\Omega+1)(\Omega+2)} \sum_{i,j} \lambda_{i,j}^2(0) \\
& + \frac{1}{8\Omega(\Omega-1)(\Omega+1)(\Omega-2)^2}\{8\Omega(m-1)(\Omega-2m+4)\mathcal{S}^2 \\
& + (\Omega+2)\left[3(m-2)m^x/2-\mathcal{S}^2\right]\left[3Y(m,S)+8\mathcal{S}^2\right]\} \sum_{i,j} \lambda_{i,j}^2(1) \\
& + \frac{\left[3(m-2)m^x/2-\mathcal{S}^2\right]X(m,S)}{4\Omega(\Omega-1)(\Omega+1)(\Omega-2)} \sum_{i,j} \lambda_{i,j}(0)\lambda_{i,j}(1) \\
& + P_2^0(m,S)\left\langle\left(V^{\nu=2,s=0}\right)^2\right\rangle^{2,0} + P_2^1(m,S)\left\langle\left(V^{\nu=2,s=1}\right)^2\right\rangle^{2,1}\;;
\end{aligned}
\tag{B2}
$$

$$
P_2^0(m,S) = \frac{\left[m^x(m^x+1)-\mathcal{S}^2\right]X(m,S)}{8\Omega(\Omega-1)},
$$

$$P_2^1(m,S) = \frac{1}{\Omega(\Omega+1)(\Omega-2)(\Omega-3)} \left\{(\mathscr{S}^2)^2(3\Omega^2-7\Omega+6)/2 + 3m(m-2)m^x(m^x-1)\right.$$

$$\left. \times(\Omega+1)(\Omega+2)/8 - \mathscr{S}^2\left[(5\Omega-3)(\Omega+2)m^x m + \Omega(\Omega-1)(\Omega+1)(\Omega+6)\right]/2\right\},$$

with

$$\begin{aligned}
&\tilde{\epsilon}_i = \epsilon_i - \overline{\epsilon}\,, \\
&\lambda_{i,i}(s) = \sum_j V^s_{ijij}\,(1+\delta_{ij}) \;-\; (\Omega)^{-1}\sum_{k,l} V^s_{klkl}\,(1+\delta_{kl})\,, \\
&\lambda_{i,j}(s) = \sum_k \sqrt{(1+\delta_{ki})(1+\delta_{kj})}\,V^s_{kikj} \;\; \text{for} \;\; i \neq j\,, \\
&V^{\nu=2,s}_{ijij} = V^s_{ijij} - \left[\langle V(2)\rangle^{2,s} + (\lambda_{i,i}(s)+\lambda_{j,j}(s))\,(\Omega+2(-1)^s)^{-1}\right], \\
&V^{\nu=2,s}_{kikj} = V^s_{kikj} - (\Omega+2(-1)^s)^{-1}\sqrt{(1+\delta_{ki})(1+\delta_{kj})}\,\lambda^s_{i,j} \;\; \text{for} \;\; i \neq j\,, \\
&V^{\nu=2,s}_{ijkl} = V^s_{ijkl} \;\; \text{for all other cases}\,.
\end{aligned} \tag{B3}$$

Equations (B2) and (B3) are tested, by using some members of the EGOE(1+2)-**s** ensemble, for all S values with $m = 6, 7$ and 8 and also for many different Ω values.