Appendex B

*Origin of the term $\frac{t_3}{4} \langle \alpha | (\rho_p \rho_n)_0 | \beta \rangle$"

We saw in chapter IV that when the density dependent Skyrme interaction is used, the variational parinciple leads to the following HF equations

$$\sum_\beta \Big[\langle \alpha | t | \beta \rangle + \sum_{j=1}^{A} \langle \alpha j | \tilde{v}_{12} + \tilde{V}'_{12}(\rho_0) | \beta j \rangle \Big] c^i_\beta$$

$$+ \frac{1}{2} \sum_{ij}^{A} \langle ij | \frac{\partial V'_{12}(\rho_0)}{\partial \rho_0} \cdot \frac{\partial \rho_0}{\partial c^{i*}_\alpha} (1 - P_M P_\sigma P_\tau) | ij \rangle$$

$$= \epsilon_i \, c^i_\alpha \qquad \text{B(1)}$$

where $(1-P_M P_\sigma P_\tau)$ takes care of the antisymmetrization.

We shall show in this appendix explicitly that the derivative term in B(1) leads to

$$1/2 \sum_{ij}^{A} \langle ij | \frac{\partial V'_{12}(\rho_0)}{\partial \rho_0} \cdot \frac{\partial \rho_0}{\partial c^{i*}_\alpha} (1 - P_M P_\sigma P_\tau) | ij \rangle$$

$$= t_3/4 \sum_\beta \langle \alpha | (\rho_p \rho_n)_0 | \beta \rangle \, c^i_\beta \qquad \text{B(2)}$$

We recall that V'_{12} is the two-body scalar band averaged density dependent part of Skyrme interaction defined by

$$V'_{12} = \frac{t_3}{6}(1+P_\sigma)\,\delta(\vec{r}_1-\vec{r}_2)\,\rho_0\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right) \qquad \text{B(3)}$$

and that the deformed states $|i\rangle$ are expanded as

$$|i\rangle = \sum_\alpha c^i_{n_\alpha l_\alpha j_\alpha} |n_\alpha l_\alpha j_\alpha, m_i \tau_i\rangle \equiv \sum_\alpha c^i_\alpha |\alpha\rangle \qquad \text{B(4)}$$

The derivative term in B(2) can be evaluated as follows: Following the definition IV(8) for $\rho_0\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$, we have

$$I = \frac{1}{2}\sum_{kj}^{A} \langle kj|\frac{\partial V'_{12}}{\partial \rho_0}\cdot\frac{\partial \rho_0}{\partial c^{i*}_\alpha}(1-P_M P_\sigma P_\tau)|kj\rangle$$

$$= \sum_\beta c^i_\beta \Big[\frac{1}{2}\sum_{kj}^{A}\langle kj|\frac{t_3}{6}\delta(\vec{r}_1-\vec{r}_2)$$

$$\cdot\frac{1}{2j_\alpha+1}\sum_{m_\alpha}\langle\vec{R}|n_\beta l_\alpha j_\alpha m_\alpha\rangle\langle n_\alpha l_\alpha j_\alpha m_\alpha|\vec{R}\rangle$$

$$\cdot(1+P_\sigma)(1-P_M P_\sigma P_\tau)|kj\rangle\Big] \qquad \text{B(5)}$$

where $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$. To avoid the confusion, we have expressed the sum by k,j. We now consider the product of operators $(1+ P_\sigma)(1- P_M P_\sigma P_\tau)$. Because of the δ-function, $P_M=1$. Since we consider only the even-even time-reversal invariant nuclei, the terms containing $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ operators would contribute identically zero becaue of the time-reversal invariance symmetry. Thus,

$$P_\sigma = \frac{1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2}{2} \Rightarrow 1/2$$

Thus we have ,

$$(1+ P_\sigma)(1 - P_M P_\sigma P_\tau) = 3/2\,(1 - P_\tau) \qquad B(6)$$

Making use of this expression in B(5), one gets after some algebra, through the δ -function and the charge exchange operator P_τ ,

$$I = \sum_\beta c^i_\beta \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{t_3}{6} \cdot X \qquad B(7)$$

where

$$\mathcal{X} = \int \Big\{ \sum_{kj}^{A} \phi^{*}_{k\tau_k}(\vec{r})\, \phi^{*}_{j\tau_j}(\vec{r})\, \phi_{k\tau_k}(\vec{r})\, \phi_{j\tau_j}(\vec{r}) - \sum_{kj}^{A} \phi^{*}_{k\tau_k}(\vec{r})\, \phi^{*}_{j\tau_j}(\vec{r})\, \phi_{k\tau_j}(\vec{r})\, \phi_{j\tau_k}(\vec{r}) \Big\}$$

$$\cdot \frac{1}{2j_\alpha+1} \sum_{m_\alpha} \langle \vec{r} | n_\beta l_\alpha j_\alpha m_\alpha \rangle \langle n_\alpha l_\alpha j_\alpha m_\alpha | \vec{r} \rangle \; d\vec{r}$$

The second term in the curly brackets arises due to the charge exchange operator P_τ. For p-p or n-n case, the curly bracket identically vanishes, however, in the p-n case, only the first term contributes. Then we have the expression,

$$I = \sum_\beta c^i_\beta \frac{t_3}{4} \cdot \frac{1}{2j_\alpha+1} \cdot \sum_{m_\alpha} \int \rho_p(\vec{r})\, \rho_n(\vec{r}) \langle \vec{r} | n_\beta l_\alpha j_\alpha m_\alpha \rangle \cdot \langle n_\alpha l_\alpha j_\alpha m_\alpha | \vec{r} \rangle \; d\vec{r}$$

$$= \sum_\beta c^i_\beta \cdot \frac{t_3}{4} \cdot \frac{1}{2j_\alpha+1} \cdot \sum_{m_\alpha} \langle n_\alpha l_\alpha j_\alpha m_\alpha | \rho_p \rho_n | n_\beta l_\alpha j_\alpha m_\alpha \rangle \qquad \text{B(8)}$$

In appendix C, we will show that only the zeroth multipole $(\rho_p \rho_n)_0$ of the product $\rho_p \rho_n$ contributes in B(8). Then we have since the integral does not depend on m_α,

$$I = \sum_\beta c^i_\beta \frac{t_3}{4} \langle \alpha | (\rho_p \rho_n)_0 | \beta \rangle \qquad \text{B(9)}$$

This is what we required to prove B(2). B(1) now can be written as,

$$\sum_{\beta} \langle \alpha | h | \beta \rangle c_{\beta}^{i} = \epsilon_i c_{\alpha}^{i} \qquad \text{B(10)}$$

$$\langle \alpha | h | \beta \rangle = \langle \alpha | t | \beta \rangle + \sum_{j\,occ}^{A} \langle \alpha j | \tilde{v}_{12} + \tilde{V}'_{12} | \beta j \rangle$$

$$+ \frac{t_3}{4} \langle \alpha | (\rho_p \rho_n)_0 | \beta \rangle \qquad \text{B(11)}$$

The equations B(10) and B(11) together define the Hartree-Fock equations for the scalar band averaged density dependent interaction defined and used in chapter IV. The integration in B(11) involving density can be easily effected making use of the Talmi integrals.