CHAPTER-2

REVIEW OF LITERATURE

2.1 Introduction

In the last few decades, some theoretical and experimental work has been carried out by investigators for predicting the stress distribution and behaviour of deep beams. A brief review of some of the works is presented here.

2.2 Theoretical Investigations

Using the classical theory of bending, the stress and deformation at any cross section may be predicated for shallow beams of constant cross section. This theory assumes that plane sections before bending remain plane after bending and stress distribution at a section is linear. Plane sections before bending do not remain plane after bending and stress distribution becomes nonlinear as span to depth ratio decreases as shown in **Fig.1.1**. The simple theory of bending does not consider the effect of normal pressure on the top and bottom edge of the beam caused by the external loads and reactions respectively. Normal pressure notably affects the bending and shear stress distribution across the transverse sections. Incorrect results will be obtained if usual concepts of flexure and shear are employed in the study of deep beams.

The pioneering work in determining the elastic stress distribution in deep beams was done by **F. Dischinger (51)**. He used trigonometric series to determine stresses in continuous deep beams and stated that if a solution can be found for Airy's stress function from the biharmonic equation $\nabla^4\phi = 0$, stress at any point (x,y) will be given by

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2}$$

$$\tau_{xy} = -\frac{\partial^2 \phi}{\partial x^2 \partial y^2} \qquad .....(2.1)$$

Only approximate solution can be obtained using Airy's stress function. For different loading cases, a particular solution for the biharmonic is chosen. Then, a polynomial is found. This polynomial cancels out stresses on the vertical edges in the particular solution selected. The main difficulty is to satisfy all the boundary conditions.

Saad and Hendry (145) analyzed deep beams under gravitational loads and under concentrated loads using finite difference method. They compared the results with photoelastic test results and concluded that for beams with $L/D \leq 1.0$, maximum stresses due to dead weight can be calculated considering dead weight as a uniformly distributed load applied at top. They also found the normal stresses given by finite difference method are reasonably accurate and simple beam theory is valid for span to depth ratios greater than or equal to 1.5.

Chow, Conway and Morgan (29) suggested a method of superimposing two stress functions. The first function was chosen in the form of a trigonometric series which satisfied all the boundary conditions except that of zero normal stress on the end of the beam. Here principle of least work was used to obtain a second stress function giving the distribution of normal stress on the ends as that produced by the first stress function. Superimpostion of the two solutions satisfied all the boundary conditions. From the two problems solved with a loading on top, it was found that for L/D = 2.0, the actual bending stresses were at par with those obtained from simple beam theory while for L/D = 1.0, actual stresses were much greater than those obtained using simple beam theory.

Raville and McCormick (136) showed that in the case of point loading the stresses in the middle third span could be obtained more easily. They chose the stress functions

$$\phi = \sum^{\alpha} f_m(y) \cdot \cos \alpha_m \qquad \text{......(2.2)}$$

where $\alpha_m = \frac{m\pi}{L}$, m =1,3,5...

The function $f_m(y)$ was determined by requiring the stress function to satisfy the biharmonic equation. They combined an expression for stresses which satisfied the boundary conditions at top and bottom but not at the vertical edges of the beam.

2.3 Photoelastic Tests

Photoleastic tests were carried out on simple deep beams subjected to point loads. Their results were found in reasonable agreement when compared with those obtained using theory of elasticity approach in most of the cases.

Casewell (26) had specimens made from annealed bakelite with span to depth ratio ranging from 1.05 to 5.32. The first test series was subjected to pure bending moment and no effect of change in span to depth ratio on stresses was observed. The second series of beams was also subjected to the same bending moment induced by a central point load. Progressive deviation of the stresses, from that obtained by simple beam theory with decrease in span to depth ratio, was found.

Kear (85) tested aluminimum and steel simply supported deep beams of varying L/D ratios and concluded that for central loading, ordinary bending formula gives erroneous results for L/D ratio less than 1.5.

Saad and Hendry (146), based on a number of tests, declared photoelastic method satisfactory for analysis of stresses in deep beams particularly with unsymmetrical loading and for beams with openings where theoretical solution becomes tedious, extremely difficult and sometimes impracticable.

2.4 Investigation by Models

Kotsovos (90,91,92) tested beams with shear span to depth ratios of 2.27 and 3.4. He tested the beams of span 918 mm and height 102 mm with effective depth of 90 mm. The width of beams was 51 mm. The tensile reinforcement used was 84.83 mm^2. For each a/d ratio and steel area, four types of beams were tested viz.; beams without shear reinforcement (type A); beams with shear reinforcement within the shear span only (type B); beams with shear reinforcement within the middle span only (type C), and beams with shear reinforcement throughout the span (type D). These beams were analysed by using finite element method.

For analysis, the author assumed that plane stress condition prevails. This lessens the significance of the small compressive or tensile stresses that develop in the transverse direction from incompatible deformational response of consecutive concrete elements. Then, based on Newton- Raphson method and residual force concept, nonlinear finite element procedure was adopted.

They observed that, irrespective of amount of tension reinforcement, the maximum load carrying capacity of type C beams was higher than that of type B beams for a/d=2.27. This meant that the beams had collapsed by failure of the compressive zone of the middle span under compression-tension stress conditions and not by failure of the region of the loading point under a wholly compressive state of stress.

They also observed that type A beams exhibited a brittle load-deflection relationship indicating shear failure. However, for over-reinforced concrete beams, all beams exhibited a brittle load-deflection relationship irrespective of shear reinforcement.

During the experimental work, type A beams could not sustain the extension of diagonal cracking through the compressive zone. For type B beams, an increase in load was observed. For type C beams, the failure was observed at next loading stage while type D beams could sustain a substantially large amount of cracking.

They concluded that for reinforced concrete beams with a/d ratio smaller than 2.5, placing shear reinforcement within the middle span rather than the shear span results in a significant improvement in both load-carrying capacity and ductility of the beams. This is so because beams with a/d < 2.5 fail due to branching of the diagonal crack within the shear span towards the compressive zone of the middle span and not due to crushing of the compressive region of the loading point. Also, the collapse of the beams always occurs before the compressive strength of concrete is exceeded anywhere within the beams.

Schlaich and Schafer (150): A unified design concept, which is consistant for all types of structures and all their parts, must be based on realistic physical models. The structural models, a generalisation of the well known truss analogy for beams, are proposed as an appropriate approach for designing structural concrete.

Strut and tie models consider the complete flow of forces within the structure rather than just the forces at one particular section. The internal load path in cracked section is approximated by an idealized truss. Here B and D regions control the design. These regions of a structure, in which linear strain distribution is assumed valid,will be referred to as B regions and the regions where internal forces or stress distribution is nonlinear will be referred to as D regions. The internal flow of forces in D regions can be reasonably well described by strut and tie models. Not much accuracy is necessary in determining the dividing sections between B and D regions. See **Fig. 2.1.**

In a strut and tie model, the strut represents concrete stress fields with prevailing compression in the direction of the strut. Accordingly the ties normaly represent one or several layers of tensile reforcement.

This method of orienting the strut and tie model along the force pattern is obviously neglected by the theory of elasticity approach.

Here struct-tie model can be developed which can provide solution nearer to the actual results. It provides realistic model for the given strut using load path methods. The strut-tie model provides adequate conceptional design for critical structural members. Strut-tie model can be formulated from experimental observations using failure crack patterns and recorded strains in concrete and steel reinforcement.

The regions of a structure in which the Bernoulli hypothesis of liner strain distribution is assumed valid will be referred to as B regions. Here, internal forces or stresses can be derived from moments, shear and axial forces analysed by means of static system.

Mohamed and Morley Tests (108,109) : Shear strength of reinforced concrete wall beams were evaulated by using improved plasticity solution. The same theory when applied to reinforced concrete beams in shear proved successful for some limited cases.

At failure a simple kinematic rigid plastic solution was derived for a stringer model with a straight "yield line". Recently, evidence has emerged that the best single yield line between two rigid wall portions may be curved and not straight. There are different stress states in yield lines and consequently three types of yield lines are identified in the analysis. The principle of rigid body plane motion is used to describe the deformation in failure mechanisms. Their proposed model predicts reasonably well the strength and mechanism for such stringer models.

This model depicts very clearly the failure mechanism. Significant of all the models is the one which uses the principle of rigid body motion with translation as well as rotation. Shear failure was observed in most of the beams.

2.5 Experimental Investigations

Previous sections summarized theoretical investigations, photoelastic tests, and physical models conducted by various investigators. In this section some important experimental investigations on behaviour and modes of failure of deep and moderately deep concrete beams are described.

de Paiva (44,45) tested thirty five simply supported reinforced concrete moderately deep beams, (thirteen statically and twentytwo dynamically) with an object to decide if failure under dynamic loading differs from that under static loading. He also tried to determine if web reinforcement prevents shear failure, and if yes, how much of it is required to do so and also suggested a formula for shear strength of reinforced concrete moderately deep beams. The beams were 710 mm long with a simply supported span of 600 mm loaded with two point loads at one-third span point. The beam depth varied from 325 mm to 175 mm with effective depths of 300 mm, 220 mm and 150 mm for span to depth ratios of 2.0, 3.0, and 4.0 respectively. To keep 'bd' equal to 371500 mm^2 constant, the beam widths were kept as 50 mm, 75 mm and 100 mm. Tension reinforcement was provided from 0.46% to 2.58% while web reinforcement was from 0% to 1.09%. The compression reinforcement was approximately equal to one half of the area of tension reinforcement. Special end anchorages were provided to prevent premature bond and anchorage failure. Eleven of the static beams were without web reinforcement and two had vertical stirrups. Half of the dynamic beams were without web reinforcement while other half were reinforced with either vertical or inclined stirrups. It was observed from the tests that in all the beams inclined cracks were formed eliminating inclined principal tensile stresses necessary for beam action and causes a redistribution of the internal stresses resulting in arch action. Thus reinforcement acts as a tension tie and portion of concrete outside inclined crack acts as an arch rib. All the beams had well developed vertical and inclined cracks at failure and three types of failures, viz., flexure, shear and flexure-shear were observed.

From the test results, de Paiva observed as under :-

(i) Moment capacity increases with the increase in beam depth provided the shear capacity is not reached.

(ii) Increase in the amount of tension steel increases the flexure capacity of beam and tended to change the mode of failure from flexure to shear.

(iii) Increase in concrete strength negligibly affects the ultimate strength of the beams failing in flexure but it increases the strength of the beam failing in shear.

(iv) The presence of nominal amount of web reinforcement has negligible effect on ultimate strength of the beam regardless of the mode of failure.

The presence of web reinforcement influences the stress distribution by reducing the degree of arch behaviour leading towards flexure behaviour with increases in deflection. De Paiva concluded that in moderately deep beams moment at flexure yielding can be predicted by conventional straight line theory while ultimate flexural strength can be predicted by standard ultimate strength procedure provided an increased value of the limiting concrete strain equal to 0.008 is used at failure.

He further noted that for shallow members, the formation of first fully developed crack is synonymous with complete collapse. For moderately deep beams, the formation of a fully developed inclined crack is well defined and the beams are able to carry some load beyond that point. However, for deep beams after the development of an inclined crack, the beams can support considerable additional load before collapse. He gave the following formula for cracking load and for ultimate shear strength.

Cracking Load :

$$\frac{V_x}{bd} = 2.14\sqrt{f_{c'}} + 4600p\left[\frac{V \cdot d}{M}\right]_x \qquad \text{......(2.3)}$$

where V_x = shear at critical section

b = width of member

d = effective depth

$f_{c'}$ = concrete strength of 150 mm x 300 mm cylinders

p = As/bd = tensile steel ratio

$\left[\frac{V \cdot d}{M}\right]_x$ = ratio of shear to moment multiplied by the effective depth computed at the location wherer V_x is computed. Always positive.

This formula gives good results when critical section is taken at middle of the shear span and can be used for beams with or without web reinforcement.

Ultimate Shear Strength :

There are two formulae; one for x/d less than 1.0, the other for x/d greater than 1.0

(A) x/d $\leq$ 1.0

$$v_s = \frac{V}{bd} = 200 + 0.188 f_{c'} + 21300 p_t \qquad .....(2.4)$$

where V = shearing force at failure at "shear proper"

v_s = normal shearing stress

$$p_t = \frac{A_s(1+\sin\alpha)}{bD} \qquad .....(2.5)$$

where A_s = total steel area crossing a vertical section between load point and support

α = angle of inclination of bent up bars

x = clear effective shear span

D = total depth of beam

The shear capacity was given by

$$P_{s''} = 0.8\left(1 - 0.6\tfrac{X}{D}\right)P_{s'}, 0 \le \tfrac{X}{D} \le 1.0 \qquad \text{....(2.6)}$$

where, Ps' = computed shear strength. Can be determined by

Laupa's formula for shear stress (v_s) (99)

$= 2\ v_s\ bD$

$$v_s = \tfrac{V}{bd} = 200 + 0.188 f_{c'} + 21300 p_t$$

where Ps" = ultimate shear strength

(B) x/d ≥ 1.0

(i) Beam without web reinforcement

$$\frac{M_s}{bd^2 f_{c'}} = (k + np')\left[0.57 - \frac{4.5 f_{c'}}{10^5}\right] \qquad \text{.....(2.7)}$$

where, Ms = moment at shear failure

b = width of beam

d = effective depth

$f_{c'}$ = concrete strength of 150 mm x 300 mm standard cylinder

p' = ratio of compression reinforcement

n = modular ratio

$$= 6 + \frac{10000}{f_{c'}}$$

k = theoretical depth of compression zone

$= \sqrt{2pn + (pn)^2} - pn$ for beam reinforced in tension only

$$= \sqrt{2[n.p + (n-1)p'(1-k')] + [(n-1)p' + np]^2} - [(n-1)p' + n.p]$$

where, $p_s = \frac{A_s}{bd}$ ratio of tension reinforcement

$$k' = \frac{d'}{d}$$

d' = distance from the centroid of the compression reinforcement to the centre of tension reinforcement

The above equation was based on the assumption that the shear strength of a beam with web reinforcement is affected not only by the amount and proportion of web reinforcement but also by the shear span of the beam.

(ii) Beam with web reinforcement

$$\frac{P_{sw}}{P_s} = 1 + \frac{2rf_{yw}}{10} \quad \text{.........} (2.8)$$

where P_s = strength of beam without web reinforcement

P_{sw} = strength of beam with web reinforcement

$r = \frac{A_w}{sb}$ = ratio of web reinforcement

A_w = area of stirrup

s = spacing of stirrup

f_{yw} = yield stress of web reinforcement

de Paiva concluded, after examining the test data,that the ultimate shear strength of the beams tested could be better related to the ultimate shear strength of the beams "shear proper" criterion than by "shear moment" criterion which is reflected in the predictions using above discussed equations.

For deep beams, "shear proper" criterion governed and the web reinforcement in the form of vertical stirrups was no longer effective.

All the beams tested statically as well as dynamically formed an arch prior to failure. So, web reinforcement was virtually ineffective in increasing the resistance of the beam to shear failure. This means a deep beam should be considered to be an "arch" rather than a "beam" to prevent a premature "shear" failure of the arch rib above the support in order to produce more ductile type of flexure failure.

Ramakrishnan and Ananthanarayan (135) tested 26 simply supported beams with L/D ratios ranging from 0.9 to 1.8 under uniformly distributed load, concentrated load at mid span and two point loads at the one-third points. The span of all the beams was kept constant (69 cm). The beams were reinforced longitudinally with plain mild steel bars and had little or no web reinforcement. The main objective of this investigation was to develop a formula for prediction of ultimate shear strength of reinforced concrete deep beams.

Four different modes of failure were observed; flexure, flexure-shear, diagonal tension and diagonal compression. Most of the beams failed in diagonal tension mode characterised by a clean and sudden fracture which occurred for concentrated loads along the line joining the support with nearest loading point, and for distributed loading along the line joining the support with nearest third span point. Diagonal compression failure was observed with an inclined crack developed along a line nearly joining the load and the support point, and is followed after a small increase in load by a second parallel crack. Failure is due to the destruction of the concrete strut between these cracks.

They concluded that the shear failure in deep beam is always initiated by splitting action, the phenomenon of failure being similar to that of cylinder under diametrical compression. Based on this, they suggested a formula to predict the ultimate shear strength of deep beams as below.

Ultimate load $P_c = 2k.f_t.BH$ (two point loading) ...... (2.9)

where k = a coefficient its value being 1.12 for beams tested

f_t = concrete splitting strength

B = width of beam

H = height of beam

Kong and Robins (87,88) tested fifty normal weight and thirty eight light weight reinforced concrete deep beams with span to depth ratios ranging from 1.0 to 3.0 and with shear span to effective depth ratio from 0.35 to 1.18. The main purpose was to understand the effect of web reinforcement in simply supported deep beams.

The simple span of specimens was 762 mm and length of 915 mm. Broadly, there were two groups : Normal weight concrete beams reinforced with one 20 mm diameter plain round bar and light weight concrete beams with one 16 mm diameter deformed bar. Depending on type and amount of web reinforcement, each group was subdivided into eight series consisting of five beams of varying depth from 254 mm to 762 mm .

The failures of the beams were usually observed in one of the following ways :

(i) Deep penetration of one or more of the existing diagonal cracks into compressive zone at the loading point or at the support causing immediate failure by crushing of concrete.

(ii) The principal mode of failure was brought about by the propagation of diagonal cracks splitting the beams approximately along the line joining the load block at the support and the loading point.

(iii) Crushing of strut like portion of concrete between two diagonal cracks.

(iv) Crushing of concrete at load bearing blocks.

(v) Vertical splitting above a support, followed by crushing of concrete near the bearing block.

In their test **Robins & Kong** recognized the diagonal cracks as the widest and the most dangerous ones, while flexure cracks remain small and quite harmless. He concluded that initial cracking load is insignificantly affected by web reinforcement but such is not the case with ultimate load.

Robins & Kong proposed the following shear strength formula :

$$V_u = C_1\left(1 - 0.35\tfrac{X}{D}\right)f_t bD + C_2 \Sigma A \tfrac{y_1}{D} \sin^2 \alpha_1 \qquad \text{....... (2.10)}$$

where V_u = ultimate shear strength of beam (N)

C_1 = empirical coefficient whose value is 1.4 and 1.0 for normal weight and light weight concrete respectively.

C_2 = empirical coefficient

X = effective clear shear span

f_t = split tensile strength for cylinder (N/mm^2)

b = breadth of the beam (mm)

D = overall depth of the beam (mm)

A = area of individual web reinforcement bar (mm^2)

Main longitudinal bars are also required to be considered in this formula

y_i = depth at which an individual web bar intersects the potential diagonal crack

α_i = angle between a web bar and line described in the definition of y_i

Potential diagonal crack is assumed to be included at an angle of $\tan^{-1}(D/X)$ to the horizontal. For uniformly distributed top loading, Robins suggested this inclination to be $\tan^{-1}(4D/L)$ to the horizontal.

To predict the flexural capacity, he put forth the following formula :

$$M = 0.87 A_{st} \cdot f_y \cdot L_a \qquad \text{........ (2.11)}$$

where L_a = lever arm

$= d_1 - (0.2D)$ for $L/D \geq 1.0$

$= d_1 - (D - 0.8L)$ for $L/D < 1.0$

Robins & Kong also concluded that the proposed formula can be used for the effects of different geometry of web reinforcement and is also suitable for light weight concrete deep beams.

Smith, and Vantsiotis (157,158) tested 52 deep reinforced cement concrete beams under two concentrated loads, spaced (4 in.) 102 mm from the mid span. For all the beams depth was the same of (14 in.) 350 mm The beams were divided into four series with a/d ratio equal to 0.77, 1.01, 1.34 and 2.01. Of the beams tested, five were without web reinforcement. Vertical web reinforcement of ($0.10\ in^2$) $165\ mm^2$ consisted of closed V-shaped stirrups while horizontal web reinforcement consisted of straight bars.

The authors observed that upto 20% of the ultimate load, no crack was formed. Then, the first vertical flexural crack developed in the region of maximum bending moment. A sudden major inclined crack in the middle of shear span formed at 40% to 50% of ultimate load. An almost stable position of all the existing cracks upto 85% to 90% of ultimate load was observed.

They also observed less damage and smaller crack widths for beams with web reinforcement as compared to the beams without web reinforcement at corresponding load levels.

They concluded that as a/d ratio increases, inclined cracking and ultimate load decreases and for a/d < 2.5, increase in ultimate load is due to arch action and mid span deflection at failure < L/200.

To determine the effect of web reinforcement, the authors suggested the equation.

$$V_s = (V_u)_T - (V_c)_{calc.} \qquad \text{......... (2.12)}$$

$$(V_c)_{calc} = \frac{(f_c')_1}{(f_c')_2} \times (V_u)'_T$$

where V_s = shear strength due to web reinforcement

$(V_c)_{calc.}$ = shear strength due to concrete.

$(Vu)_T$ = measured ultimate shear strength of beam with web reinforcement.

$(f_c')_1$ = concrete compressive strength of beam with web reinforcement.

$(f_c')_2$ = concrete compressive strength of beam without web reinforcement at same a/d ratio.

$(V_u)'_T$ = measured ultimate shear strength of beam without web reinforcement at same a/d ratio.

Patel and Damle (127,128) : In December 1976, they statically tested twenty reinforced concrete simply supported deep beams with span to depth ratio from 1.0 to 2.5 and shear span to depth ratio ranging from 0.33 to 1.25. Ten beams were loaded by concentrated load at midspan and other ten beams were loaded by two point loads at one-third points. Four beams from each had no web reinforcement, three were reinforced with horizontal web reinforcement while the other three had inclined web reinforcement. Longitudinal reinforcement in all the beams consisted of 16 mm diameter plain round mild steel bars. Electrical strain gauges were fixed on the web reinforcement and deflection, crack patterns, cracking loads and ultimate loads were studied.

They suggested that shear failure was actually a diagonal splitting failure and based on the splitting of an elliptical section whose major axis lies on a line joining the load point and support. They suggested a new expression for ultimate shear strength of reinforced concrete deep beam without web reinforcement using following assumptions :

(i) The approximate direction of the diagonal crack is the line joining the load point with the support point.

(ii) The shear strength of a deep beam is dependent upon the splitting strength of the concrete.

(iii) The main longitudinal bars are also considered as web bars in calculating the ultimate shear strength.

(iv) Total shear capacity of a reinforced concrete deep beam is obtained by summing up shear capacity of web reinforcement and shear capacity of concrete.

(v) Shear capacity of concrete is dependent on the splitting of an elliptical section whose major axis lies on the line of diagonal crack defined in (i).

(vi) The effectiveness of a web bar increases with the depth at which it intersects the line of diagonal crack and is dependent on the yield stress of the web reinforcement.

For shear failure, ultimate load capacity of a reinforced concrete deep beam can be given by

$$V_u = V_{uc} + V_{us}$$

$$W = 2V_u = \frac{3.0 \cdot f_t \cdot b \cdot d}{\sqrt{\left(1+0.75(a/d)^2\right)}} + \frac{f_y}{\sqrt{\left(1+(a/d)^2\right)}} \cdot \times \; \Sigma\left[\frac{y_i}{d} + 0.40\right] \times A_{si} sin(\alpha_i + \theta)$$

.....(2.13)

For flexure failure, another major mode of failure, they put forth a new formula on the basis of stress strain relation of second degree polynomial, to predict ultimate flexure strength of reinforced concrete moderate deep beams.

Shear failure occurs in beams with a/d raitos less than one and if there is no web reinforcement, this happens with rather a sudden diagonal cracking. Web bars intercept the diagonal crack line, prevent sudden crack propagation and increase shear capacity. For beams with a/d ratios appro..ximately equal to or greater than one, mode of failure is flexure. Thus, the crack pattern in a deep beam depends more on shear span to depth ratio than span to depth ratio.

2.6 Fibre reinforced concrete

General

Fibre reinforced concrete can be defined as the cement based matrix incorporated with short discrete fibres. The matrix can be either a cement paste, mortar or concrete.

The idea of incorporating strong thin fibres for strengthening brittle matrices is not new. The concept is more than 4500 years old.[165] Straw was used in bricks long before the use of ordinary Portland cement cocnrete itself. Wood and bamboo are the best examples of naturally available fibre reinforced construction materials. However, excluding asbestos, only in the past two decades serious consideration has been given to the use of synthetic fibres in the conventional mouldable construction materials like gypsum plaster, cement paste and concrete to improve their performance. The work of Romualdi and Batson (16,141) in 1963 is considered to be the pioneering one, though Porter envisaged as early as 1910 the idea of reinforcing concrete by thin wires. So the use of short, discrete fibres as a crack arrest mechanism in concrete is relatively new.

Mainly three types of fibres (glass, polypropylene and steel) are currently being investigated as concrete reinforcement. Due to low effectiveness, poor alkaline resistance or high cost, use of other fibres such as nylon, rayon, carbon, etc., has been almost ruled out after initial investigations. The use of strong and stiff fibres in concrete improves the post-cracking performance of concrete considerably. After micro-cracking, fibres spanning the cracks control crack propagation and reduce the rate of widening of cracks under tensile loading. This role of fibres imparts ductility to the concrete and delays its failure which otherwise would have occurred almost immediately after microcracking. After sufficient widening of the cracks at a relatively higher load, the short fibres start pulling out and the load gets reduced gradually. The process of fibre pullout absorbs lot of energy and hence the toughness of concrete and its impact resistance are considerably increased. Research work carried out in the past decade keeping in mind the above property of fibre reinforced concrete transformed it into a practical reality from a laboratory curiosity.

A critical review of the existing literature has revealed that the presence of steel fibres changes the brittle failure to ductile failure by imparting ductility. Increase in ultimate load capacity is significant. Use of steel fibres in conventional reinforced concrete beams has attracted the attention of the structural engineers.

Shah and Rangan (153) carried out different tests on different types of specimens to study various properties.

Flexure Tests: They tested concrete beams with cross-section(2 in x 2 in) 50·8 mm X 50·8 mm in flexure with three-point loading and span (10 in.) 254 mm Rate of cross-head motion was 0.01 in/min. Sand-cement, gravel-cement and w/c ratios were 2.0, 3.0 and 0.6 respectively. Low-carbon, smooth steel fibres with cross section (0.01 in x 0.01 in) 0·25 mm X 0·25 mm with 2% yield strength of 7740 kgf/cm^2 and tensile strength of 120 kgf/cm^2 were used. Different aspect ratios of 25, 50, 75 and 100 with 0.25, 0.50, 0.75 and 1.0 percentages of fibres by volume were used.

The authors observed that increasing volume fraction of fibres increases strength and toughness. Increase in toughness is about 20 times for 0.25% of fibres while increase in strength is less than two times. Also they concluded that increasing the length i.e. aspect ratio upto a point increases toughness substantially whereas increase in strength is not so much.

Tensile Tests : Dowel-shaped(12 in.) 300 mm long specimens with central test region(4 in.) 102 mm long and of cross section(2 in. x 1 in.) 50·8 mm X 25·4 mm were used. Load was applied through studs embedded in the end sections at a constant rate of(0.05 in /min) 0·125 mm / min The specimens were tested after 7 days of moist curing. Atleast four specimens for each variable, viz., orientation, aspect ratio, volume percentage and spacing were tested.

Mortar specimens with 0.5% volume of fibres with alignment in the direction of the load, in the direction perpendicular to load and with random orientation was more improved than increase in tensile strength. Also, perpendicular fibres showed no resistance after tensile strength of concrete is exceeded.

To study the effect of aspect ratio,mortar specimens with 0.5% volume of randomly oriented fibres with aspect ratio 100, 75, 50, and 25 were tested. It was observed that the length of fibres had very little influence on tensile strength but it significantly influenced postcracking resistance, for fibres with aspect ratio 100. The postcracking resistance was lesser than that for aspect ratio 75. This was attributed to non-uniform distribution of fibres due to more length.

Mortar and concrete specimens with 0.5, 1.0 and 1.5% volume of fibres aligned in the direction of tensile load were tested and approximately linear increase in the tensile strength and toughness of the composite was observed with increasing fibre volume.

Keeping the same 0.5% vol. of steel, different geometric arrangements of one wire, six wires and aligned parallel fibres were tested and it was observed that the influence of spacing was less for smaller spacing than for large spacing.

Compression Tests : Concrete specimens(2in. x 2in. x 10 in.)with 0.5% volume of
50.8 mm X 50.8 mm X 254 mm
steel, either in the form of randomly oriented fibres or closely spaced stirrups were tested under uniaxial compression. The authors concluded that addition of fibres or stirrups spaced at(1 in.) or less were better for increase in ductility and toughness.
25.4 mm

Kukreja (95,97) had tested eleven beams of different series classified as $A_s, B_s, C_s, \ldots, K$. In each series an average of two beams of size 101.6 mm wide x 152.4 mm deep in cross section and 1220 mm long were used. The first series was for reinforced concrete but without fibres and the remaining series for reinforced concrete with steel fibres and the last series was with reinforced concrete without fibres but with shear stirrups.

Fibres of mild steel with 0.46 mm diameter and aspect ratio 100, 80 and 60 were used. Three volume fractions, that is 0.5%, 1.0% and 1.5% were used throughout the investigations. For main reinforcement 12 mm diameter deformed bars and plain mild steel of 6 mm diameter with yield and ultimate values of 4100 kg/cm^2 and 5600 kg/cm^2 respectively was used for shear reinforcement.

He concluded that the first crack strength increased by maximum 66.66 percent in beams having fibres of aspect ratio 100 and volume fraction of 1.5 percent. Due to the inclusion of fibres, shear strength improved. Optimum percentage of fibres is 2.0% which gives the maximum increase of 66.12% in shear strength for fibres having aspect ratio 80.

The ultimate resistance to shear offered by the fibres was given by

$$V_{fu} = k_b \cdot \sigma_b \cdot D\sqrt{V_f \cdot l_f} \qquad \text{......(2.14)}$$

where k_b = bond coefficient

σ_b = bond stress

D = over all depth of the section

V_f = volume of fibres

l_f = length of fibres

The author observed that for smaller volume percentage of fibres, strength increase with increase in aspect ratio. For higher volume percentage, that is 1.0 and 1.5, the shear strength increases from aspect ratio 60 to 80 and after 80 it starts decreasing. The maximum increase of strain in concrete of 100% was obtained for beams FS having fibres in 1.0% volume and aspect ratio 80.

It was observed that for beams containing fibres, actual ultimate measured deflection at the load section increases. This increase was maximum about two times for beams having fibres with aspect ratio 80 and volume percentage 1.5.

Further, he observed that initially a few flexural cracks developed in the pure moment zone. Then, the diagonal tensile crack developed in shear span. As the diagonal crack started extending both ways, flexural cracks stopped developing further.

Einiema Test (57) : The effect of steel fibre on the behaviour and strength of reinforced concrete beams under shear is studied for nine reinforced beams and one plain reinforced concrete beam under two point loading on a span of 1800 mm. Crimped steel fibres with three different aspect ratios were used in varying percentage of fibres from 0.4 to 1.0 percent.

The author observed that the improvement of first crack load and ultimate load over that of plain concrete was noted. The first crack strength ratio of fibrous and plain beams was a function of fibre spacing . Increase in cracking load of a fibrous beam compared with a beam without fibres is as large as 65 percent, especially when the aspect ratio is high. Increase in ultimate load was also noted, but to a much lesser extent, with a maximum value of 22.5 percent.

He observed from test results that the crushing strength increased with increasing fibre volume, especially for the specimens with high aspect ratio. Maximum increase was approximately thirteen percent. The strain gauge reading for steel strain in longitudinal reinforcement and stirrups indicated a decrease in strain as the aspect ratio increased for constant volume percentage of fibre. It can be observed that the increase in toughness is due to the presence of fibres. This is indicated by the increase in area under load defection curve.

Swamy and Al-Noori (166),from their tests on reinforced concrete beams with fibre (0.5 mm x 50 mm crimped square fibre, 1% volume) concrete in the tension zone or compression zone or as a tensile skin,have concluded the following :

The ratio of the first visible crack moment to the design and ultimate moment was about 50% higher for beams with fibre concrete in the tension zone. These beams showed less deflection particularly in the post-cracking stage. For a given crack width higher stress could be permited in the main tension steel compared to beams without fibres. Crack heights were lesser with fibre inclusion. Fibre concrete in the compression zone is shown to improve effectively the performance of structural members. It prevents disintegration and preserves the integrity of the compression zone. It shows higher degree of compressibility and enables the beam to develop plastic deformations at failure. The provision of fibre concrete in the form of a tensile skin is just as beneficial as providing fibre concrete in the whole of the tension zone, with the additional advantage of preferential orientation. With steels of characteristic strength 700 N/m^2, the crack width at design loads (f_s = 160 to 210 N/mm^2) varied from 0.10 to 0.19 mm. Beams with high strength steel and a tensile skin of fibre concrete were able to develop plastic deformation characteristics at failure similar to that in steel structures. The provision of a tensile skin of fibre concrete transforms a conventionally over-reinforced beam to behave like an under-reinforced beam with ductile characteristics.

Byung Hwan Oh'S (24) tested nine fibre reinforced concrete beams. The width of the beams was 120 mm, height 180 mm, effective depth 140 mm and span 1800 mm. The tensile reinforcement for the two series of singly reinforced concrete beams was 0.4 b and 0.65 b respectively. Tensile and compressive reinforcement ratio for the doubly reinforced concrete beam was 0.90 b and 0.0085 b respectively where b represents balanced reinforcement ratio. Round and straight fibres with aspect ratio 57 were used. Fibre contents were varied from 0% to 20% and w/c ratio used was 0.40.

The author observed that the ultimate resistance of fibre reinforced concrete beam is remarkably increased with an increase of fibre content and the rate of increase of maximum load capacity reaches upto 50% for the fibre content of 2.0%. He also observed that the effect of the steel fibre is more pronounced for the case of lightly reinforced concrete beams.

The author concluded that the crack width increases almost linearly with the increase of steel stress and the crack width of the same loading stage is remarkably reduced when the fibre content in the beam increases. Also, due to steel stress, crack spacing is reduced as the amount of fibres in the beam increases. It was concluded that the fibres play a very effective role in curbing the opening of the cracks and contribute to distributed cracking. He also observed that the beam with 2.0% fibre addition shows considerably less cracking at each loading stage indicating remarkable resistance against cracking.

Based upon composite materials concept, the author proposed that

$$\sigma_{ct} = \sigma_{mt}\rho_m + \sigma_f\rho_f \qquad \text{...... (2.15)}$$

where σ_{ct} = flexural strength of fibre reinforced composit
σ_{mt} = flexural strength of matrix
σ_f = strength of fibres
ρ_m = fibre volume ratio
ρ_f = volume ratio of matrix = $(1 - \rho_m)$

Neglecting the strength contribution of concrete mix at ultimate state, he finally arrived at the following equations :

$$\sigma_t = 2\alpha_0 \cdot \alpha_1 \cdot \alpha_b \cdot \rho_f \cdot J\left(\frac{l_f}{d_f}\right) \qquad \text{..... (2.16)}$$

Neutral axis depth $C = \frac{A_s\sigma_y + \sigma_t bh}{0.85 f'_c \beta b + \sigma_t b}$

Flexural capacity $M_n = A_s\sigma_y\left(d - \frac{a}{2}\right) + \frac{\sigma_t \cdot b(h-c)(h+c-a)}{2}$ ..... (2.17)

where α_0 = orientation factor = 0.41 for uniformly dispersed fibres

α_b =bond efficiency factor = 1.04 for straight fibres

α_1 = length - efficiency factor

$$\alpha_1 = 1 - \frac{\tan\left(\beta \cdot \frac{L_f}{2}\right)h}{\left(\beta \cdot \frac{L_f}{2}\right)}$$

$$\beta = \left[\frac{2\pi G_m}{E_f A_f l_n\left(\frac{S}{x_f}\right)}\right]^{1/2}$$

G_m = shear modulus of matrix

l_n = effective length of fibre

x_f = effective spacing of fibre

L_f = length of fibre

V_f = volume of fibre

d_f = diameter of fibre

s = average spacing of fibres

$$= 25\left[\frac{L_f}{V_f d_f}\right]$$

Shanmugam and Swaddiwudhiopong (151) carried out experimental investigations on fibre reinforced concrete deep beams and observed failure load significantly increasing due to addition of fibres. They used steel fibres of mean diameter 0.5 mm length 50 mm and aspect ratio 100. Concrete mix used was 1:1.8:2.9.

They tested three series of beams, each series consisting of six beams with L/D ratio from 1.0 to 3.5. The effective span was 600 mm while thickness was 60 mm. The beams were tested in Universal Testing Machine with a midspan concentrated load and simply supported ends. Longitudinal surface strains were measured at various points along the depth at mid-span of all the beams. Vertical deflection of the centre of the beam was measured.

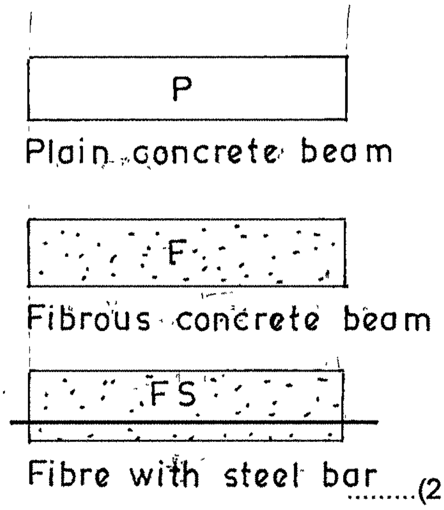
Using idealized stress-strain diagram and following equations, failure load of the beams was calculated shown below :

$$\frac{\epsilon_s}{\epsilon_{cu}} = \frac{d_{ef}-x}{x}$$

$$0.6 f_{cu} \cdot b \cdot x = f_s A_s + f_{ct} \cdot b(d - x)$$

$$M_u = f_s A_s\left(d_{ef} - \frac{X}{2}\right) + fct \cdot b(d - X)\frac{d}{2}$$

$$P_u = \frac{4M_u}{L} \qquad \text{.........(2.18)}$$



define

The authors observed that "P" and "F" beams failed in flexure while "FS" beams showed shear mode earlier and then developed flexure cracks.

Load-displacement curves showed linear variation in the initial stage and became non-linear later. Also, fibre reinforced beams showed larger deflections than plain beams indicating higher post cracking ductility of fibres.

Also, the authors observed higher increase in ultimate load of fibre reinforced concrete beams over plain concrete beams compared to that observed in shallow beams. They suggested that this may be due to larger amount of fibres present in tensile zone of a deep beam.

They concluded that the steel fibres increase ultimate flexural capacity significantly with more increase for deep beams. Also, the failure observed is more ductile and gradual.

Shanmugam and Swaddiwudhiopong Tests (152) : An empirical formula to predict the ultimate strength of fibre reinforced concrete deep beams contanining openings was proposed. Experiments, that have been carried out to study the effect of the position of openings and shear span to effective depth ratio on the strength of such beams, were described. The beams were simply supported and tested for failure under two point loading.

The results show that the ultimate strength primarily depends upon the extent to which the opening intercepts the natural load path.

According to Kong and Sharp [87], the ultimate shear capacity of deep beams with openings may be calculated by using the idealised structural model was modified by them for such beams.

All the tested beams failed in diagonal tension. In most of the beams, the first crack originated from the two corners of the openings and propogated with increasing load. Load deflection graphs were found to be nearly linear for the initial stage of loading which indicates elastic behaviour of the beams prior to the occurence of cracks. At the later stage of loading, deflection increased at higher rate as more cracks were developed. It was observed that the opening located in the tension zone affects the behaviour and strength of beams more than those located in the compression zone.

Batson, et al. Test (16) : Batson Jenkings and Partney tested a number of beams under central point load. The size of beams were 101 mm wide x 152 mm deep x 1980 mm length. The beams were designed to fail in shear.

Authors observed two types of failures : (i) Shear failure when cracks developed along the compressive trajectories in the shear span. (ii) Initial moment failure when vertical cracks were first observed in the moment span.

Authors calculated the minimum amount of fibres to prevent shear failure under constant a/d ratio. They also observed from their test that the replacement of vertical stirrups by round, flat or cripped steel fibres produces effective reinforcement against shear failure and the shear span ratio decreases with increasing steel content. They suggested that, there was a considerable increase of shear strength for $a/d < 3.0$ than for $a/d > 3.0$.

Hughes and Fattuhi (72,73) carried out tests on fibre reinforced concrete beams. They studied the effect of beam dimension and concrete filling sequence on the flexural strength of steel fibre reinforced concrete beams. Effects of beam dimension alongwith span to depth ratio are presented in detail. From their work, they plotted curves for flexural stresses versus deflection for beams of cross section 102 mm x 102 mm with spans equal to 1050 mm, 1275 mm and 2250 mm. The curves indicate that more gradual and ductile type of failure occures in longer span beams compared to the small span beams.

From the graph of modulus of rupture versus depth of the beam it can be seen that the flexural strength tends to increase with increasing beam depth for fibre reinforced sections as well as without fibre reinforced sections.

It is also observed that with the increase of span to depth ratio, strength of fibre reinforced and without fibre reinforced concrete beams decreases in terms of modulus of rupture. For span to depth ratio less than 6.0 a sharp increase in flexural strength was observed in case of fibre reinforced concrete deep beams compared to the reinforced concrete beam.

Sabapathi and Achyutha (147,148) carried out the compression and flexure tests as per IS:516-1959 and a new flexure theory is proposed for steel fibre reinforced concrete beams based on the available load-slip curve obtained from pull-out tests on fibres. It is possible to predict by the proposed theory the post cracking behaviour of SFRC beams in the entire range. Moreover, it can be extended easily towards the analysis of steel fibre reinforced concrete beams.

They had proposed theory on the basis of load-slip curve. They concluded that minimum fibre content required for flexural strength is 0.4 percent by volume at the corresponding optimum aspect ratio of 86 from their study. The most general range of fibre content has been varied from 0.5 percent to 1.5 percent by volume at the corresponding optimum aspect ratio range of 65 to 85.

Dwarkanath and Nagaraj Test (54,55) : In this investigation, a comperative study of the full depth inclusion of fibre and half depth inclusion of steel fibres towards its deformation behaviour of conventionally reinforced concrete beam is reported. The beams having cross section of 100 mm wide x 200 mm deep over a span of 1800 mm were used in the investigation. Two percentages of conventional reinforcing steel (0.77% and 1.28%) were used. For each percentage of reinforcing steel, two fibre contents with two different conditions of dispersions (Half depth inclusion and full depth inclusion) were used. Beams were tested over a simply supported condition with effective span of 1500 mm. Steel fibres are effective in bringing about the desired modifications in the deformational characteristics of conventionally reinforced concrete beams for almost the entire range of loading from beginning to ultimate.

Under-reinforced beams are adequately ductile in themselves and the addition of fibres is benifical in improving deformational characteristics of the beams. The fibre beams behave stiffer, particularly during service loads. The improvements in these characteristics are reflected in terms of reduced strain in steel, reduced curvatures and reduced deflections. Reduced steel strains are indicative of reduced crackwidths and such modifications are much desirable in slender structures.

The steel fibres have been added in two types of inclusions full depth fibre inclusion and half depth fibre inclusion. It is found that half depth fibre inclusion is practically as effective as full depth fibre inclusion in bringing about the desired modification in the deformational charateristics. An engineering parmeter for tensile strain enhancement factor has been identified for desinging such beams in analysis of conventionally reinforced fully fibrous and partially fibrous normal beams, the effect of the presence of fibers on the behaviour of such section was investigated. The fibres contribution is significantly useful in maintaining structural integrity of the beam. It is further suggested that the lower volume fractions, improved geometry, as well as surface characteristics of the fibres are the ruling criteria for fibres to be included in conventional reinforced concrete beams for obtaining desired modification. They have also concluded that addition of fibre upto 1.0% of concrete volume is beneficial.

2.7 Reinfoced Cement Concrete Deep Beam Design

Uhlmann (176), using finite difference method,analyzed deep beams and developed following design procedure for several loading arrangements. He assumed that concrete cannot take tension and the whole tension is required to be resisted by tension reinforcement.

The minimum thickness of beam is given by

$$b \geq 0.06\frac{L}{\sqrt{K}} \quad \text{....... (2.19)}$$

where, L = span of beam

k = a constant given in Table-1 of Uhlmann's paper for different L/D ratios.

For simply supported beams loaded on top with uniformly distributed load, the area of tension reinforcement is given by

$$A_{st} = \frac{M}{f_s \cdot l_a} \quad \text{..... (2.20)}$$

where, M = bending moment at that section

f_s = permissible steel stress

l_a = lever arm

The main steel bars are to be placed on the lower edge and no web reinforcement was necessary here. As per Uhlmann, for deep beams loaded at lower edge, in addition to tension reinforcement, hanging steel was necessary.

Area of hanging steel $A_W = \frac{P}{f_s}$

where, P = total load between the support

For uniformly distributed load, the support bars are to be carried up vertically and then bent outward in accordance with Fig. 13 of Uhlmann's paper.

In deep beams, stresses depend also on the normal pressure apart from shear force and bending moment. Therefore, for deep beams loaded at the bottom edge, the stress distribution is quite different from that caused by identical loads on the top edge.

Darwish and Narayanan Tests (40) : For estimating the strength of the steel fibre reinforced concrete element in shear, they suggested empirical equations based on experimental data which are suitable for design office use.

From the equations, they prepared design charts for the ultimate shear resistance of fibre reinforced concrete deep beams.

And total shear strength $v_u = v_{u1} + v_{u2}$ .... (2. 21)

$$v_{u1} = \frac{V_{u1}}{bh} = 0.7\left[1 - 0.35\frac{x_c}{h}\right] \cdot f_{spf}$$

$$v_{u2} = \frac{Vu2}{bh} = 1.95 \cdot \frac{100 \cdot As \cdot d}{bh^2}\left[\sin\left(\tan^{-1}\frac{1}{\frac{xc}{h}}\right)\right]^2$$

The first term represents the contribution of the fibre concrete v_{u1} and second term gives the contribution of the main longitudinal reinforcement v_{u2} (using deformed bars only).

The value of v_{u1} corresponding to the grade of concrete, the fibre factor and $\frac{X_c}{h}$, can be read directly from the graph on design chart **Fig.7** ; and value of v_{u2} corresponding to the $\frac{X_c}{h}$ and 100 $A_s d/bh^2$ can be read from the graph on design chart **Fig.8** in their paper referred above.

The design charts stated are applicable if the following conditions apply :

(i) $\frac{l}{h} > 2$

(ii) Only static loads occur and are applied to the top of beam only.

(iii) Positive anchorage is provided to main reinforcement.

(iv) $\frac{X_c}{h}$ is not vastly outside the range of 0.23 to 0.7

Portland Cement Association Provisions (130)

Using Dischingers's (51) work, the Portland Cement Association (130) has proposed some design rules for reinforced concrete deep beams. The method can be used for L/D less than 2.0 for continuous girders and for L/D less than 1.25 for single span girders. The methodology is as under.

For an assumed loading, once the section of the beams has been chosen, the designer calculates two characteristic ratios, which are needed to use the stress curves. These ratios are the height to span ratio (D/L) and the support to span ratio (C/L). The curves provided are then used to check the stress level in concrete. If this is satisfactory, the total tensile force, T is determined from another graph. The total area of tensile reinforcement A_{st} required is then calculated.

$$A_{st} = \frac{T}{f_s} \qquad \text{...... (2.22)}$$

where f_s the allowable working stress in steel.

Recommendations are given for positioning of the main steel. For a single span beam loaded on the top edge, the Portland Cement Association stated that the tensile reinforcement should be placed as near to the bottom as possible.

Examination of the direction of diagonal tension cracks in deep beams showed that the direction of principal tensile stress was nearly horizontal. From this, it was concluded that vertical stirrup was virtually useless as shear reinforcement and that the horizontal tensile stress could be carried by the longitudinal steel. The paper pointed out that without modification the customary shear investigation was not applicable to the relative deep beams. In the absence of data on shear strength it was tentatively recommended that the unit shear could be computed as

$$V_s = \frac{8V}{7 \cdot B \cdot D} \qquad \text{..... (2.23)}$$

where B = width of beam
D = depth of beam

and that a stress of $V_c\left[\frac{1+5\frac{D}{L}}{3}\right]$ be allowed where V_c is the permissible shear stress for shallow beams.

Current Practice of Deep Beam Design

CIRIA Guide No.2 1977 (34)

The CIRIA guide applies to beam having an effective span/depth ratio i.e. l/h less than 2.0 for single-span beam and less than 2.5 for continous beam. The CIRIA guide was intended to be used in conjunction with British code CP 110:1972.

The guide considers that active height "h_a" of a deep beam is limited to a depth equal to the span; that part of the beam above this height is taken merely as a load bearing wall between support.

(a) Flexural strength $M_u = 0.12 f_{cu} \cdot b \cdot h_a^2$ .... (2.24)

where f_{cu} - concrete strength

b - width of beam

Area of main longitudinal reinforcement

$A_{st} > \frac{M}{0.87 f_y Z}$ .... (2.25)

where M - applied moment

f_y - steel characteristic strength

z - lever arm which is to be taken as follows

$z = 0.2 \cdot l + 0.4 \cdot h_a$ for single span .... (2.26)

$z = 0.2 \cdot l + 0.3 \cdot h_a$ for continuous beam .. .. (2.27)

Distribute the main steel over a depth of $0.2h_a$. Anchor the reinforcement bars to develop at least 80% of the maximum ultimate force beyond the face of the support.

(b) Shear strength : Bottom loaded beams

Shear capacity $V_u = 0.75 \cdot b \cdot h_a \cdot V_a$ .. .. (2.28)

where V_u - maximum shear stress taken from Table-6 of CP 110

for normal weight concrete. For light weight concrete Table-26 should be used.

Provide hanger bars in both faces to support the bottom loads, using a design stress of $0.87 f_y$. The hanger bars should be anchored by a full bond length above the active height h_a or alternatively anchored as links around longitudinal bars at the top.

Provide nominal horizontal web reinforcement over the lower half of the active height h_a and over a length of the span to 0.4 h_a measured from each support.

(c) Shear strength : Top loaded beams

$$V_u = 2.0bh_{a^2} \cdot \frac{V_c}{X_c} \text{for} \frac{h_a}{b} < 4 \qquad \text{.. ...... (2.29)}$$

$$V_u = 1.2bh_{a^2} \cdot \frac{V_c}{X_c} \text{ for} \frac{h_a}{b} \geq 4 \qquad \text{.. ...... (2.30)}$$

$$V_u = 6.0h_a \cdot v_c \qquad \text{.. ...... (2.31)}$$

where X_c - effective clear shear span

v_c - shear stress value taken from Table-5 of CP 110-1972

for normal weight concrete and Table-25 for light weight concrete

Provide normal web reinforcement in the form of a rectangular mesh in each face. The amount of this nominal reinforcement should not be less than that required for a wall by clauses 3.11 and 5.5 of CP 110-1972.

Bearing strength for deeper beams (l/h < 1.5), the bearing capacity may well be the governing design crierion particularly for those having shorter shear span. The bearing stress so calculated should not exceed 0.4 f_{cu} and support length should be "c" or 0.2 l_o which ever is the less c is support length.

ACI Building Code 318-1989 (5)

Defines a deep beam as a beam in which the ratio of the clear span l_o to the overall depth h is less than the limits in equations.

Simple spans : $\frac{l_0}{h} < 1.25$

Continous span : $\frac{l_0}{h} < 2.5$

Minimum tension reinforcement : The main steel ratio shall not be less than given below

$$\rho_{min} = \frac{200}{fy}$$

where $\rho_{min} = \frac{A_s}{bd}$

A_s - main tension reinforcement

b - width of beam

d - effective depth

f_y - steel strength

ρ_{min} - about 0.3%

Web reinforcement : An orthogonal mesh of web reinforcement is required. The minimum areas of the vertical and horizontal bars shall be

$$\frac{A_v}{b \cdot s_v} \geq 0.15\%$$

$$\frac{A_h}{b.s_h} \geq 0.25\%$$

where A_v - area of the vertical bars within the spacing s_v.
and A_h - area of the horizontal bars within the spacing s_h apart from the above requirement ACI code does not give further detailed guidelines. It merely states that account shall be taken of the nonlinear distribution of strain and lateral buckling.

Shear strength : The shear provisions of ACI code 318-89 apply to top loaded simple or continuous beams having a clear span/effective depth ratio $\frac{l_0}{d}$ less than 5.0.

For simply supported deep beams calculations are carried out for the critical section defined as follows. For uniformly distributed loading, the critical section is taken as $0.15 \cdot l_o$ from the face of the support. For concentrated load, it is taken as halfway between the load and the face of the support. The design is based on

$$V_u < \phi V_n$$
$$V_n = V_c + V_s$$

where V_u - design shear force at the critical section

V_n - nominal shear strength

ϕ - capacity reduction factor for shear, taken as 0.85

V_c - shear strength provided by concrete

V_s - shear strength provided by steel

V_n should not exceed the following :

$$V_n < 8\sqrt{f_{c'}} \cdot b \cdot d \quad \text{for } \frac{l_o}{d} < 2 \qquad \text{.. ...... (2.32)}$$

$$V_n < \left(\frac{2}{3}\right)\left(10 + \frac{l_0}{d}\right)\sqrt{f_{c'}} \cdot bd \text{ for } 2 \le \frac{l_0}{d} < 5 \qquad \text{.. ...... (2.33)}$$

where $f_{c'}$ - concrete cylinder compressive strength

b - beam width

d - effective depth

The shear provided by concrete is calculated from

$$V_c = \left[3.5 - 2.5\frac{M_u}{V_u}\right] d \left[1.9\sqrt{f_{c'}} + 2500\rho\frac{V_u \cdot d}{M_u}\right] bd \qquad \text{.. ...... (2.34)}$$

where M_u - design bending moment which occurs simultaneously with V_u at the critical section

ρ - ratio of the main steel area to the area of the concrete section $\left(\rho = \frac{As}{bd}\right)$

The first term of the right-hand side is multiplier to allow for strength increase in deep beams, subject to the restrictions that follow

$$\left[3.5 - 2.5\left(\frac{M_u}{V_u d}\right)\right] < 2.$$

$$V_c < 6\sqrt{f_{c'}} \cdot b \cdot d$$

In case where V_u exceeds ϕV_c, a system of orthogonal shear reinforcement must be provided to carry the excess shear. The contribution V_s of shear reinforcement is given by

$$\frac{V_s}{f_y d} = \frac{A_v}{S_v}\left(\frac{1 + \frac{l_o}{d}}{12}\right) + \frac{A_h}{S_h}\left(\frac{11 - \frac{l_0}{d}}{12}\right) \qquad \text{...... (2.35)}$$

where A_v - area of vertical web reinforcement within a spacing S_v

A_h - area of horizontal web reinforcement within aspacing S_h

f_y - strength of the web steel which should not be taken as more than 410 N/mm^2

S_v - spacing of the vertical web bars

S_h - spacing of the horizontal web bars

The quantities $\left[\left(1+\frac{l_0}{d}\right)/12\right]$ and $\left[\left(11-\frac{l_0}{d}\right)/12\right]$ represent weighting factors for the relative effectiveness of the vertical and horizontal web bars. ACI Code 318-1989 rightly considers that horizontal web reinforcement is more effective than vertical web reinforcement. As l_o/d ratio decreases, horizontal web bars become increasingly more effective compared with vertical web bars.

Canadian Code CAN 3-A23.3-M84 (25)

For flexural design the Canadian Code define a deep beam as a beam in which the ratio of the clear span l_0 to the overall depth "h" is less than the limits in equation.

Simple span $l_{0/h}$ < 1.25

Continous span $l_{o/h}$ < 2.5

Minimum tension reinforcement : The main steel ratio shall not be less than $\rho_{min} = \frac{1.4}{f_y}$

where $\rho \min = \frac{A_s}{bd}$

A_s - area of the main tension reinforcement

b - beam width

d - effective depth

f_y - steel strength

Web reinforcement : A system of othogonal web reinforcement is required with bars in each face. The minimum areas of the vertical and horizontal reinforcement shall satisfy.

$$\frac{A_v}{b \cdot s_v} \geq 0.2\%$$

$$\frac{A_h}{b \cdot s_h} \geq 0.2\%$$

where A_v - area of the vertical web reinforcement within the spacing S_v which shall exceed neither $d/5$ nor 300 mm

A_h - area of the horizontal web reinforcement within the spacing S_h, which shall exceed neither $d/3$ nor 300 mm

Apart from the above requirements, the Canadian code does not give further detailed guidelines. It merely states that account shall be taken of the nonlinear distribution of stress, lateral buckling and the increased anchorage requirements.

Shear strength : The Canadian code uses the concept of the shear span/depth ratio rather than the span/depth ratio. The shear provisions of the Canadian code apply those parts of the structural member in which (i) the distance from the point of zero shear to the face of the support is less than 2d or a load causing more than 50% of the shear at a support is located at less than 2d from the face of the support.

The calculations are based on truss model consisting of compression struts and tension tie. Nodal zone is defined as the regions where the strut and tie meet. Here compressive stresses should not exceed : $0.85\phi_c \cdot f_c{}'$ in nodal zones where ϕ_c is a material resistance factor = 0.6 for concrete and $f_c{}'$ is the cylinder compressive strength of concrete.

The main tension tie reinforcement is determined from the tensile tie force. These reinforcing bars should be effectively anchored to trasfer the required tension to the lower nodal zones of the truss to ensure equilibrium.

Checking of the compressive struts against possible crushing of concrete as follows $f_2 < f_2$ max where f_2 is the maximum stress in the concrete strut and f_2 max is the diagonal crushing strength of the concrete given by

$$f_2 \text{ max} = \frac{\lambda \phi_c f_c{}'}{0.8 + 170\epsilon_t} \qquad \text{.. ...... (2.36)}$$

where λ - a modification factor to be taken account of the type of concrete (λ = 1.0 for normal weight concrete)

$\in_t$ - principal tensile strain

For the design porpose $\in_t$ may be computed from

$$\in_t = \in_x + \frac{\in_x + 0.002}{\tan^2\theta} \qquad \text{.. ...... (2.37)}$$

where $\in_x$ - longitudinal strain

θ - angle of inclination of the diagonal compressive stresses to the longitudinal axis of the member.

An orthogonal system of web reinforcement must be provided. This shall meet the requirements.

Draft Eurocode and CEB-FIP Model Code - 1990 (32)

The CEB-FIP Model Code provide guidelines for design of deep beams. It applies to simply supported beams of span/depth i.e. l/h less than 2.0 and to continuous beams of l/h less than 2.5.

Flexural strength : simply supported deep beams.

The area of the longitudinal reinforcement is calculated from the equation

$$A_s = \frac{M}{\frac{fy}{rm}} \cdot z \qquad \text{.. ...... (2.38)}$$

where M - largest applied bending moment

f_y - reinforcement characteristics strength

r_m - partial safety factor

z - lever arm which is to be taken as follows :

$z = 0.2(l + 2h)$ for 1 < (1/h) < 2

$z = 0.6 \cdot l$ for (l/h) < 1

The main longitudinal reinforcement should extend without curtailment from one support to the other and be adequately anchored at the ends. According to this model code vertical hooks cause the development of cracks in the anchorage zone and should be avoided. The required steel should be distributed uniformly over a depth of (0.25h-0.05l) from the soffit of the beam.

Shear strength and web reinforcement

Design shear strength should not exceed

$$0.10b \cdot h\left(\frac{f_c'}{r_m}\right)$$

where b - width

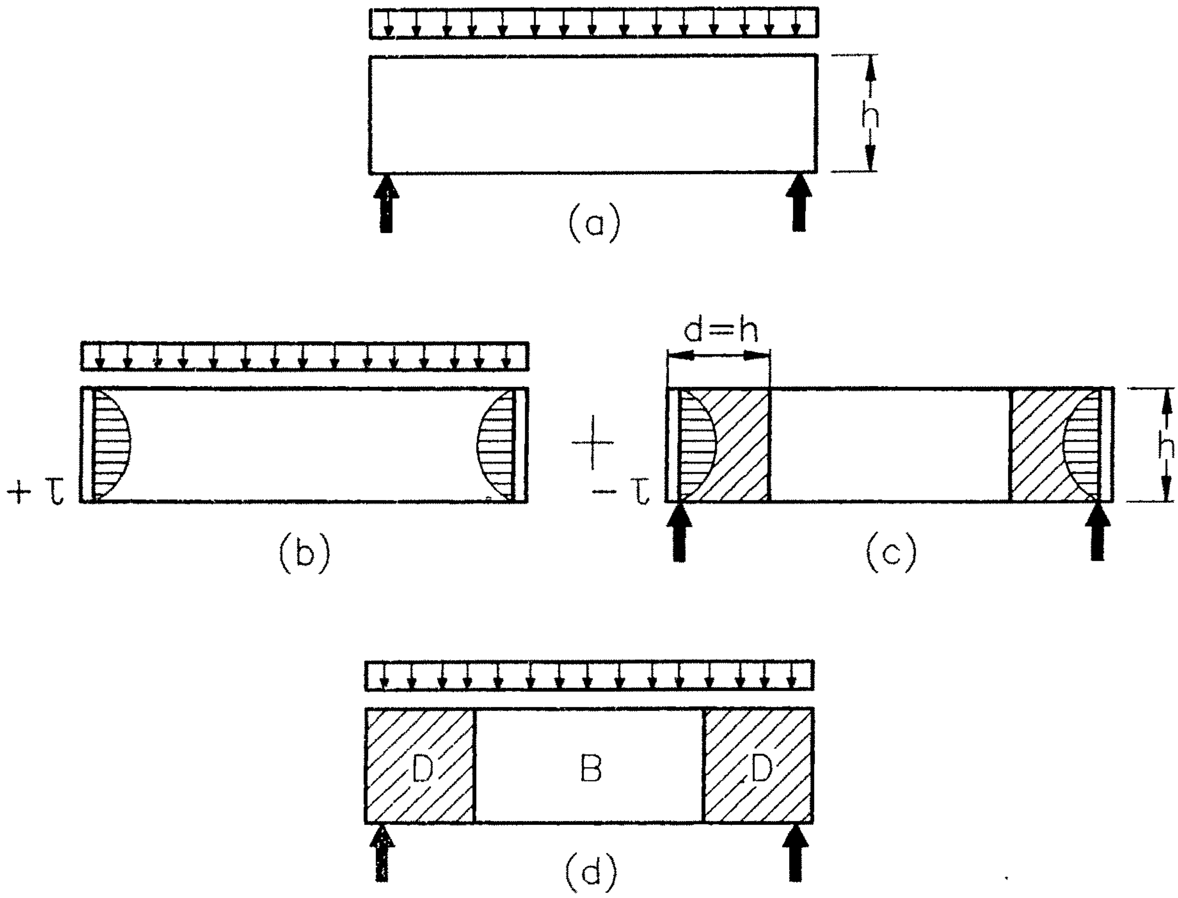
h - beam depth

f_c' - characteristics cylinder strength of concrete

r_m - partial safety factor for material

The web reinforcement is provided in the form of a light mesh to orthogonal reinforcement consisting of vertical stirrups and horizontal bars placed near each face. The web steel ratio should be about 0.20% in each direction near each face for smooth round bars and 0.25% for high bond bars. Additional bars should be provided near the support, particularly in the horizontal direction.

The aim of the web reinforcement is mainly to limit the crack widths which may be caused by the principal tensile stresses. For beams loaded at the bottom edge, vertical stirrups are required to transmit the load into the upper portion of the beam; this is in addition to the orthogonal web reinforcement.



(a) Structure with real loads
(b) Loads and support reaction accordance with the Bernoulli hypothesis
(c) Self-equlibrathing state of stress
(d) Real structure with B and D regions

Fig. 2.1 SUBDIVISION INTO B AND D REGIONS.