Appendix I

Fixed-(m, M) occupation numbers

Our purpose here is to derive a simple expression for the occupation probabilities $\left\langle n_{m_{z_i}} \right\rangle^{mM}$ for m fermions in N sp states labeled by J_z quantum number m_{z_i}. Here, M are the eigenvalues of the J_z operator. As $\left\langle n_{m_{z_i}} \right\rangle^{mM}$ is an expectation value, we can write a polynomial expansion in terms of the J_z operator [Dr-77],

$$\left\langle n_{m_{z_i}} \right\rangle^{mM} = \sum_{\mu} \left\langle n_{m_{z_i}} P_\mu(\widehat{J}_Z) \right\rangle^{m} P_\mu(\widehat{M}) \,, \tag{I1}$$

where $\widehat{M} = M/\sigma_{J_z}(m)$, $\widehat{J} = J_z/\sigma_{J_z}(m)$ and $P_\mu(M)$ are orthogonal polynomials defined by the density $\rho_{J_z}(M)$ which is close to a Gaussian. Retaining terms up to order 2, the expansion is,

$$\begin{aligned}
\left\langle n_{m_{z_i}} \right\rangle^{mM} &= \left\langle n_{m_{z_i}} \right\rangle^{m} + \left\langle n_{m_{z_i}} \widehat{J}_Z \right\rangle^{m} \widehat{M} + \frac{\left(\left\langle n_{m_{z_i}} \widehat{J}_Z^2 \right\rangle^{m} - \left\langle n_{m_{z_i}} \right\rangle^{m}\right)(\widehat{M}^2 - 1)}{\langle \widehat{J}_Z^4 \rangle^{m} - 1} \\
&= \left\langle n_{m_{z_i}} \right\rangle^{m} + \frac{\left\langle n_{m_{z_i}} J_Z \right\rangle^{m} M}{\sigma^2_{J_z}(m)} \\
&+ \frac{1}{2}\left(\frac{\left\langle n_{m_{z_i}} J_Z^2 \right\rangle^{m}}{\sigma^2_{J_z}(m)} - \left\langle n_{m_{z_i}} \right\rangle^{m} \right)\left(\frac{M^2}{\sigma^2_{J_z}(m)} - 1 \right) .
\end{aligned} \tag{I2}$$

In the above expression we used $\langle \hat{J}_z^4 \rangle^m = 3$, the value for a Gaussian $\rho_{J_z}(M)$. Now the formulas for the traces in Eq. (I2) are as follows. Firstly,

$$\left\langle n_{m_{z_i}} \right\rangle^m = \frac{m}{N} \left\langle \left\langle n_{m_{z_i}} \right\rangle \right\rangle^1 = \frac{m}{N} . \tag{I3}$$

This implies $n_{m_{z_i}}^{\nu=0} = \hat{n}/N$. Also,

$$\left\langle n_{m_{z_i}} J_z \right\rangle^m = \frac{m(N-m)}{N(N-1)} \left\langle \left\langle n_{m_{z_i}} J_z \right\rangle \right\rangle^1 , \tag{I4}$$

with $\left\langle \left\langle n_{m_{z_i}} J_z \right\rangle \right\rangle^1 = m_{z_i}$. Unitary decomposition of the number operator gives,

$$\left\langle n_{m_{z_i}} J_z^2 \right\rangle^m = \left\langle n_{m_{z_i}} \right\rangle^m \langle J_z^2 \rangle^m + \left\langle n_{m_{z_i}}^{\nu=1} J_z^2 \right\rangle^m , \tag{I5}$$

$$\left\langle n_{m_{z_i}}^{\nu=1} J_z^2 \right\rangle^m \quad = \quad \frac{m(N-m)(N-2m)}{N(N-1)(N-2)} \left\langle \left\langle n_{m_{z_i}}^{\nu=1} J_z^2 \right\rangle \right\rangle^1 . \tag{I6}$$

Now,

$$\left\langle \left\langle n_{m_{z_i}}^{\nu=1} J_z^2 \right\rangle \right\rangle^1 = \left\langle \left\langle (n_{m_{z_i}} - n_{m_{z_i}}^{\nu=0}) J_z^2 \right\rangle \right\rangle^1 = m_{z_i}^2 - \frac{1}{N} \langle \langle J_z^2 \rangle \rangle^1 , \tag{I7}$$

where we have used the result that $n_{m_{z_i}}^{\nu=0} = \hat{n}/N$ deduced from Eq. (I3). Thus,

$$\left\langle n_{m_{z_i}} J_z^2 \right\rangle^m / \sigma_{J_z}^2(m) \quad = \quad \left\langle n_{m_{z_i}} \right\rangle^m + \frac{(N-2m)(m_{z_i}^2 - \langle J_z^2 \rangle^1)}{(N-2)N\langle J_z^2 \rangle^1} . \tag{I8}$$

Substituting above traces in Eq. (I2) we have,

$$\left\langle n_{m_{z_i}} \right\rangle^{m,M} = \overline{m} + \frac{m_{z_i} M}{N\langle J_z^2 \rangle^1} - \frac{(\overline{m} - 1/2)(m_{z_i}^2 - \langle J_z^2 \rangle^1)(M^2 - \sigma_{J_z}^2(m))}{N^2(\langle J_z^2 \rangle^1)^2 \overline{m}(1 - \overline{m})} , \tag{I9}$$

where $\overline{m} = m/N$. The expression for the occupation number $\left\langle n_{m_{z_i}} \right\rangle^{mM}$ is close to that obtained in [Mu-00, Ze-04] where statistical mechanics approach has been employed. Thus, we have successfully reproduced the previously obtained results using moment method formalism.