CHAPTER II

EFFECTIVE INTERACTIONS

II.0 Introduction

In this chapter we shall review and discuss properties and some applications of Skyrme interaction and briefly discuss the Sussex interaction. These are the effective interactions employed to study intrinsic as well as spectroscopic properties of nuclei in this thesis.

Two nucleons interact differently in the presence of other nucleons than in free space. Therefore the calculation of the effective interaction between two nucleons in the nucleus in terms of the interaction between to nucleons in free space is required. It turns out that this interaction is density dependent. It can be shown that this is a natural consequence of Pauli's exclusion principle[7]. After defining the Skyrme interaction[8,9], we shall review the efforts to show its close connection with the realistic forces[16,17]. We shall very briefly discuss the several variants of the Skyrme interaction recently obtained by Beiner et al.[18] and inter-relationship of the various interaction parameters. We shall see why a two-body density dependent force which simulates a three-body contact force in Skyrme interaction[20,21] is preferred. We shall then discuss pairing in

Skyrme interaction and its suitability for spectroscopic calculations. We shall show there that a proper choice of the value of oscillator parameter (b) leads to proper pairing trend. Finally, we shall briefly discuss the salient features of Sussex interaction[23,24].

II.1 The origin of density dependence in effective interaction in nuclei.

The effective interaction between two nucleons in the presence of other nucleons in terms of free nucleon-nucleon interaction is derived by Brueckner-Goldstone theory[1-7]. From this theory, it is quite straightforward to see that the density dependence of the effective interaction is a natural consequence of the Pauli's exclusion principle.

The basic quantity of Bethe-Brueckner-Goldstone theory is the reaction matrix G which defines the effective interaction between two nucleons in the presence of others. It satisfies the matrix (integral) equation[5,6,7]

$$G = V - V \frac{Q}{e} G \qquad \text{II(1)}$$

where V is the free nucleon-nucleon interaction. More explicitly this can be written,

$$\langle kl \mid G \mid ij \rangle = \langle kl \mid V \mid ij \rangle$$

$$- \sum_{mn} \langle kl \mid V \mid mn \rangle \frac{Q(mn)}{E(m)+E(n)-W} \langle mn \mid G \mid ij \rangle \qquad \text{II(2)}$$

In II(1) and II(2), Q is the Pauli operator. Since in a nucleus, two nucleons can not scatter into orbits already occupied by other nucleons, it is equal to 1 if both the intermediate states $|m\rangle$ and $|n\rangle$ are unoccupied, otherwise zero. In the energy denominator,

$$e = E(m) + E(n) - W \qquad \text{II(3)}$$

E(m) and E(n) are the energies of the intermediate unoccupied states $|m\rangle$ and $|n\rangle$ and W is the starting energy. If both the states $|i\rangle$ and $|j\rangle$ are occupied,

$$W = E(i) + E(j) \qquad \text{II(4)}$$

For given W, G matrix is Hermitian. G always operates on two-particle wave function, i.e. on

$$|\phi_{ij}\rangle = \mathcal{A}\,|\phi_i(1)\rangle|\phi_j(2)\rangle \qquad \text{II(5)}$$

where $\mathcal{A}$ is the antisymmetrization operator. This is called the two-body "unperturbed" wave function. Because of the presence of other nucleons, there are correlations introduced in it and we have the "correlated" two-body wave function $|\psi_{ij}\rangle$ given by the equation,

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \frac{Q}{e} G\,|\phi_{ij}\rangle \qquad \text{II(6)}$$

It is then easy to show that,

$$V|\psi_{ij}\rangle = G|\phi_{ij}\rangle \qquad \text{II(6a)}$$

The operator G can thus be considered as the "effective interaction". Eqn.II(6) can be written as,

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \frac{Q}{e} V|\psi_{ij}\rangle \qquad \text{II(6b)}$$

From eqn.II(1) and II(2), it is evident that the effective interaction G between two nucleons depends very much on the presence of other nucleons and thus on their density through the Pauli exclusion operator Q because the other nucleons prevent them from scattering into the states which they occupy. This makes G a very complicated function. The simplifying assumption made for G is that the effect of other nucleons is accounted for by representing G as a density dependent function of the two nucleon co-ordinates. We may have,

$$G \sim V_{eff}(\vec{r}_1, \vec{r}_2, \rho)$$
$$= V_1(\vec{r}) + V_2(\vec{r})\ f[\rho(\vec{R})] \qquad \text{II(7)}$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$ and $\rho(\vec{R})$ is the density of nuclear matter at the point $\vec{R}$. In a finite nucleus, we assume that two nucleons interact as in nuclear matter whose

density equals the density at the mean position of the two nucleons. This approximation is known as the local density approximation.

One thus sees that the density dependence of the G-matrix or the effective interaction is a feature that naturally arises because of the Pauli's exclusion principle. Calculations with such realistic forces are quite complicated and thus a simpler G-matrix is desirable. Skyrme proposed a simple parametrization of G-matrix[8]. It is purely a phenomenological fit to the G-matrix with zero range forces and a three-body force simulating a two-body density dependent force. Due to its simplicity, the Skyrme force has proved to be quite attractive and popular. There are several versions of Skyrme force which we shall briefly review in the next section.

II.1.1 The Skyrme Interaction

The Skyrme interaction[8] has a two-body part V_{12} and a three-body part V_{123},

$$V = V_{12} + V_{123} \quad . \qquad \text{II(8)}$$

Skyrme used a short range expansion of two-body interaction. For Hartree-Fock calculations only low-momentum matrix elements ($k, k' \leq 2\ k_F$) are required where $\vec{k}$ and $\vec{k}'$ are relative wave-vectors of the two nucleons. In momentum space, the matrix

elements of the two-body term between $\vec{k}$ and $\vec{k}'$ are given by

$$\begin{aligned}\langle \vec{k}' | V_{12} | \vec{k} \rangle = {} & t_0 (1 + x_0 P_\sigma) \\ & + \tfrac{1}{2} t_1 (k^2 + k'^2) \\ & + t_2 \vec{k} \cdot \vec{k}' \\ & + iW (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{k} \times \vec{k}')\end{aligned} \qquad \text{II(9)}$$

This can be shown to correspond to a short range expansion of two-body interaction[8,9]. In the configuration space, V_{12} can be written as,

$$\begin{aligned}V_{12} = {} & t_0 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \\ & + \tfrac{1}{2} t_1 [\delta(\vec{r}_1 - \vec{r}_2) k^2 + k'^2 \delta(\vec{r}_1 - \vec{r}_2)] \\ & + t_2 \vec{k}' \cdot \delta(\vec{r}_1 - \vec{r}_2) \vec{k} \\ & + iW (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k}\end{aligned} \qquad \text{II(10)}$$

The operator $\vec{k} = \dfrac{\vec{\nabla}_1 - \vec{\nabla}_2}{2i}$ acts on the right while $\vec{k}' = -\dfrac{\vec{\nabla}_1 - \vec{\nabla}_2}{2i}$ acts on the left.

By considering the matrix elements of II(10) in a state of relative motion $\psi(\vec{r}) = R(\vec{r}) Y_{lm}(\theta, \phi)$, it is easy to see that the matrix elements of the first two terms are proportiona

to $|\psi(0)|^2$ and $\psi(0)\nabla^2\psi(0)$ respectively and hence these two terms correspond to s-wave interactions. The matrix elements of the remaining two terms are proportional to $|\vec{\nabla}\psi(0)|^2$ and hence they correspond to P-wave interactions. The last term is the zero range limit of a two-body spin-orbit force[11].

For the three-body force, Skyrme assumed a contact force,

$$V_{123} = t_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3) \qquad \text{II(11)}$$

It was shown by Vautherin and Brink[9,10] that in Hartree-Fock calculations of even-even nuclei such a term is equivalent to a two-body density dependent interaction

$$v_{12} = \frac{t_3}{6}(1+P_\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right) \qquad \text{II(12)}$$

This is the form of interaction we shall use in this work.

The density dependent two-body force in Skyrme interaction describes how the interaction between two nucleons is modified due to the presence of other nucleons. Skyrme interaction is a phenomenological representation of G-matrix which includes the effects of short range correlations particularly through the density dependent term.

There are several versions of Skyrme interaction. For example Moszkowski[12,13] used the form:

$$V = -\alpha\,\delta(\vec{r}_1 - \vec{r}_2) + \beta\,\vec{p}^{\,2}\,\delta(\vec{r}_1 - \vec{r}_2) + \gamma\,\rho^{n}\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\delta(\vec{r}_1 - \vec{r}_2)$$

II(12a)

This interaction differs from that given by eq.II(8) by the absence of p-wave interactions. The index n is taken to be 2/3 or 1. The value n=2/3 was suggested by Bethe[14]. Thus there are only three interaction parameters for a fixed n, viz. α , β and γ .

Another form of Skyrme interaction was used by Krewald[25] et al. and Liu[26]. They have terms in addition to the terms of Skyrme interaction (eq.II(10) and II(12)) which have density dependence along with momentum dependence. We shall not consider these forms of Skyrme interaction.

II.1.2 Significance of various parameters of Skyrme Interaction

The significance of various parameters in the Skyrme interaction can be best visualized in the energy density formalism for the variational calculations of finite nuclei, as given

by Vautherin and Brink[9]. From their expressions for the Hamiltonian density $H(\vec{r})$ (the expectation value of total energy $E = \int H(\vec{r})\, d^3r$), one can see that the parameter $(9t_1 - 5t_2)$ is important for surface effects, since it determines the importance of the term $(\vec{\nabla}\rho)^2$ in the energy density. Larger values of $(9t_1 - 5t_2)$ give larger surface thicknesses. They show that the parameter $(3t_1 + 5t_2)$ is important for single particle energy levels. Also for a given value of energy per nucleon E/A and the Fermi momentum k_F in nuclear matter, the coefficient t_3 increases linearly with the nuclear matter incompressibility K, where

$$K = k_F^2 \frac{\partial^2 (E/A)}{\partial k_F^2} \qquad \text{II(13)}$$

The last term in II(9) and II(10) gives rise to the single-particle spin-orbit force.

It may be mentioned that the parameter x_0 takes care of the neutron-proton asymmetry properties, since $x_0 P_\sigma \delta \equiv -x_0 P_\tau \delta$ while operating on antisymmetrized states.

II.2 Relation between Skyrme force and realistic forces.

With only six parameters the Skyrme interaction is able to reproduce binding energies, sizes and other intrinsic properties

of nuclei all over the periodic table. Quality of Hartree-Fock calculations performed with this interaction is quite high compared with the simplicity of the force. The realistic G-matrices derived via the local density approximation[15] are shown to describe accurately the interaction between two nucleons in a nucleus, while maintaining the contact with nuclear matter theory based on a realistic interaction. Results obtained with these forces are quite close to the predictions made with the Skyrme force. If one tries to relate these two theories via a moment expansion, one finds that this does not allow truncation of the reaction matrix . Therefore a rearrangement of the expansion was carried out by Negele and Vautherin[16,17] so that the effect of the long range part of the force on the nuclear matter density matrix could be included exactly to lowest order. To perform such a rearrangement, it is not convenient to consider the Taylor series for the interaction in momentum space. Hence one considers the equivalent short-range series for the density matrix defined by

$$\rho(\vec{R}_1, \vec{R}_2) = \sum_{i=1}^{A} \langle \phi_i | \vec{R}_1 \rangle \langle \vec{R}_2 | \phi_i \rangle \qquad \text{II(14)}$$

and then this is expanded in powers of the relative co-ordinate $\vec{S} = \vec{R}_1 - \vec{R}_2$ and the coordinate $\vec{R} = \frac{\vec{R}_1 + \vec{R}_2}{2}$, so that

$$\rho(\vec{R}_1, \vec{R}_2) = \rho(\vec{R} + \vec{s}/2, \vec{R} - \vec{s}/2)$$

II(15)

It has been shown in Ref.(16) that in the case of finite nuclei the first two terms of the expansion give a very good approximation to the density matrix both in the interior of the nucleus as well as on the surface. Considering the fact that the nucleons interact via a G-matrix, this approximation can be used to derive expression for the energy density of a nucleus which is formally identical to that obtained with the Skyrme force. From these density-matrix expansion (DME) calculations, it is possible to extract the numerical value of the Skyrme parameters (except W). It is clear that there is a close connection between the Skyrme interaction and the DME considering the two nucleons interact via a G-matrix. This is the reason why the results obtained with either Skyrme or realistic forces are quite close to each other.

II.3 Variants of the Skyrme Interaction.

Skyrme adjusted the parameters in his force by fitting the binding energy and density of nuclear matter and also binding

energy differences of some light nuclei calculated with oscillator wave functions. These parameters gave too small radii for heavy nuclei. To reproduce these properties as well, Vautherin and Brink carried out a readjustment of these parameters[9]. They obtained two sets of parameters by fitting properties of ^{16}O and ^{208}Pb. Both these sets provide a good description of binding energies and radii of doubly-closed-shell nuclei. Recently a careful study of these parameters was made by Beiner et al.[18]. By requiring that these parameters accurately reproduce the total binding energies and charge radii of magic nuclei in spherical self-consistent calculations, they showed that several parameter sets or variants of Skyrme interaction can satisfy these requirements. They report four sets of parameters SIII to SVI (besides the set SII of Vautherin and Brink) corresponding to quite different values of t_3 and hence covering a wide range of density dependence. These variants are shown in Table II.1. The set SV has $t_3=0$ and hence it is a simple two-body force with no density dependence. These variants differ essentially by the single particle spectra they give. It was also shown by Flocard and Quentin[19] that simple Hartree-Fock calculations using the interactions SIII and SIV are able to give correctly some static properties like total binding energies, radii and deformations of the ground state of s-d shell nuclei. In fact, good overall agreement is obtained for various experimental data all over the periodic

Table II.1

Parameters of the Skyrme Interactions

	t_0 MeV fm^3	t_1 MeV fm^5	t_2 MeV fm^5	t_3 MeV fm^6	χ_0	W MeV fm^5
SI	-1057.3	235.9	-100.0	14463.5	0.56	120.0
SII	-1169.9	586.6	-27.1	9331.1	0.34	105.0
SIII	-1128.75	395.0	-95.0	14000.0	0.45	120.0
SIV	-1205.6	765.0	35.0	5000.0	0.05	150.0
SV	-1248.29	970.56	107.22	0.0	-0.17	150.0
SVI	-1101.81	271.67	-138.33	17000.0	0.583	115.0

table with all the variants SII to SVI.

II.3.1 Interrelationship of Skyrme Parameters

The variants SII to SVI of the Skyrme interaction have the strength parameter t_3 for the density dependence of the interaction ranging from 0.0 to 17000 MeV fm^6. It would be quite instructive to study if there is any functional relationship among the parameters of various sets. If we plot the various parameters for the sets SII to SVI (Figs.II(1) to II(3)) as a function of t_3, we see that the parameters t_0, t_1 and t_2 are linearly related to t_3 and hence to each other. Hence given a value of t_3, we can fix these parameters. The parameters x_0 and W, however, would need to be adjusted since they do not have a linear dependence on t_3. (The variant SI of Vautherin and Brink[9] does not fall on the straight line since it gives too small radii[25]). It would be interesting to carry out calculations with such an intermediate set of parameters and compare results with those of the variants SII to SVI. This would enable one to perform calculations with any value of t_3 and make a systematic study of the density dependence of the interaction on various properties of nuclei.

Vautherin and Brink[9] showed that this sort of linear relationship among the parameters of the Skyrme force exists in the case of nuclear matter. The variants SII to SVI yield

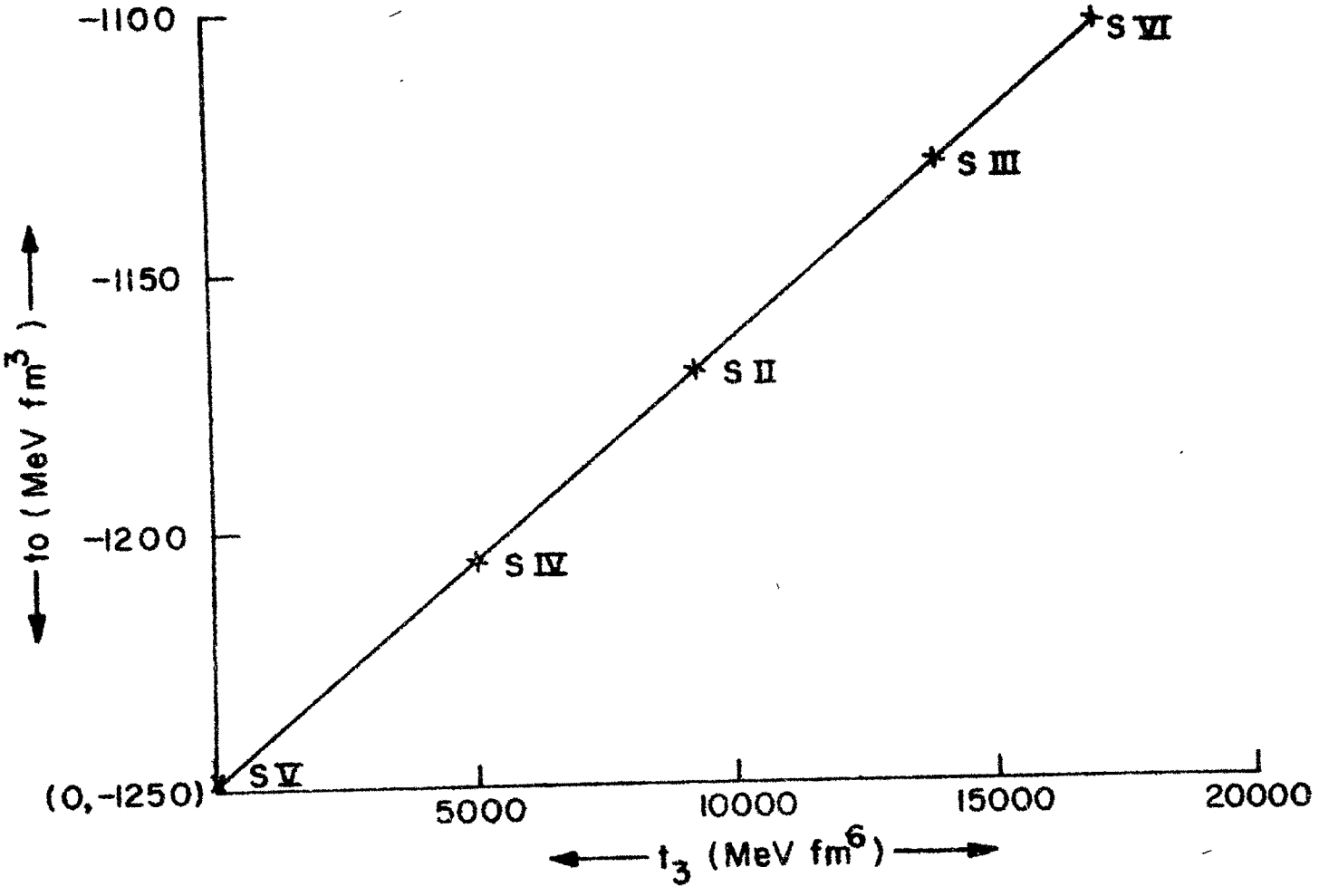

Fig.II(1): t_o plotted as a function of t_3 for Skyrme variants SII to SVI.

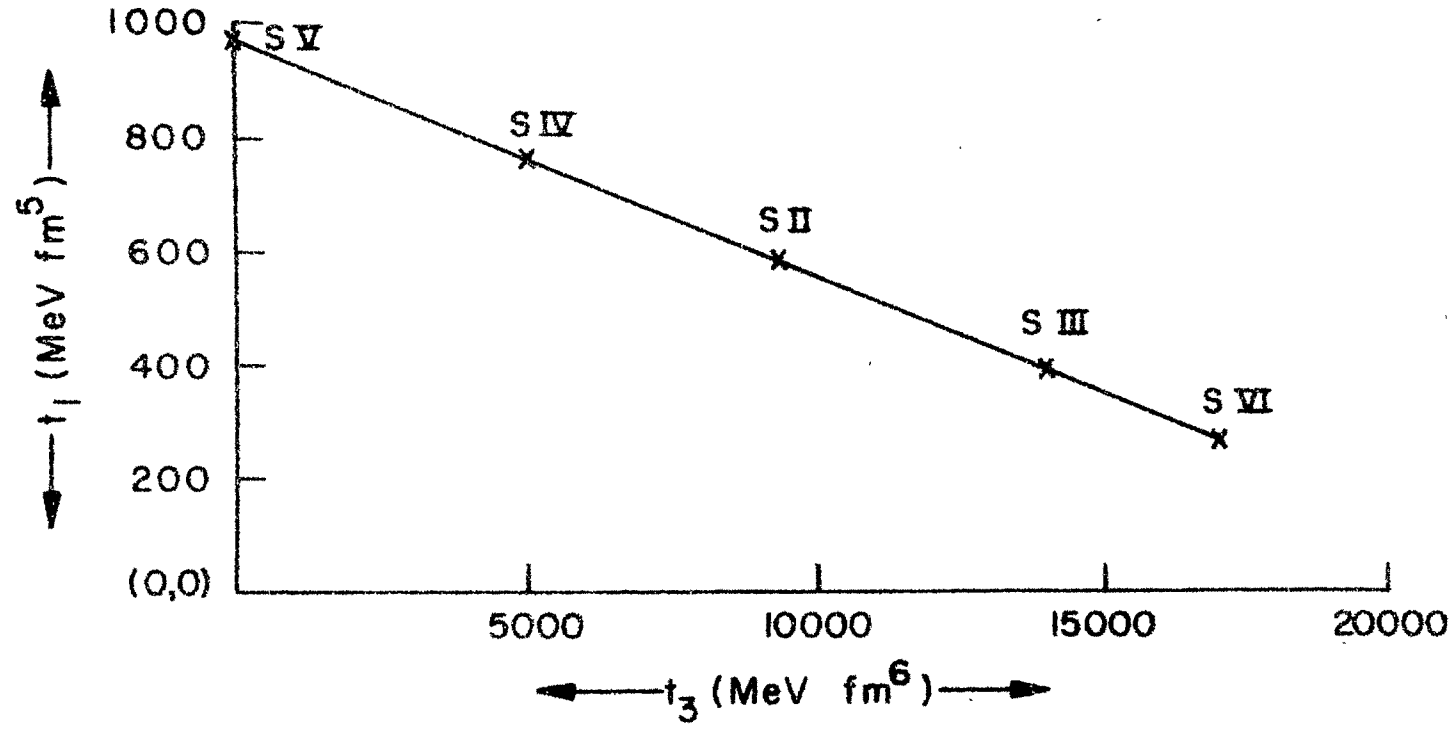

Fig.II(2): t_1 plotted as a function of t_3 for Skyrme variants SII to SVI.

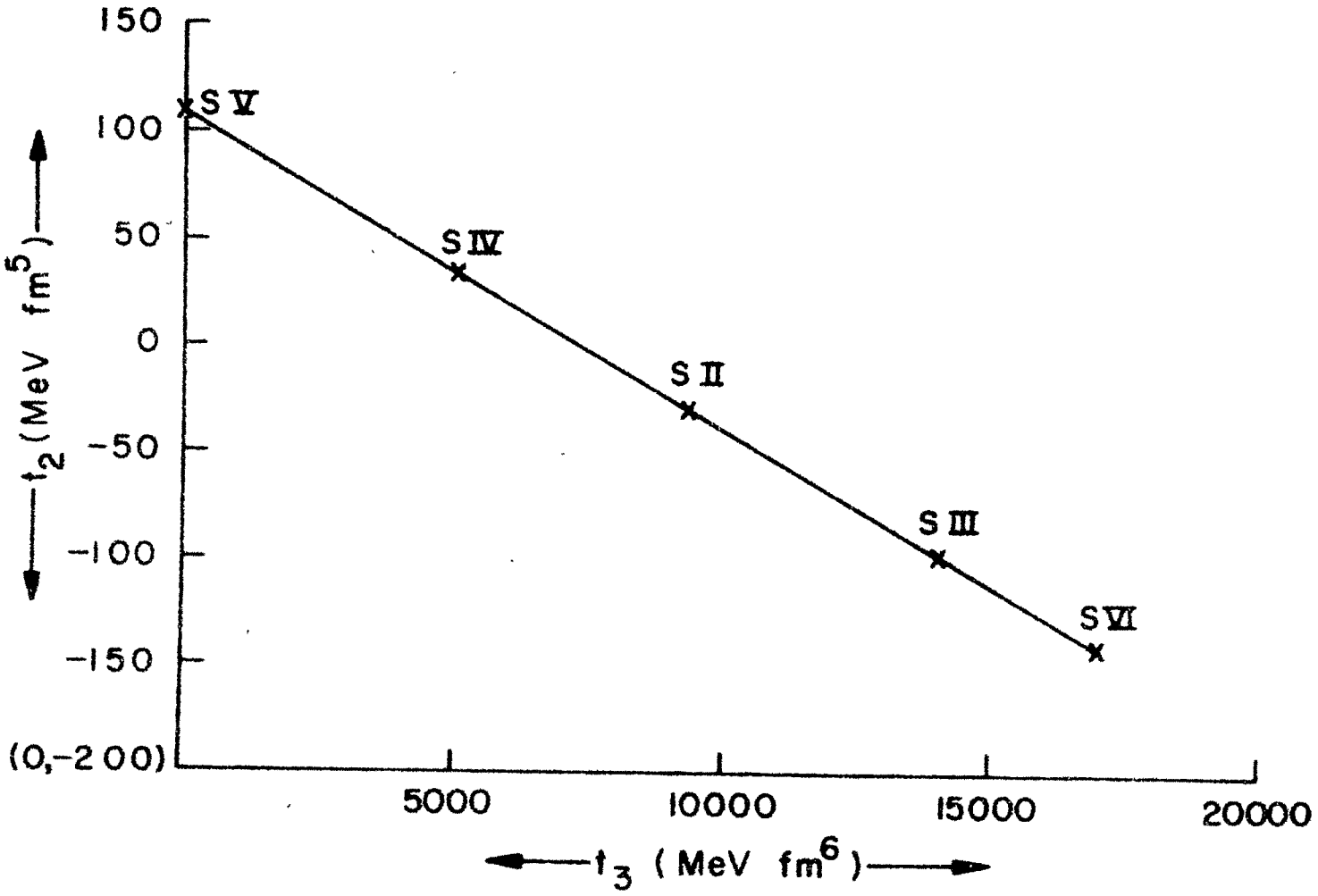

Fig.II(3): t_2 plotted as a function of t_3 for Skyrme variants SII to SVI.

good agreement for nuclear matter as well as finite nuclei. This is quite significant since in a finite nucleus, the surface effects are quite important which tend to reduce the binding energy per particle compared to the nuclear matter value. We can thus have a variant of the Skyrme force for any desired value of the strength parameter t_3 though W and χ_0 will have to be treated as free parameters. Although the several variants of Skyrme interaction reproduce bulk properties of nuclei quite well, very few attempts are directed towards the study of spectroscopic properties. In fact, spectroscopic calculations would provide best criteria to choose among the several variants of Skyrme interaction and therefore such calculations are quite essential.

II.4 Some limitations of the Skyrme interaction

Despite the success achieved in reproducing bulk properties of nuclei all over periodic table with the Skyrme force, it has its own limitations. We shall briefly discuss them in this section.

II.4.1 Calculations for nuclear matter

Although the variants of Skyrme interaction give a value for binding energy per particle in nuclear matter close to the accepted value of -16 MeV, the value for Fermi momentum at saturation is always smaller than the usual value of k_F=1.36 fm^{-1}. Also the incompressibility coefficient K is always found to be

larger than 300 MeV compared to the value of 200 MeV given by realistic forces[18,26].

II.4.2 Density dependent two-body versus three-body δ-interaction

When three-body version of Skyrme interaction is used (eq. II(11)) instead of two-body density dependent one (eq.II(12)), the variants with strong three-body terms (the variant SIII in particular) can lead to spin-aligned Hartree-Fock solutions for odd and odd-odd nuclei[20]. It gives spin-aligned solutions also for nuclear matter and even-even nuclei if the time-reversal invariance symmetry is not imposed[20,21]. The nuclei become overbound by some 100 M.eV. The reason for this catastrophe is the use of three-body contact force in the Hamiltonian. It is shown in the Refs.(20) and (21) that the contribution of the three-body term to total energy vanishes if the nucleus is spin-aligned, instead of contributing to saturation. This difficulty can be overcome by using the two-body density dependent force (eq.II(12)) instead of three-body contact force. Both the versions are equivalent in the Hartree-Fock calculations of even-even nuclei with time-reversal invariance. The contribution to total energy due to the two-body density dependent term does not vanish for spin-aligned solutions and thus contributes to saturation. Two-body density dependent force is therefore always preferred to three-body contact force.

II.5 Pairing in Skyrme interaction

Uptill now our discussion was mainly centred about the connection of Skyrme interaction with realistic interactions and its success in reproducing the static ground state properties of the nuclei without any reference to spectroscopic properties. It is necessary to test the suitability of the interaction for this purpose.

Sharp and Zamick[22] calculated particle-particle two-body coupled pairing matrix elements using harmonic oscillator wave functions for the state $(0f_{7/2})^2$ for the configuration in ^{42}Ca, important for the ground state energy spectrum. They report that this pairing matrix element for the Skyrme interaction (eq.II(10)) gives a repulsive value for the J=0, T=1 and higher states. This would lead to an unphysical energy spectrum for ^{42}Ca. This suggests that one should check such pairing matrix elements for Skyrme interaction before calculating spectroscopi properties.

We show in this section that this difficulty can be overcome by a proper choice of the value of the oscillator parameter b. We studied the pairing matrix elements $(0s_{1/2})^2$, $(0p_{3/2})^2$, $(0d_{5/2})^2$ and $(0f_{7/2})^2$ for J=0, T=1 states as a function of the oscillator parameter b for the Skyrme variant **SIV.**

For the $(0s_{1/2})^2$ and $(0d_{5/2})^2$ matrix elements (m.e.), only t_0 and t_1 terms contribute while for $(0p_{3/2})^2$ and $(0f_{7/2})^2$ m.e., the t_2 and W terms also contribute a little. In either case, the most contribution to these matrix elements is due to t_0 and t_1 terms. Only t_0 and t_1 m.e. would decide whether the total m.e. would be attractive or repulsive.

We have plotted the pairing matrix elements (J=0,T=1) for $(-t_0)$ and t_1 terms for the $(0s_{1/2})^2$ and $(0d_{5/2})^2$ states and also the total m.e. (Fig.II(4) and II(5)). The density dependent part of the Skyrme interaction will not contribute to the matrix elements for T=1 states due to antisymmetry requirements.

For the $(0s_{1/2})^2$ J=0, T=1 m.e. (Fig.II(4)), one can see that the t_0 m.e. are always larger than the t_1 m.e. for any value of b amd hence this m.e. shall always be attractive. Similar features are exhibited by $(0p_{3/2})^2$ m.e.

The situation is rather interesting for the $(0d_{5/2})^2$ m.e. (Fig.II(5)). For small values of b, the t_0 m.e., though attractive is numerically smaller than t_1 m.e. and the total m.e. is therefore positive. After a critical value of b, the t_0 m.e. becomes numerically larger than t_1 m.e. for all values of b and the total m.e. now becomes attractive. In the calculations of nuclear properties, the value of b must be larger than the critical b (b=1.7 fm in this case), otherwise the results would be unphysical. Similar behaviour is exhibited by $(0f_{7/2})^2$ m.e.

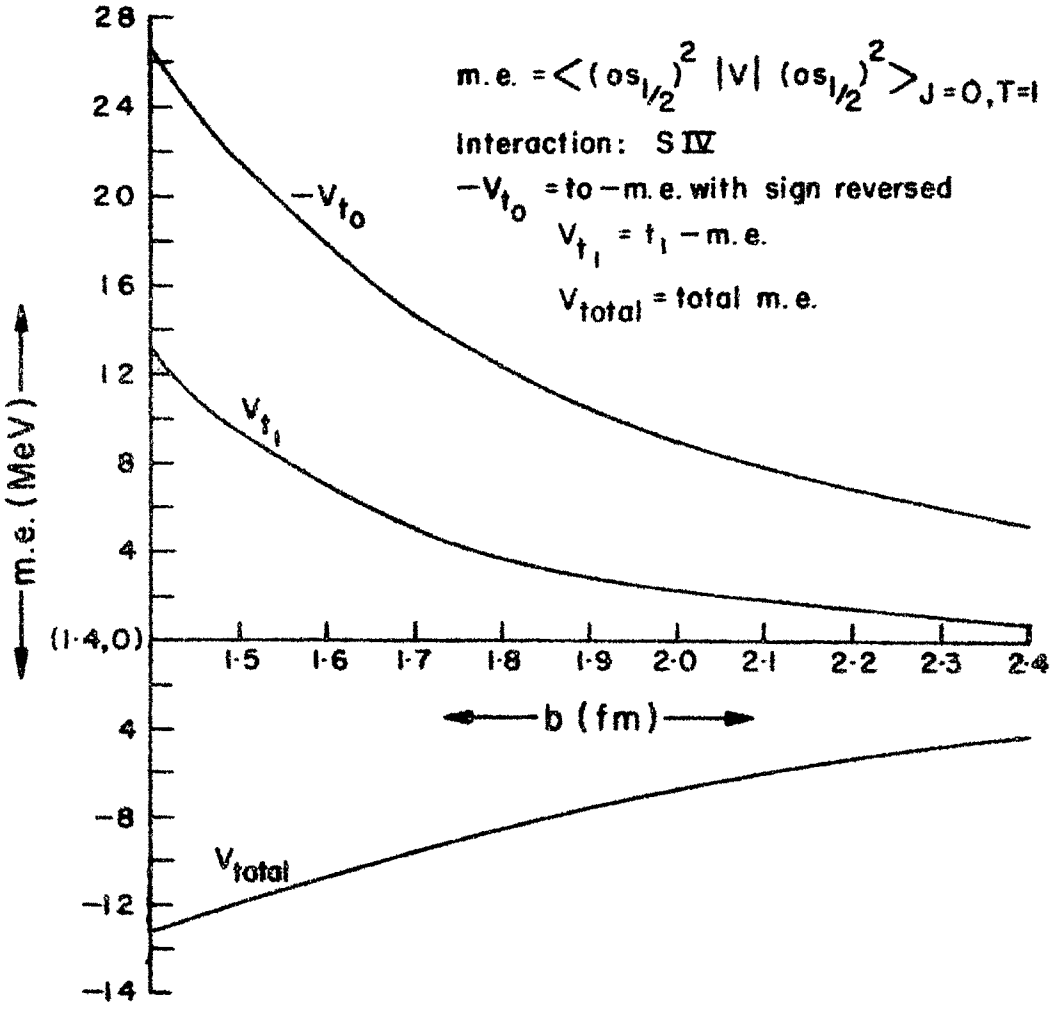

Fig.II(4): The matrix element $\langle (0s_{1/2})^2 \mid V \mid (0s_{1/2})^2 \rangle_{J=0,T=1}$ plotted as a function of the oscillator parameter b.

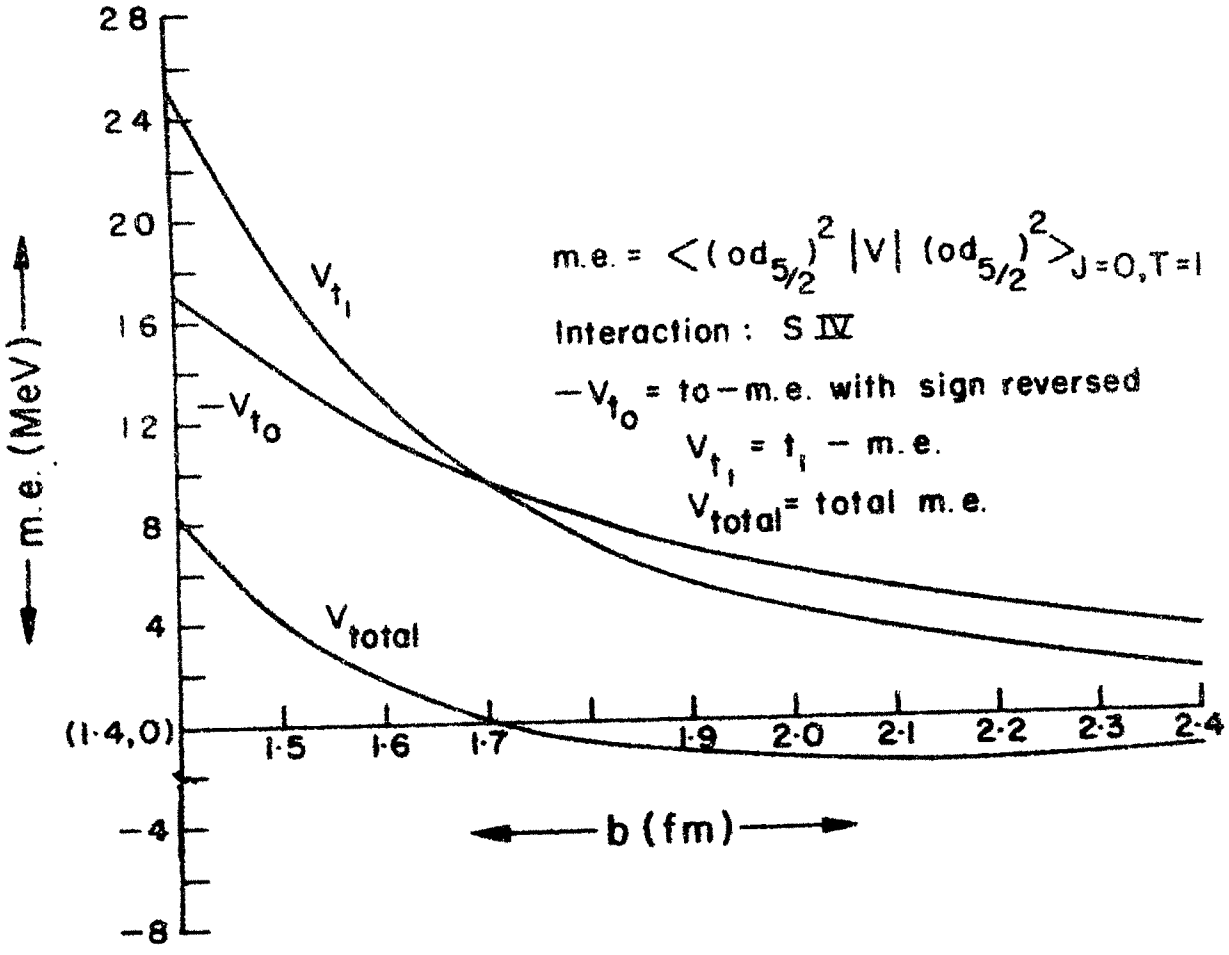

Fig.II(5): The matrix element $\langle (0d_{5/2})^2 | V | (0d_{5/2})^2 \rangle_{J=0,T=1}$ plotted as a function of the oscillator parameter b.

The source of this behaviour is the severe b dependence of the m.e. of various parts of the Skyrme force. The t_0 m.e. fall off as b^{-3} while others as b^{-5} except t_3 m.e. which fall off as b^{-6}. There is a competition between the t_0 and t_1 m.e. which leads to the pairing or the antipairing of the m.e. For b value larger than the critical value, the m.e. of t_1 etc. fall off much faster than t_0 and the total m.e. becomes attractive.

We state that the difficulty raised by Sharp and Zamick can be remedied by optimizing b for every nucleus in question. One can then proceed to the calculation of nuclear properties.

II.6 A note on Sussex interaction

Another interaction that we have used in this work for comparison with the results of Skyrme interaction is the Sussex interaction deduced directly from the nucleon-nucleon phase shifts ~ without specifying the potential in the harmonic oscillator basis. This method is essentially a distorted wave Born approximation. The assumptions made were, (i) the potential is non-singular and (ii) the information concerning scattering at energies above 300 MeV is not required for an understanding of low-energy nuclear properties and spectra.

Elliott et al.[23] derived the set of relative radial integrals $\langle nl(S)j \,|\, V \,|n' l' (S)\, j\rangle_T$ where $\vec{l}$ is the relative angular momentum between two nucleons and $\vec{j} = \vec{l} + \vec{S}$. These radial

integrals can be readily employed to calculate two-body coupled matrix elements.

The potential is divided into two parts, viz.

$$V = V_0 + V_1 \qquad \text{II(16)}$$

where V_0 is the auxiliary potential and V_1 is supposed to be small. If δ is the experimentally observed phase-shift and δ_0 that calculated for the auxiliary potential V_0 at the same energy and if at this energy V_1 is small enough to be treated in Born approximation, then it can be shown that,

$$\int_0^\infty \bar{u}_k(r)\, V_1\left(r, \frac{\partial}{\partial r}\right) u_k(r)\, r^2\, dr = -\frac{\hbar^2 k}{m} \tan(\delta - \delta_0) \qquad \text{II(17)}$$

where $u_k(r)$ is the radial wave function for scattering by the auxiliary potential. $\vec{k}$ is the relative wave vector. For an auxiliary potential of the cut-off oscillator type,

$$V_0 = -\frac{\hbar^2}{m}\left(\alpha - r^2/4b^4\right) \quad \text{for } r < a$$
$$= 0 \quad \text{for } r > a \qquad \text{II(18)}$$

(where α is the depth, a the range and $b = \sqrt{\hbar/m\omega}$ the oscillator parameter), one can have under certain conditions for $r < a$,

$$u_k(r) = A\, R_{nl}(r) \qquad \text{II(19)}$$

where A is a constant. Then one can write,

$$\int_0^\infty R_{nl}(r)\, V\, R_{nl}(r)\, r^2\, dr = -\frac{\hbar^2}{m}\int_0^a R_{nl}^2(r)\,(\alpha - r^2/4b^4)\, r^2\, dr$$

$$-\frac{\hbar^2 k}{m A^2}\tan(\delta - \delta_0) \qquad \text{II(20)}$$

The off-diagonal m.e. are calculated by differentiating the diagonal m.e. (eq.II(20)) by b and obtaining a recurrence relation. This, however, is not adequately reliable.

The errors arise in deducing this realistic bare G-matrix essentially from the assumption that $(V-V_o)$ be small to be treated in the Born approximation, and deducing off-diagonal m.e. Another source of uncertainty is the experimental error in determining the phase-shifts.

Although the Sussex interaction[23] gives reasonable results for spectroscopic properties of nuclei, they are quite

underbound, the r.m.s. radii are quite small and do not correspond to correct saturation densities. This interaction was therefore modified[24] to include in relative s-states a central spin-independent potential, V_c which has an infinite hard core plus an attractive outer region. The parameters of the attractive part were chosen so that the two-nucleon phase-shift fit of the original Sussex interaction was maintained while the core-radius, c, remained a free parameter which could be adjusted to give the correct saturation densities for the nuclei. One is forced to use Brueckner's theory so that a finite G-matrix could be used in the Hartree-Fock calculations, because of the presence of the hard core. Energy spectra for p-shell nuclei have been calculated using this interaction by Dirim et al.[27] using the value for core-radius c=0.3 fm. These calculations are rather cumbersome and so we have chosen to use the original Sussex interaction[23].

II.7 Summary

In this chapter we discussed the Skyrme interaction in detail and made some brief comments about Sussex interaction. We remarked that the Skyrme interaction has been quite successful in reproducing bulk properties of nuclei all over the periodic table only with six parameters. There are several variants of the Skyrme force all of which more or less reproduce ground state binding energies, r.m.s. radii etc. quite well. We discussed the

inter-relationship of various parameters of the Skyrme interaction. We showed that by optimizing the value of the oscillator parameter b, one has desired property of pairing for ($J=0, T=1$) m.e. We also discussed the relation of Skyrme interaction to realistic interactions.

There are however very few attempts to calculate spectroscopic properties with the Skyrme interaction. In fact, spectroscopic calculations will provide best criteria to choose among the sets of parameters of the Skyrme interaction. We have therefore undertaken to study this aspect in this work. Spectroscopic calculations with Skyrme interaction forms the theme of this thesis.

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