CHAPTER-5

ANALYTICAL FORMULATION

5.1 General

Analytical prediction of increased load carrying capacity of concrete deep beams and moderate deep beams due to addition of steel fibres requires a clear understanding of the behaviour of members in compression, splitting, direct tension and flexure. Hence, before attempting to evolve a procedure for analysis of such beams, the basic behaviour of steel fibre reinforced concrete in compression, splitting, direct tension and flexure is highlighted in this chapter using the experimental results of the present and earlier investigators.

In present investigations, the possibility of identifying appropriate engineering parameters of steel fibre reinforced concrete which can be incorporated in the analysis of structural response of deep beams and moderate deep beams has been explored. The possible increase in moment capacity corresponding to various depths of fibrous concrete is investigated.

5.2 Behaviour in Compression

The behaviour of fibre reinforced concrete in compression is analysed using experimental results of the fibrous concrete in compression. Cubes with fibre casted along with beams were tested in compression. Their results are presented in **Table-3.2.** The general trend is that the inclusion of steel fibres cause lowering of the compressive strength of concrete marginally. The above conclusion agrees with that of most of the earlier investigators [3,7,70,80,95,168]. The reason that can be attributed for the little reduction in the compressive strength is due to the decrease of degree of compaction because of addition of fibres in concrete. Final strength of steel fibre reinforced concrete is marginally lower or higher than that of plain concrete.

From the results of the present investigations, the ratio of the cylinder compressive strength to the cube compressive strength shows a mean value of 0.789. The above results clearly indicate that the ratio of cylinder compressive strength to cube strength of concrete does not get altered due to the presence of steel fibres in concrete. There is no significant difference between the initial tangent modulus of elasticity of steel fibre reinforced concrete and plain concrete. This agrees with the conclusions arrived by earlier investigators [70,80,95,179].

5.3 Behavior in Splitting Tensile Strength

The present test indicates that splitting tensile strength of fibre reinforced concrete is greatly enhanced due to addition of steel fibres. The rate of increase of tensile strength was more than 160%, when the fibre content was about 1% by volume. This agrees with conclusions arrived by earlier investigators [23,66,67,95,163,179]. Rate of strength increase due to fibre addition is greatest in splitting tensile strength. This means that steel fibres greatly improve the resistance to cracking.

5.4 Behaviour in Flexure

In the pre-cracking stage, flexural behaviour of fibre reinforced concrete does not differ significantly from that of plain concrete. For post-cracking behaviour of fibre reinforced concrete, it is totally different than that of plain concrete. The results show that for addition of 1% fibre by volume, increase in flexural strength is about 95 percent. The addition of 1.5% of fibres by volume increases the flexural strength by about 125 percent over plain concrete. This is for plain round steel fibres. The above conclusion agrees with that of most of the previous investigators [3,23,66,67,161,163].

5.5 Behaviour in Direct Tension

The knowledge of the direct tension test shows that the fibres at the cracked section will undergo the process of pulling out. The amount of pullout undergone by any fibre depends on the magnitude of the strain in the cross section at the location of fibre.

It is evident from direct tension test results that due to the presence of fibres, brittle matrix transforms into a ductile one which shows significant enhancement in the tensile strain capacity. See **Fig.5.1** and **Table-5.1**. From the above fact it is logical that for any analytical formulation pertaining to the full fibrous concrete or partial fibrous concrete section the strain enhancement need to be established along with other parameters. (54,55)

The concept of strain enhancement is used for predicting the flexural behaviour of fibre reinforced concrete. Over the entire post-cracking range, it has not been attempted so far in deep beams and moderate deep beams. Based on this strain enhancement concept, a flexural theory for steel fibre reinforced concrete member has been proposed.

For steel fibre reinforced concrete deep beams and moderate deep beams, the ultimate flexural capacity of such beams depends on L/D ratio, compressive strength of concrete, percentage of steel fibres, tensile strength of fibre reinforced concrete, areas of tension and compression reinforcement, the yield strength of reinforcement alongwith the strain enhancement factor and moment capacity enhancement factor.

5.6 Moment Capacity Enhancment Factor

Analytical investigations pertaining to the structural response of partially steel fibre reinforced concrete beams as well as for fully steel fibre reinforced sections are presented in the following.

The main aim of this investigation is to explore the possibility and potential use of fibres in the zones of concrete structural members where tensile stresses are likely to be induced, in a manner just similar to that of conventionally reinforced concrete members. Here, the appropriate engineering parameter of steel fibre reinforced concrete which can be incorporated in the analysis of structural response of deep beams and moderate deep beams is obtained. The possible increase in moment capacities corresponding to different depth condition has been analytically computed using suitable parameter such as aspect ratio of fibres, volume fraction of fibres, grade of concrete which can be arrtibuted towards the enhancement in moment capacity of such sections. It has already been established that incorporation of steel fibres in concrete matrices results in substained increase in post-cracking ductility in addition to the increase in its tensile strength.

Figure 5.3 shows the longitudinal section as well as cross section with possible strain distribution behaviour of a partially fibrous concrete section subjected to two point loading. As per assumptions, the fibrous zone will attain a stress lėvel equal to the modulus of rupture of the plain concrete at the extreme tension layer. As load increases gradually, there is progressive development of microcracks on tension side of beam. At ultimate load, entire fibrous zone is in the ductile stress condition. After this stage, load begins to decrease gradually with fibres being pulled out across the crack. Due to increase in tensile strains, the netural axis begins to shift more and more towards compression side. When the pull out of the fibres from matrix is complete, the composite is considered to have failed.

5.7 Possible Modes of Failure

The partially fibre reinforced concrete members undergoing flexural behaviour have three possible modes of failures.

(1) Failure may take place at the interface when plain concrete reaches its ultimate strain in compression before the fibrous reinforced zone reaches its ultimate strain in tension. This mode of failure is undesirable since the failure is of brittle nature.

(2) The second mode of failure is considered when the depth of fibre reinforced concrete is too large than that required for compatibility of ultimate strains and the associated parameters. Here,fibre reinforced section reaches its flexural tensile strain before the plain concrete fails at the interface. This mode of failure is preceded by cracking showing good ductile behaviour similar to the fully fibrous section. Although this mode of failure satisfies the performance of fibres in the entire depth of the section, it becomes an uneconomical proposition.

(3) The third mode of failure is considered when fibre reinforced concrete reaches its ultimate strain at the tensile extremity simultaneously when the plain concrete reaches its limiting strain at the interface. This mode of failure satisfies the structural performance and also ensures optimum use of steel fibres.

The distribution of compressive stress in concrete may be taken as any suitable shape that results in a reliable prediction of the flextural strength of the members. Several relationships were proposed to characterise this stress-strain behaviour. They are usually in the form of linear, bilinear, parabolic and combination of the previous various curves. The often used stress-strain curve consisting of a second degree parabola upto the maximum stress at the strain of 0.002 and then a linearly falling branch upto an assumed limit of useful concrete strain is used in the following derivation.

5.8 Derivation of Flexural Formula

Assumptions:

1. Strain distribution is linear throughout the depth of the section at all stages of loading.
2. The fibres are uniformly distributed and randomly oriented in the concrete matrix.

3. Before crack is developed, contribution of the fibres to the internal moment of resistance is negligible.

4. For the quantity of fibers used in the investigation, Young's Modulus for fibrous concrete is not significantly different from that of plain concrete. Modulus of elasticity is identical in both compression and tension.

5. Cracking stresses and strains for plain concrete and for fibrous concretes upto first cracking are the same and equal to those of plain concrete.

6. The contribution of the fibres dispersed in a concrete matrix can be represented in the form of a triangular block of suitable stress intensity.

7. A functional form of stress-strain relationship of second degree polynomial upto the point of maximum stress(σ_u) is considered and beyond that point of ultimate load this stress-strain relationship is neglected.

8. The shape of the stress-block in compression has been assumed to be the same as the shape of stress-strain curve of concrete upto ultimate load.

9. A fibre becomes ineffective when the strain in the beam section at the fibre location is "t_f" times the strain in the fibre at its maximum stress.

10. It is assumed that the tension steel stress is at or above yield stress at ultimate load as most of the moderate fibre reinforced concrete deep beams are reinforced with only a little amount of tension steel.

The above assumptions proposed are valid for the prediction of ultimate post-cracking flexural strength only. It is not useful to obtain the contribution of fibres at any intermediate stage.

Considering the assumed stress-strain relation in a functional form as a parabolic relation, ~~as shown in Fig. 5.4,~~

$$\sigma = A \in + B \in^2 \qquad \text{......... (5.1)}$$

where A and B are constants. The constants are determined by considering different boundary conditions. see Fig 5.4

Initial tangent modulus of concrete does not vary significatly due to the inclusion of steel fibres. Hence, the initial stage of the ascending portion of stress-strain curve of steel fibre reinforced concrete remains nearly the same as that of an identical plain concrete. The strain at maximum stress increases with the inclusion of steel fibres. This increase is more for higher value of aspect ratio and volume fraction of fibres. The stress strain curve generally becomes flatter in descending part with the increase in aspect ratio and volume fraction.

It is reasonable to expect certain upper limits for aspect ratio and volume of fibres upto which the ductility of concrete could increase. After such a limit, there will be problems in mixing, workability, compaction, etc., due to bunching or balling of fibres in concrete.

The analytical formulations involve several ratios, and coefficients related to the tensile stress-strain relationship of reinforcing steel bars and compressive stress-strain curves of fibrous concrete. These parameters and ratios are now defined with repect to the idealized stres-strain curve of steel and concrete and the contribution of the fibres.

The functional form of stress-strain relationship for fibrous concrete in compression is expressed as

$$\sigma = A \in + B \in^2$$

where A and B are constants.

At origin

$$\in = 0; \frac{d\sigma}{d\in} = E_o$$

$$\therefore\ A = E_o$$

At ultimate

$$\in = \in_u\ ; \frac{d\sigma}{d\in} = 0$$

$$\therefore\ B = -\frac{E_o}{2\in_u}$$

The resulting equation for stress-strain for fibrous concrete in compression is

$$\sigma = E_o \in -\frac{E_o \in^2}{2\in_u} \qquad \text{................(5.2)}$$

At ultimate stage $\in = \in_u$ and $\sigma = \sigma_u$;

Hence, $\sigma_u = \frac{E_o \in_u}{2}$

$$\in_u = \frac{2\sigma_u}{E_o}$$

Substituting in Eq. (5.2), the resulting equation is

$$\sigma = E_o \in -\frac{E_o^2 \in^2}{4\sigma_u} \qquad \text{....... (5.3)}$$

On integrating the curve between the limits zero and ultimate, the total compressive force C is obtained for various conditions.

(a) Derivation for Lower Half Depth Fibrous Concrete Section

For lower half depth fibrous concrete (see **Fig.5.6 (b)**) total compressive force C can be written as

$$C = C_1^{'} + C_2^{'}$$

$$C = \left[\frac{E_o k^2 bd^2}{2R} - \frac{E_o^2 k^3 bd^3}{12\sigma_u R^2}\right] + A_s^{'} f_s^{'} \qquad \text{....... (5.4)}$$

Total tensile force T can be written as

$$T = T_1 + T_2$$

$$T = A_s f_s + \tfrac{1}{2} bd(1-k) f_r \qquad \text{...... (5.5)}$$

Defining α = rotational factor = $\frac{E_o d}{\sigma_u R}$, and equating C = T,

$$\frac{E_o k^2 bd^2}{2R} - \frac{E_o^2 k^3 bd^3}{12\sigma_u R^2} = \tfrac{1}{2}(1-k) \cdot f_r bd + A_s f_s - A_s^{'} f_s^{'}$$

Reaaranging the above expression

$$\left[\frac{\alpha k^2}{2} - \frac{\alpha k^3}{12}\right]\frac{\sigma_u}{p} - \tfrac{1}{2}(1-k) \cdot \frac{f_r}{p} = f_s - \frac{p^{'}}{p} f_s$$

where p = As/bd

$p^{'}$ = A's/bd

f_s - stress in tensile reinforcement

$f_s^{'}$ - stress in compression reinforcement

f_r - modulus of rupture of fibrous concrete

At ultimate,

$$\sigma_u = \frac{E_o \in_u}{2}$$

Defining α_u as a rotation factor at ultimate load

$$\alpha_u = \frac{2d}{\in_u R}$$

$$\in_u = \frac{k_u d}{R}$$

$$\alpha_u = \frac{2}{k_u}$$

$$f_{s;=}\left[\frac{\alpha k_u^2}{2} - \frac{\alpha k_u^3}{12}\right]\frac{\sigma_u}{p} - \frac{1}{2}(1-k_u)\cdot\frac{f_r}{p} + \frac{p'}{p}f'_s \quad \text{... (5.6)}$$

Assuming $\sigma_u = 0.85 f_c$, and substituting in the Eq.(5.6) and simplifying

$$f_s = 0.67\frac{k_u}{p} \times 0.85 f_c - 0.5\frac{f_r}{p}(1-k_u) + \frac{p'}{p}\cdot f'_s \quad \text{.... (5.7)}$$

$$k_u = \frac{f_s p + 0.5 f_r - f'_s p'}{0.57 f_c + 0.5 f_r}$$

Assuming f_r=0.15f_c for plain concrete

$$k_u = 1.55\cdot\frac{f_s}{f_c}\cdot p + 0.12 - 1.55\cdot\frac{f'_s}{f_c}\cdot p' \quad \text{.....(5.8)}$$

Taking moment of all the forces about the centre of gravity of compressive force due to partial fibrous concrete, the following expression is obtained.

$$M_{HFL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \tfrac{3}{8}k_u\right) + A_s' f_s' d' + \tfrac{1}{2} f_r b\left[\tfrac{2}{3}(d - k_u d)\right] + \tfrac{5}{8}k_u d$$

..... (5.9)

Rearranging the above Eq. (5.9)

$$M_{HFL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \tfrac{3}{8}k_u\right) + A_s' f_s' d' + \tfrac{0.5}{24} f_r b d^2 (1 - k_u)(16 - k_u)$$

...... (5.10)

Let $h_f D$ be the depth of fibrous tensile zone to achieve a balance section condition for optimising the use of material to its desirable possible values of $\in_{cu}$ and $\in_{mu}$

i.e. ultimate strain of plain concrete = ultimate strain of matrix

$$\in_{cu} = \in_{mu}$$

$$\text{and} \quad \sigma_t = \sigma_{mu}$$

From the strain distribution diagram of **Fig. 5.3**

$$\frac{[1-k_u-h_f]}{[1-k_u]} = \frac{\in_{mu}}{\in_{cu}} = \frac{1}{t_f} \qquad \text{....... (5.11)}$$

Let

$$\in_{cu} = t_f \in_{mu} \qquad \text{......(5.12)}$$

where t_f is tensile strain capacity enhancement factor.

It depends on aspect ratio of fibre, volume fraction of fibres and grade of concrete. This t_f can be obtained from direct tension test of the plain and fibrous concrete.

From the stress distribution diagram of **Fig.5.3**, flexural tensile stress of concrete is equal to the ultimate flexural tensile stress of matrix.

$$\sigma_t = \sigma_{mu}$$

$$\sigma_t h_f D + \tfrac{1}{2}\sigma_t D(1 - k_u - h_f) = \tfrac{1}{2}\sigma_c k_u D$$

$$k_u = \frac{(1+h_f)}{(\sigma_t+\sigma_c)}\sigma_t$$

Taking

$$r' = \frac{\sigma_c}{\sigma_t}$$

$$k_u = \left[\frac{1+h_f}{1+r'}\right]$$

$$\beta_f = 2[1 + h_f - k_u] \qquad \text{..........(5.13)}$$

where β_f is defined as moment capacity enhancement factor

Assuming $f_r = 0.15 f_c$ for plain concrete Eq.(5.10) reduces to

$$M_{HFL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \tfrac{3}{8}k_u\right) + A_s' f_s' d' + \tfrac{0.064}{24} \cdot \beta_f \cdot f_c b d^2 (1 - k_u)(16 - k_u) \qquad \text{....... (5.14)}$$

(b) Derivation For Full Depth Fibrous Concrete Section

For full depth firbous concrete (see **Fig.5.6 (a)**) total compressive force C can be written as

$$C = C_1' + C_2' + C_3'$$

$$C = \left[\frac{E_o k^2 b d^2}{2R} - \frac{E_o^2 k^3 b d^3}{12\sigma_u R^2}\right] + \tfrac{1}{2} bkdf_r + A_s' f_s' \qquad \text{.... (5.15)}$$

Total tensile force T can be written as

$$T = T_1 + T_2$$

$$T = A_s f_s + \tfrac{1}{2} bd(1 - k) f_r \qquad \text{......(5.16)}$$

Equating C = T and substituting α = rotational factor $=\frac{E_o d}{\sigma_u R}$

$$\left[\frac{\alpha k_u^2}{2} - \frac{\alpha^2 k_u^3}{12}\right]\frac{\sigma_u}{p} - \frac{1}{2}(1-2k_u)\frac{f_r}{p} = f_s - \frac{p'}{p}f_s'$$

$$f_s = \left[\frac{\alpha k_u^2}{2} - \frac{\alpha^2 k_u^3}{12}\right]\frac{\sigma_u}{p} - \frac{1}{2}(1-2k_u)\frac{f_r}{p} - \frac{p'}{p}f_s'$$

Substituting in the above Eq. and assuming σ_u = 0.85f_c

$$k_u = \frac{f_s p + 0.5 f_r - f_s' p'}{0.57 f_c + f_r}$$

Assuming $f_r = 0.15 f_c$ for plain concrete

$$k_u = 1.38 \cdot \frac{f_s}{f_c} + 0.10 - 1.38 \cdot \frac{f_s'}{f_c} p'$$

Taking moment of all the forces about the centre of gravity of compressive force,

$$M_{FL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \frac{3}{8}k_u\right) + A_s' f_s' d'$$

$$+\frac{1}{2} f_r b\left[\frac{2}{3}(d - k_u d)\right] + \frac{5}{8}k_u d$$

Rearranging the above Eq.

$$M_{FL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \frac{3}{8}k_u\right) + A_s' f_s' d'$$

$$+\frac{0.5}{24} f_r b d^2 (1 - k_u)(16 - k_u) \quad \text{...... (5.17)}$$

Considering enhancement in moment capacity due to full depth fibrous concrete, the moment capacity enhancement factor β_f is utilised as a fibre parameter assuming

$$\beta_f = 2(1 + h_f - k_u)$$

f_r = 0.15f_c for plain concrete

$$M_{FL} = \left[A_s f_s - A_s' f_s'\right] d\left(1 - \tfrac{3}{8}k_u\right) + A_s' f_s' d'$$

$$+\tfrac{0.064}{24} \cdot \beta_f \cdot f_c b d^2 (1 - k_u)(16 - k_u)$$

(c) For without fibre condition

Eq.(5.10) of lower half depth fibre condition can be used with modification in β_f and k_u value.

Here β_f =1 as no enhacement in moment capacity

$$k_u = 1.56\tfrac{f_s}{f_c}p - 1.56 \cdot \tfrac{f_s'}{f_c}p' + 0.1 \qquad \text{....... (5.18)}$$

Substituting the value of k_u from Eq.(5.18)

$$M_{FL} = \left(A_s f_s - A_s' f_s'\right) d\left[1 - \tfrac{0.6}{f_c}\left(f_s p - f_s' p'\right)\right]$$

$$+A_s' f_s' d' + 2.65 \times 10^{-3} f_c \cdot bd$$

$$\times\left[0.9 - \tfrac{1.6}{f_c}\left(f_s p - f_s' p'\right)\right]\left[15.9 - \tfrac{1.6}{f_c}\left(f_s p - f_s' p\right)\right] \qquad \text{....... (5.19)}$$

In Eqs.(5.10 and 5.17), all the quantities except the steel stresses f_s and f_s', are known for any given beam. These stresses are governed by the stress-strain characteristics of the reinforcement. When considering low-carbon steel with a definite yield point, f_s and f_s' are easily determined for under reinforced beams if the tensile reinforcement has not gone into strain hardening, i.e.

$$f_s = f_y \qquad \text{......... (5.20)}$$

$$f_s' = E_s \in_u \frac{[k_u - \alpha_c]}{k_u}$$

$$f_s' = 2m \cdot \sigma_u \left[\frac{k_u - \alpha_c}{k_u}\right] \qquad \text{....... (5.21)}$$

When strain hardening is reached or the steel exhibits a non-linear stress-strain curve, assuming plane sections remain plane the following is obtained.

$$f_s = 0.625 \cdot k_u \cdot \frac{f_c}{p} - 0.075 \cdot \frac{f_c}{p} + \frac{p'}{p} \cdot f_s'$$

$$= 0.64 \cdot \frac{f_c}{p} \cdot \left[\frac{\in_u}{\in_u - \in_s}\right] - 0.075 \cdot \frac{f_c}{p} - \frac{p'}{p} \cdot f_s' \qquad \text{....... (5.22)}$$

The stress in steel can only be determined if the stress-strain curves for the tension and compression reinforcement are known i.e.

$$f_s = f \cdot (\in_s) \qquad \text{....... (5.23)}$$

$$f_s' = f \cdot \left(\in_s'\right) \qquad \text{....... (5.24)}$$

By the simultaneous solution of Eqs.(5.23) and (5.24), the values of f_s and f_s' can be determined. Due to the nature of most steels, Eq. (5.23) and Eq. (5.24) cannot be easily expressed as a single continuous function. Therefore, if a curve of f_s is plotted for the specified ultimate load parameters $f_c, \in_u$, p and $p^{/}$ on the stress-strain curve of tensile reinforcement, f_s can be obtained from the intersection of the curves (37) as shown in **Fig.5.9**.

Equation (5.22) is valid for planer sections. However, it can be modified for non-planer sections by the introduction of the strain compatibility factor (SCF).

$$SCF = \frac{k_d \in_s}{\in_c (d-kd)} \quad \text{....... (5.25)}$$

For ultimate conditions

$$SCF = \frac{K_u d \cdot \in_s}{\in_u (d-k_u d)} \quad \text{....... (5.26)}$$

The expression for stress in steel for non-planer section becomes

$$f_s = 0.64 \cdot \frac{f_c}{p}\left[\frac{(SCF)\in_u}{(SCF)\in_u + \in_u}\right] - 0.075 \cdot \frac{f_c}{p} + \frac{p^{/}}{p} \cdot f_s^{/} \quad \text{........ (5.27)}$$

It is interesting to note that after consideration of strain compatibility factor, an equivalent strain or $SCF \times \in_u$ for deep beams is identical to that for shallow beams. The increased limiting strain and the effect of non-planarity in deep beams have a compensating net result.

$(SCF) \times \in_u = 0.003$ For Plain Concrete.

= 0.007 For Fibrous Concrete

Formulae proposed herein for flexure capacity of fibrous moderate deep beams can be used directly provided the value of f_c and moment capacity enhancement factor β_f according to the depth of fibres are known. There is no necessity of assuming values of constants, k_1, k_2 and k_3 as is usally done in various known formulae of other investigators.

5.9 Derivation of Shear Strength Formulae

The analysis of the ultimate strength of a steel fibre reinforced concrete deep beam and moderate deep beam failing in shear is based on the following assumptions :

(1) The shear strength of deep beam is dependent on the splitting strength of fibrous concrete.

(2) The approximate direction of the diagonal crack is the line joining the load point with the support point.

(3) The main longitudinal bars are also considered as web bars in calculating the ultimate shear strength.

(4) Total shear capacity of steel fibre reinforced concrete deep beams and moderate deep beams is obtained by a superposition of two components namely shear capacity of fibrous concrete and shear capacity of the web reinforcement.

(5) Shear capacity of fibrous concrete is dependent on the splitting of an elliptical section whose major axis lies on the line of diagonal crack defined in (2) above.

(6) The effectiveness of a web bar increases with the depth at which it intersects the line of diagonal crack and is dependent on the yield stress of the web reinforcement.

In deep beams, failure is actually a diagonal splitting failure. This phenomenon is similar to that exhibited by specimens in the Brazilian splitting test failing under diametrical compression.

Brock (18) pointed out the possibility of predicting the ultimate shear strength of reinforced concrete beams on the basis of the splitting strength of concrete. This is substantiated by the work of Ramkrishnan and Anantnarayan (135), Prakash Desai (49), Patel S. N. (127) and others.

Using a similar approach, a method is proposed for calculating shear capacity of steel fibre reinforced concrete deep beams and moderate deep beams using splitting analogy see **Fig.5.7**. With addition of fibres, shear capacity improves. Shear resistance is built up through fibres crossing a major diagonal crack or any such similar crack. The total shear capacity of a such beam is made up of the shear capacity of concrete, shear capacity of web reinforcement together with the resistance offered by the steel fibres (96,97) in the concrete.

Total Shear Capacity	=	Shear capacity of Concrete	+	Shear capacity of web Reinforcement	+	Shear capacity of Fibres

$$V_u = V_{uc} + V_{us} + V_{uf}$$

$$V_{uf} = k_b \cdot \sigma_b d\sqrt{V_f \cdot l_f}$$

where,

k_b - coefficient of bond stress variations.

σ_b - interfacial bond stress between fibres and matrix.

l_f - length of steel fibres.

V_f - volume percentage of steel fibres.

If θ is the inclination of the crack with the axis of the beam, the splitting component of the load will be $\frac{W}{2}\operatorname{cosec}\theta$ as shown in **Fig. 5.7**. It is in the direction of the crack causing splitting. This component is resisted by fibrous concrete and reinforcing steel crossing the line of splitting. At ultimate, the shear capacity of the concrete is assumed to be due to splitting of an elliptical section whose major axis is along the line joining the support and the nearest load point. Further, maximum tensile stress that develops in the elliptical section is assumed to be equal to the tensile strength of the matrix.

Thus

Tensile strength of concrete = $\frac{V_{uc} \cdot co\sec\theta}{(\text{constant})(Width \cdot of \cdot beam)(length \cdot of \cdot major \cdot axis \cdot of \cdot an \cdot elliptical \cdot section)}$

Hence,

$$f_t = \frac{V_u cose\theta}{(\text{constant})(b)(2A_o)} \qquad .... (5.28)$$

where A_o is the semi-major axis of the elliptical section, From Appendix (B)

$$A_o^2 = \frac{\frac{d^2}{4a^2}[a^2+d^2]}{\left[\frac{d}{a}\right]^2+[1-e^2]} \qquad 0 < e < 1 \qquad .... (5.29)$$

By taking the value of e close to one it is readily noticed from the above expression that the ellipse tends to degenerate into an elliptical strip, while the value of e close to zero tend the ellipse very near to a circle. Hence for moderate ellipses, assuming the value of e to be 1/2 one gets.

$$2A_o = d\sqrt{\frac{\left(1+\frac{a^2}{d^2}\right)}{1+0.75\left(\frac{a^2}{d^2}\right)}} \qquad .... (5.30)$$

The constant in the equation depends on the ratio of length of major axis to length of minor axis of an ellipse. Its value is 1.483 and is calculated as given in reference (12). Using values of $2A_o$ from the above equation and simplifying by substituting above said values

$$V_{uc} = \frac{1.483 \cdot ft \cdot bd}{\sqrt{1+0.75 \cdot \left(\frac{a^2}{d^2}\right)}} \qquad .... (5.31)$$

Shear capacity of web reinforcement can be obtained by considering the strain in web steel placed at given level in the tested beams. This variation of strain in the web steel is assumed to be given by a straight line.

$$\frac{Y}{d} = N_1 \in_s + \frac{N_2}{d} \quad \text{.... (5.32)}$$

where N_1 and N_2 are constants and $\in_s$ is the strain in the web steel at any level Y from the top surface of the beam. Using the method of least squares of all the average maximum strain readings given in **Table-5.3**.

$$N_1 \in_s = \frac{Y - N_2}{d}$$

$$5.8 \frac{f_s}{E_s} = \frac{Y - 382}{d}$$

$$f_s = \frac{E_s}{5.8}\left[\frac{Y - 382}{d}\right] \quad \text{...... (5.33)}$$

Hence, stress in web steel f_{si} at any depth Y_i is given by

$$f_{si} = \frac{E_s}{5.8}\left[\frac{Y_i - 382}{d}\right] \quad \text{........ (5.34)}$$

As the strain in web steel at y/d = 1.0 has an average maximum value of strain of about 70% of yield strain of the steel used, the stress in web steel at y/d = 1.0 can be safely assumed to be equal to $0.7 f_y$. Hence

$$0.7 f_y = \frac{E_s}{5.8}\left[\frac{Y_i - 382}{d}\right] \quad \text{......(5.35)}$$

Rearranging we have Eq.(5.33)

$$E_s = \frac{5.8\times 0.7f_y}{\left[\frac{Y_i-382}{d}\right]}$$

Substituting the above value in Eq.(5.34)

$$f_{si} = \frac{5.8\times 0.7f_y}{5.8\left[\frac{Y_i-382}{d}\right]}\times\left[\frac{Y_i-382}{d}\right]$$

$$f_{si} = 0.7f_y\left[\frac{Y_i}{d}\right] \qquad \text{..... (5.36)}$$

Now $V_{us}cose\theta = \Sigma f_{si}A_{si}\sin(\alpha_i+\theta)$ see Fig 5.8 (c) ... (5.37)

Substituting the value of f_{si} in Eq.(5.37)

$$V_{us} = \frac{0.7f_y}{\sqrt{1+\left[\frac{a^2}{d^2}\right]}}\Sigma\frac{Y_i}{d}\cdot A_{si}\cdot\sin(\alpha_i+\theta) \qquad \text{.... (5.38)}$$

. The total shear capacity of a steel fibre reinforced concrete deep beam and moderate deep beam is given by

$$V_u = \frac{1\,483{\cdot}f_t{\cdot}bd}{\sqrt{1+0\,75\left[\frac{a^2}{d^2}\right]}} + \frac{0.7{\cdot}f_y}{\sqrt{1+\left[\frac{a^2}{d^2}\right]}}\Sigma\frac{Y_i}{d}\cdot A_{si}\cdot sin(\alpha_i+\theta) + k_b\sigma_b\cdot d\sqrt{V_f\cdot l_f}$$

...... (5.39)

and ultimate load W for a steel fibre reinforced concrete deep beams in shear can be approximated as

$$W = 2V_u = \frac{3.0{\cdot}f_t{\cdot}bd}{\sqrt{1+0\,75\left[\frac{a^2}{d^2}\right]}} + \frac{1\,4{\cdot}f_y}{\sqrt{1+\left[\frac{a^2}{d^2}\right]}}\Sigma\frac{Y_i}{d}\cdot A_{si}\cdot sin(\alpha_i+\theta)$$

$$+2K_b\cdot\sigma_b\cdot d\sqrt{V_f\cdot l_f} \qquad \text{...... (5.40)}$$

$$W = 2V_u = \alpha_1 \cdot f_t \cdot bd + \alpha_2 \cdot f_y \Sigma \frac{Y_i}{d} \cdot A_{si} \cdot sin(\alpha_i + \theta)$$
$$+2k_b \cdot \sigma_b \cdot d\sqrt{V_f \cdot l_f} \qquad \text{..... (5.41)}$$

where

$$\alpha_1 = \frac{3.0}{\sqrt{1+0.75\left(\frac{a^2}{d^2}\right)}}$$

$$\alpha_2 = \frac{1.4}{\sqrt{1+\left(\frac{a^2}{d^2}\right)}}$$

α_1 and α_2 are known from the loading and geometry of the sections.

The shear strength Eq.(5.39) is derived for the case of deep beams with two points loads. Deep beams with only one point load are rarely met in the practice. Normally, these will be subjected to uniformly distributed loads or a conbination of concentrated and distributed loads. Hence, it is essential to derive methods for calculating the shear strength of these beams. The theory developed previously is extended to the case of deep beams subjected to uniformly distributed loads. See **Fig.5.8.**(d)

Let dQ be the elemental splitting force in the direction of the potential crack and F be the resultant splitting force acting at an angle ψ with the horizontal as shown in **Fig.5.8.** (d)

$$dQ = \frac{W \cdot cos\psi \cdot dx}{Sin(\theta+\psi)} \qquad \text{..... (5.42)}$$

Resolving dQ in the direction of potential crack and summing up one gets

$$F = w \int \frac{\cos\Psi \cdot \cos(\Psi-\theta)}{\sin \cdot (\theta+\Psi)} \cdot dx$$

$$F = \frac{wL}{6d}[L \cdot cos\psi + 3d \cdot sin\psi] \qquad \text{..... (5.43)}$$

For F to be maximum

$$\frac{dF}{d\psi} = 0$$

$$\tan\psi = \frac{3d}{L}$$

This fixes the direction of the diagonal crack which is inclined at an angle of $\tan^{-1}(3d/L)$ to the horizontal. Thus shear span "a" in case of deep beams with uniformly distributed load is L/3. Substituting this value of shear span in Eq.(5.41) the following formula is obtained.

$$W = \alpha_1' \cdot f_t \cdot bd + \alpha_2' \cdot f_y \Sigma \frac{Y_i}{d} \cdot A_{si} \cdot \sin(\alpha_i + \theta) + 2k_b \cdot \sigma_b \cdot d\sqrt{V_f \cdot l_f} \quad \text{....... (5.44)}$$

where

$$\alpha_1' = \frac{3.0}{\sqrt{1+\frac{L^2}{12d^2}}}$$

and

$$\alpha_2' = \frac{1.4}{\sqrt{1+\frac{L^2}{9d^2}}}$$

5.10 Computation of Maximum Crack Width Formula

In recent years the control of crack widths in reinforced concrete design has become an important design consideration. Cracks can be developed in a reinforced concrete structure as the internal stresses exceed permisible tensile strength of the concrete. See **Fig.5.10**. With the introduction of high tensile strength deformed bars, the problem of control of cracking has become more complex.

(116)

Assurance of strength adequacy of structural elements is no longer sufficient for aesthetic or safe performance acceptance. Serviceability conditions have to be met involving cracking performance at normal load conditions. In the limit state design of concrete structures, the limit state of cracking is one of the criteria which the design has to satisfy. This check is required not only for reinforced concrete members, but also for prestressed concrete members which come under Class-3 structures of the CEB-FIP classification.(32)

Two approaches are available for estimation of maximum crack width viz. steel stress approach and fictitious tensile stress approach. Samarai and Elvery (149) compared reinforced tensile specimens with different fibre contents, and showed that the fibres reduced crack width and crack spacing with longer and thinner fibres being more effective. Swamy and Al Noori (166) studied the effect of fibres in beams and found that the addition of fibres reduced the crack widths and the provision of fibre concrete in the form of a tensile skin was just as benifical as providing fibre concrete in the whole tension zone.

In the present investigations, the variation of crack width with the load and the effect of quantum of fibres on crack width is studied. In order to comply with the design requirements of the present codes of practice the servicability limit state must be fulfilled.

Crack spacing as such whether minimum, mean or maximum is of little practical significance. Similalry minimum and mean crack widths are not of much importance from practical design considerations. Nawy (116) also reported similar opinion. Only the maximum crackwidth is limited to certain specified values at service load, depending on the exposed environment in the design codes.

It has been observed, in the present investigations, that the crackwidth at a point in a cracked section is proportional to the distance of the point from the neutral axis. The same observation was made by many of the earlier investigators (9,19,30,48). Also the variation in the maximum spacing was not found to be significant due to inclusion of steel fibres in the present investigation, though the crackwidth was reported to very linearly with load as well as steel strain in general.

IS:456-1978 (76) does not refer to any computations of crack widths. In section 5 under the clause 35,3.2, it is specified as a guide line that the surface width of the cracks should not exceed 0.3 mm. In the field, estimation of crack widths are usually done based on empirical formulae developed by ACI Committee 318-1989 (5), CP-110-1972 (36) and CEB-FIP (32). Out of these formulae, normally ACI formula gives very near solution for reinforced concrete structures. The ACI formula is based on the work of Gergeley and Lutz (63). They have suggested a formula to predict maximum crack width in inches at the extreme concrete tensile fibre at the bottom, based on computer statistical analysis of beam test results from Karr, Mattock, Clark, Rusch and Rahimi. (30,83,106,131)

The formula for maximum crack width in reinforced concrete beams under flexure is given by Gergeley and Lutz (63) as below :

$$w_{b\max} = 10.8 \times 10^{-6} \cdot \left(\frac{h_1}{h_2}\right) f_y \cdot \sqrt[3]{A_{ct} \cdot d_c} \qquad \text{....... (5.45)}$$

where h_1 ,h_2 and A_{ct} are variables. All the other parameters in the equation are constants. A formula is proposed to obtain a functional representation of maximum crack width in fibre reinforced concrete beams under flexure. Due to the presence of fibres and different L/D ratios the above formula is modified using least square method of standard optimization, (160). The existing formula is modified as under :

$$w_{b\max} = 10.8 \times 10^{-6} \left(\frac{h_1}{h_2}\right) f_y \cdot \sqrt[3]{A_{ct} \cdot d_c} \left[A\left(\frac{L}{D}\right) - B\right] [1 - \alpha V_f] \qquad \text{....... (5.46)}$$

where h_1 - distance from neutral axis to the soffit of the beam

h_2 - distance from neutral axis to the centroid of steel reinforcement

A_{ct} - area of concrete in tension zone

d_c - clear cover distance

where the term in the first square bracket gives the effect of L/D ratios and the term in the next bracket gives the effect of percentage of fibres in concrete. Expression for estimating maximum crack width Eq.(5.45) was modified using the results obtained in these investigation. A statistical analysis of data of L/D varying from 3.0 to 7.0 enabled the determination of coefficients in the expression. From the experimental results, the constants A = 0.09 B = 0.36 and α = 0.176 are obtained.

B = -.36

In absence of fibres in the reinforced concrete sections, $V_f = 0$ and hence the

$[1 - \alpha V_f]$ is reduced to 1.0.

When L/D = 7.0 or more, then the first factor $\left[A\left(\frac{L}{D}\right) - B\right]$ is to be taken as 1.0. Hence, the formula will reduce to the original formula which is applicable to normal shallow beams.

TABLE - 5.1

DIRECT TENSION TEST RESULTS OF CONCRETE SPECIMENS

$V_f = 0\%$ | $V_f = 0.5\%$

Load N	Stress N/mm²	Average Strain x10⁻⁶	Load N	Stress N/mm²	Average Strain x 10⁻⁶
1000	0.300	13.80	1000	0.300	11.20
2000	0.601	26.40	2000	0.600	22.10
3000	0.913	29.92	3000	0.910	33.12
4000	1.220	54.78	4000	1.230	45.80
5000	1.529	59.40	5000	1.530	54.70
6000	1.830	67.32	6000	1.820	62.40
7000	2.080	78.16	7000	2.090	71.23
8000	2.290	88.00	8000	2.280	88.10
8900 *	**2.480 ***	**99.00 ***	9000	2.450	112.16
			9300 *	**2.580 ***	**120.60 ***

$V_f = 0.75\%$ | $V_f = 1.0\%$

Load N	Stress N/mm²	Average Strain x 10⁻⁶	Load N	Stress N/mm²	Average Strain x10⁻⁶
1000	0.27	10.10	1000	0.270	9.0
2000	0.58	20.30	2000	0.550	18.0
3000	0.88	32.20	3000	0.840	30.5
4000	1.18	41.30	4000	1.130	36.8
5000	1.47	52.40	5000	1.410	49.9
6000	1.68	60.00	6000	1.700	57.7
7000	1.94	65.16	7000	1.990	59.1
8000	2.24	83.05	8000	2.220	78.0
9000	2.55	98.50	9000	2.500	84.5
9500	2.68	103.00	9500	2.660	121.3
10000	2.81	123.00	10000	2.780	151.0
10600 *	**2.92**	**135.00 ***	10500	2.860	173.0
			11300 *	**3.135 ***	**206.6 ***

*** Average values**

TABLE - 5.1 (Contd.)

DIRECT TENSION TEST RESULTS OF CONCRETE SPECIMENS

V_f = 1.5%

Load N	Stress N/mm^2	Average Strain $\times 10^{-6}$
1000	0.28	9.0
2000	0.56	17.0
3000	0.83	24.0
4000	1.10	35.0
5000	1.40	44.0
6000	1.68	54.0
7000	1.94	62.4
8000	2.25	74.3
9000	2.54	81.1
10000	2.80	119.3
11000	2.98	160.4
11500	3.21	237.0
12100 *	**3.37 ***	**276.0 ***

* **Average values**

TABLE 5.2

COMPUTATION OF ENHANCEMENT FACTORS

(A) STRAIN ENHANCEMENT FACTOR "t_f"

V_f %	Load N	Maximum stress N/mm^2	Maximum strain $\times 10^{-6}$	t_f
0.00	8900	2.48	99.0	1.00
0.50	9300	2.58	120.6	1.22
0.75	10600	2.92	135.0	1.37
1.00	11302	3.13	202.0	2.04
1.50	12100	3.37	296.0	2.98

TABLE 5.2

COMPUTATION OF ENHANCEMENT FACTORS

(B) MOMENT CAPACITY ENHANCEMENT FACTOR "β_f"

$t_f = \frac{\in_{cut}}{\in_{mut}}$	$r = \sqrt{(2t_f - 1)}$	$k_u = \frac{r}{t_f - r}$	$h_f = \frac{t_f - 1}{t_f + r}$	$\beta_f = 2[1 + h_f - k_u]$
1.00	1.000	0.500	0.000	1.000
1.20	1.183	0.496	0.084	1.175
1.40	1.342	0.489	0.146	1.313
1.80	1.612	0.473	0.234	1.524
2.00	1.732	0.464	0.268	1.608
2.20	1.844	0.456	0.297	1.682
2.50	2.000	0.444	0.333	1.778
3.00	2.236	0.427	0.382	1.910
3.50	2.449	0.412	0.420	2.017
3.75	2.550	0.405	0.437	2.064
4.00	2.646	0.398	0.451	2.107
4.50	2.828	0.386	0.478	2.183
5.00	**3.000**	**0.375**	**0.500**	**2.250**
5.50	3.162	0.365	0.519	2.309
6.00	3.317	0.356	0.537	2.361
7.00	3.606	0.340	0.566	2.452
7.50	3.742	0.333	0.578	2.491
8.00	3.873	0.326	0.590	2.527
9.00	4.123	0.314	0.610	2.591
10.00	4.359	0.304	0.627	2.646
12.00	4.796	0.286	0.655	2.739
14.00	5.196	0.271	0.677	2.813
16.00	5.568	0.258	0.695	2.875
18.00	5.916	0.247	0.711	2.927
20.00	**6.245**	**0.238**	**0.724**	**2.972**
25.00	7.000	0.219	0.750	3.063

TABLE- 5.3

MAXIMUM STRAIN IN WEB STEEL

Beam No.	Dist. from top y	Effect depth d	y/d	Maximum strain	Max. average strain
F1.0 D60	55	55	1	1500	
F1.5 D60	55	55	1	1337	1970.0
H1.0 D60	55	55	1	2100	
H1.5 D60	55	55	1	2943	
F1.0 D50	45	45	1	2040	
F1.5 D50	45	45	1	2591	2771.7
H1.0 D50	45	45	1	3529	
H1.5 D50	45	45	1	2927	
F1.0 D40	36	36	1	1344	
F1.5 D40	36	36	1	1770	1538.5
H1.0 D40	36	36	1	1100	
H.15 D40	36	36	1	1940	
F1.0 D30	26	26	1	1204	
F1.5 D30	26	26	1	2390	1942.55
H1.0 D30	26	26	1	2045	
H1.5 D30	26	26	1	2130	
F1.0 D20	16	16	1	2067	
F1.5 D20	16	16	1	1434	1371.75
H1.0 D20	16	16	1	1280	
H1.5 D20	16	16	1	0706	
F1.0 D15	11	11	1	1610	
F1.5 D15	11	11	1	1335	1910.5
H1.0 D15	11	11	1	2130	
H1.5 D15	11	11	1	2567	
F1.0 D12	08	08	1	1436	
F1.5 D12	08	08	1	1356	1516.5
H1.0 D12	08	08	1	1628	
H1.5 D12	08	08	1	1646	
F1.0 D10	06	06	1	960	
F1.5 D10	06	06	1	1445	1373.0
H1.0 D10	06	06	1	1470	
H1.5 D10	06	06	1	1617	

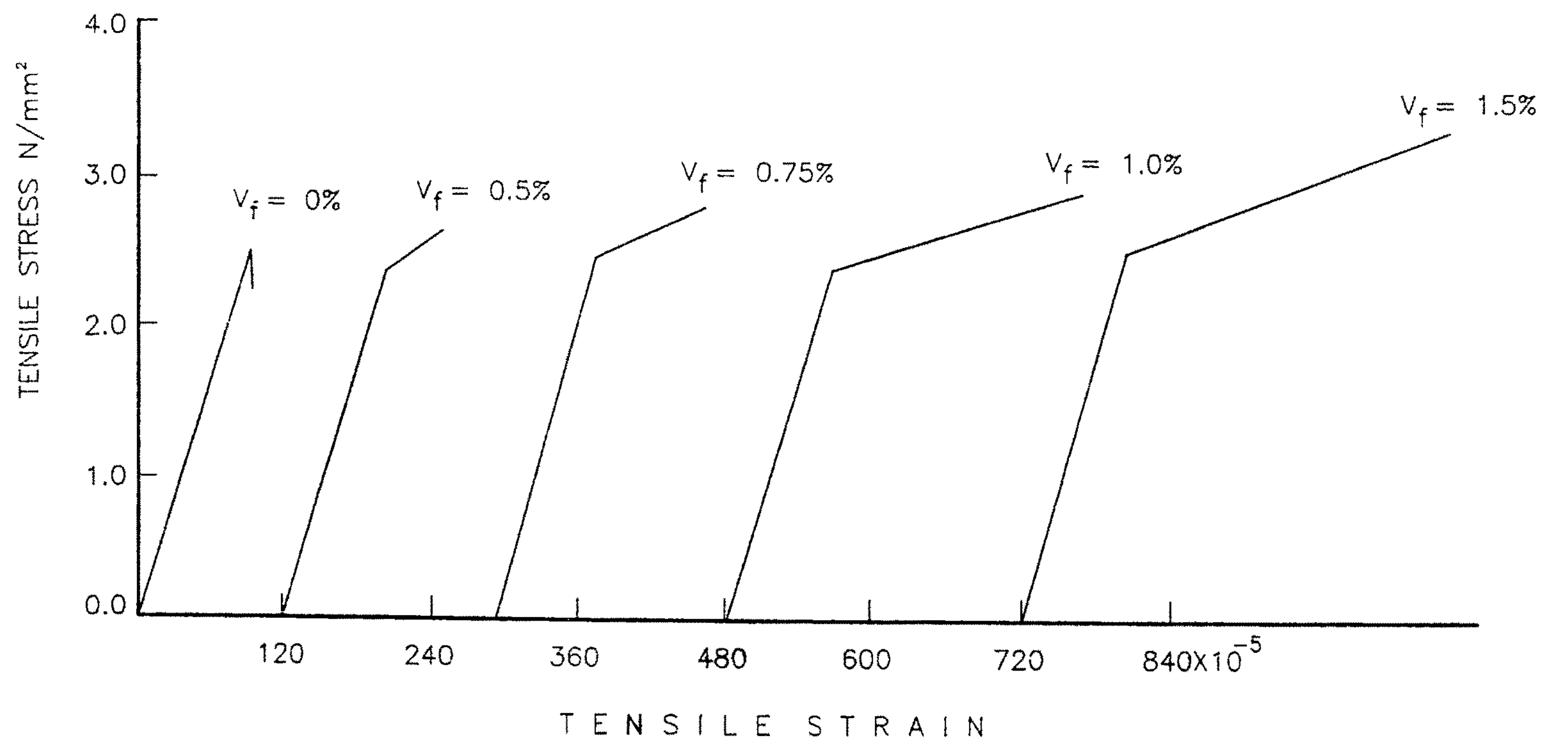


Fig. 5.1 TENSILE STRESS STRAIN CURVES FOR CONCRETES IN A DIRECT TENSION.

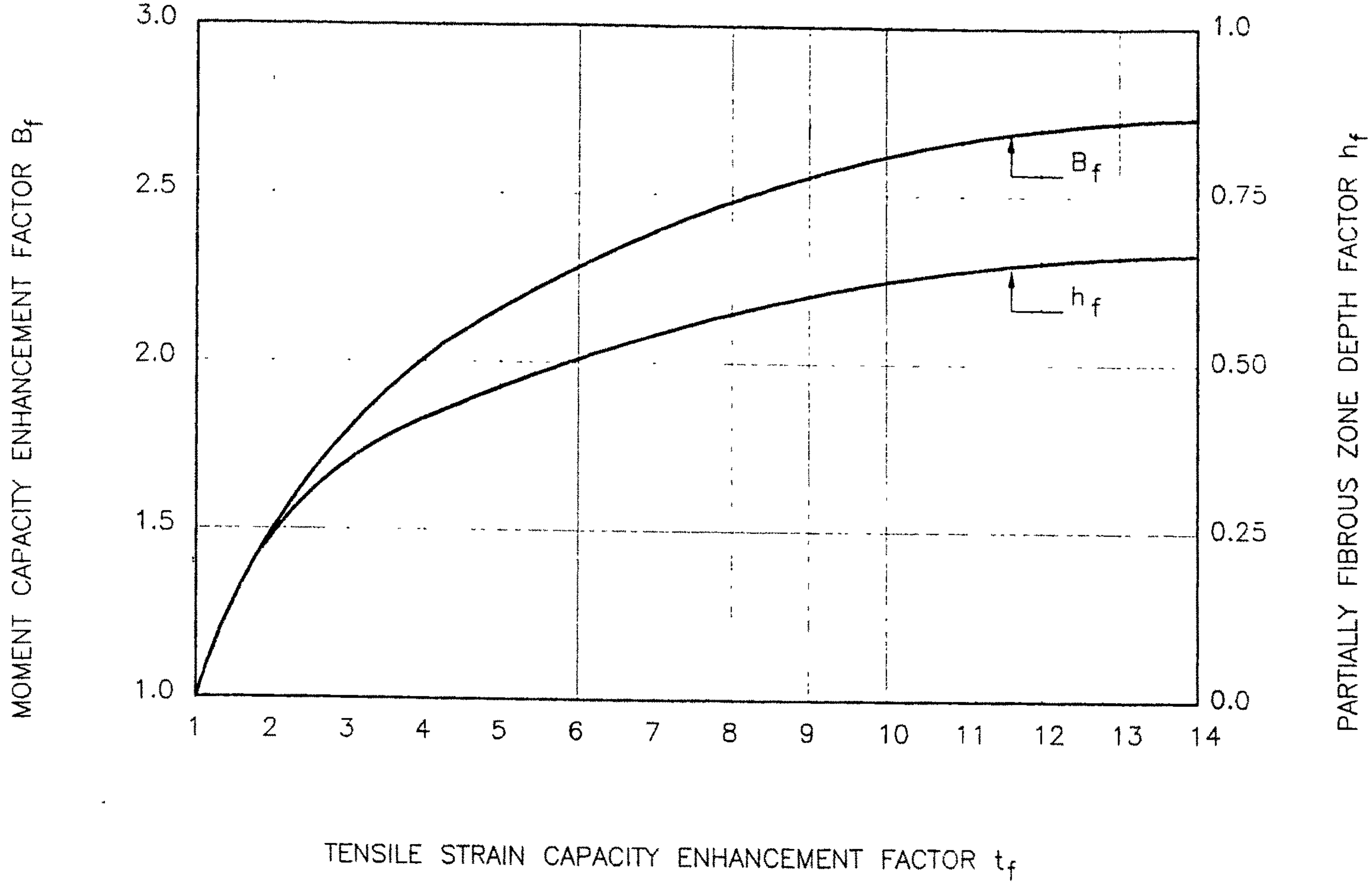
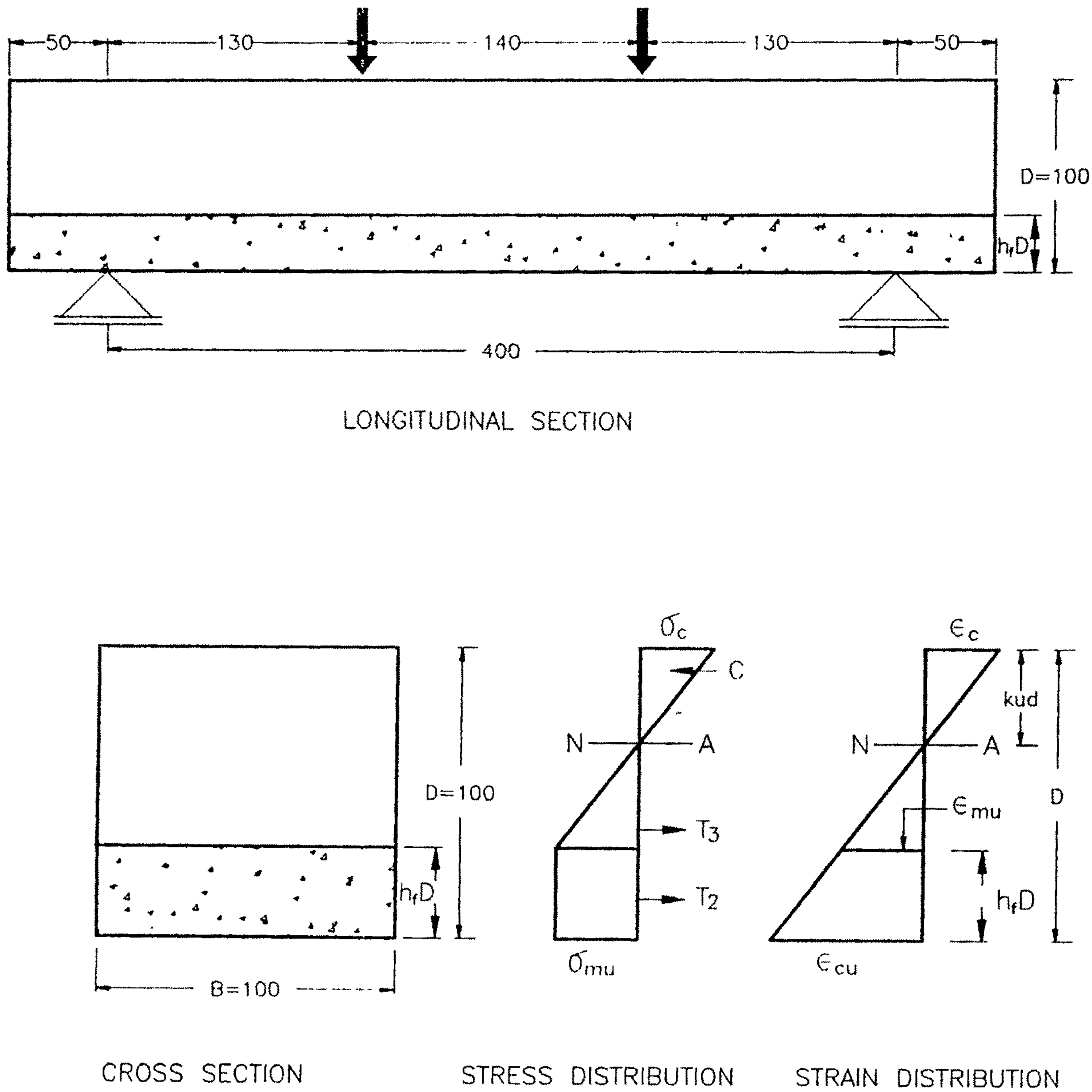


Fig. 5.2 VARIATION OF h_f AND B_f WITH t_f



ALL DIMENSIONS IN MM

Fig. 5.3 SECTIONAL VIEWS OF A PARTIALLY FIBROUS CONCRETE BEAM.

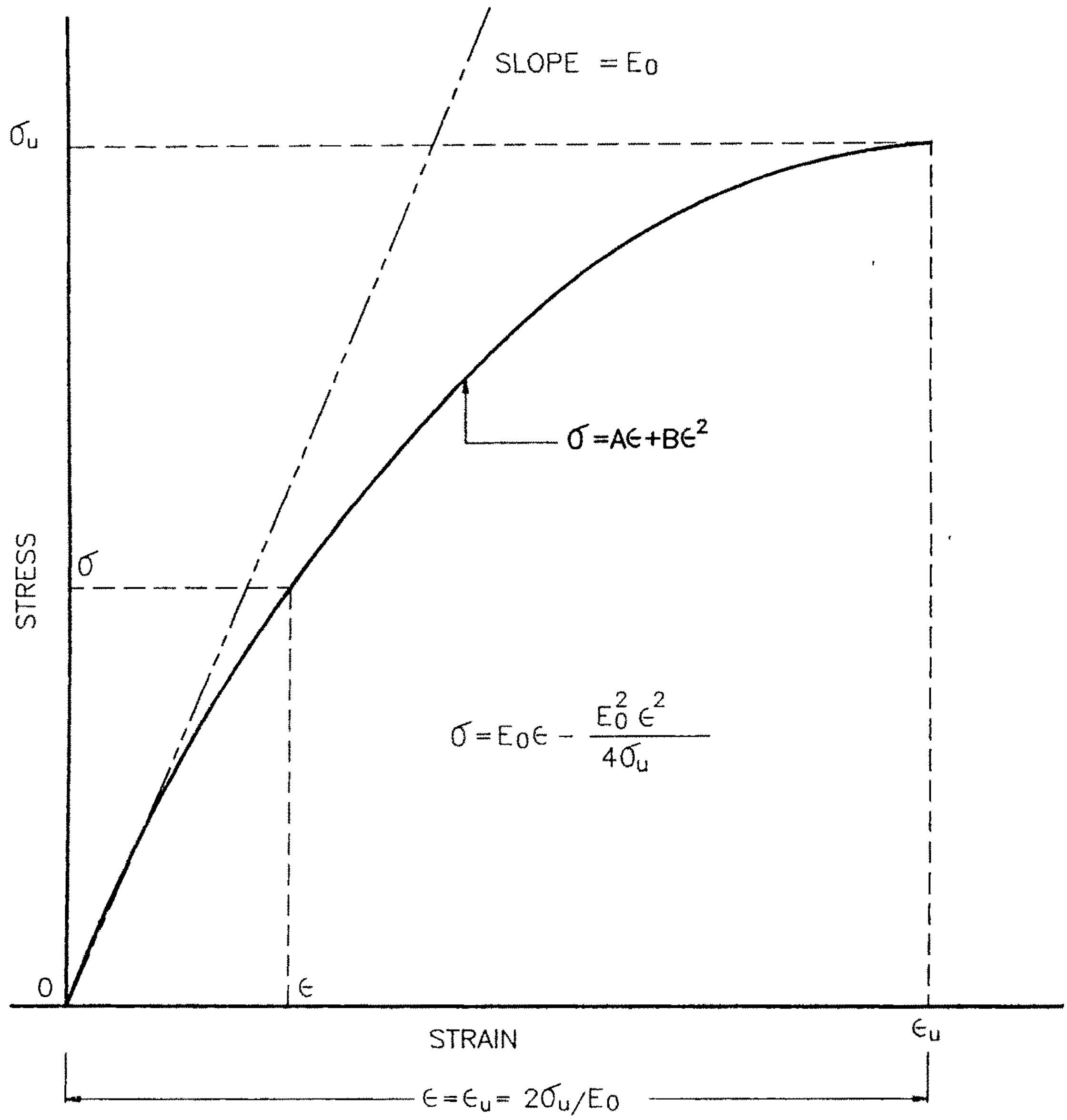


Fig. 5.4 ASSUMED STRESS−STRAIN RELATIONSHIP FOR CONCRETE

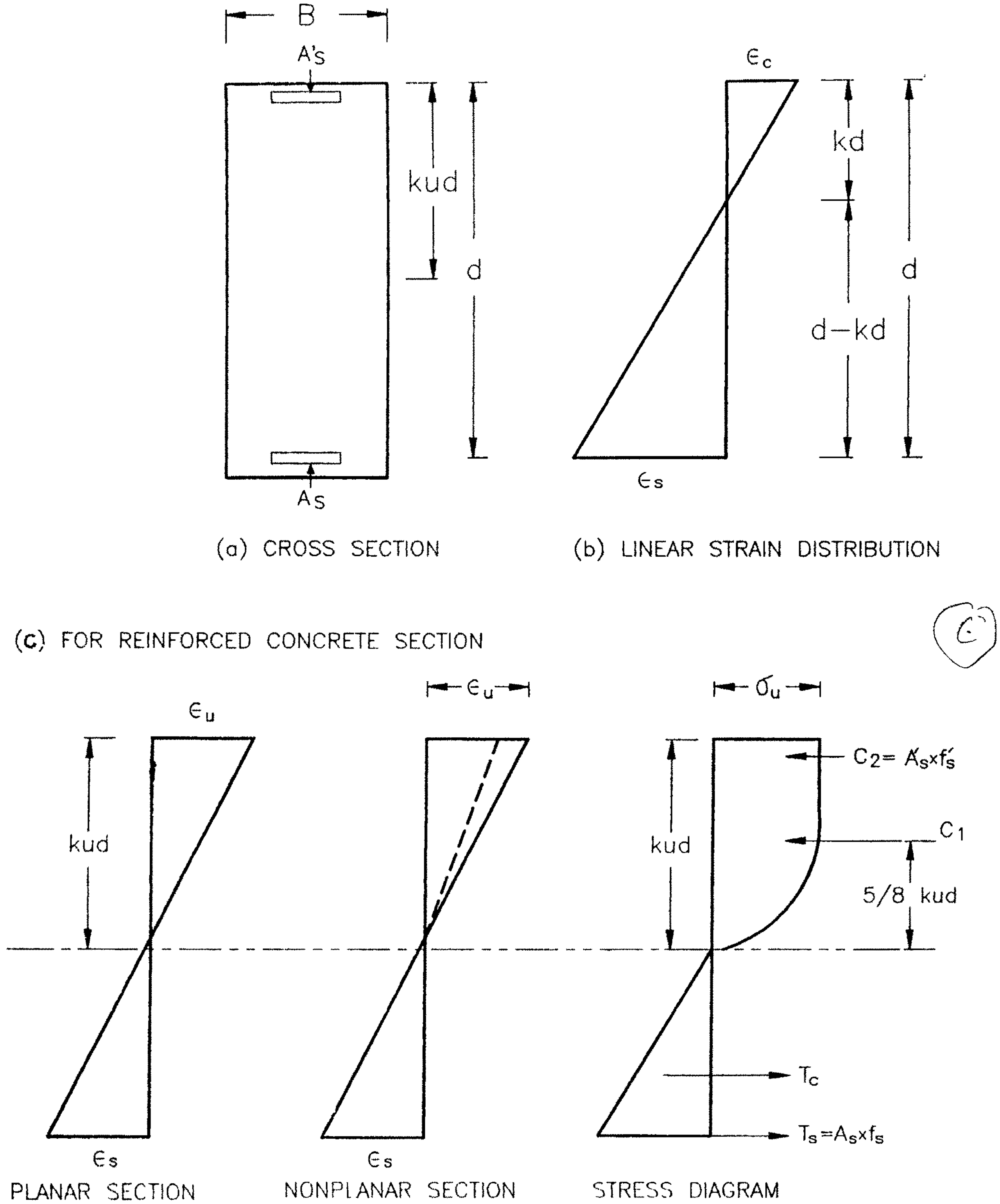
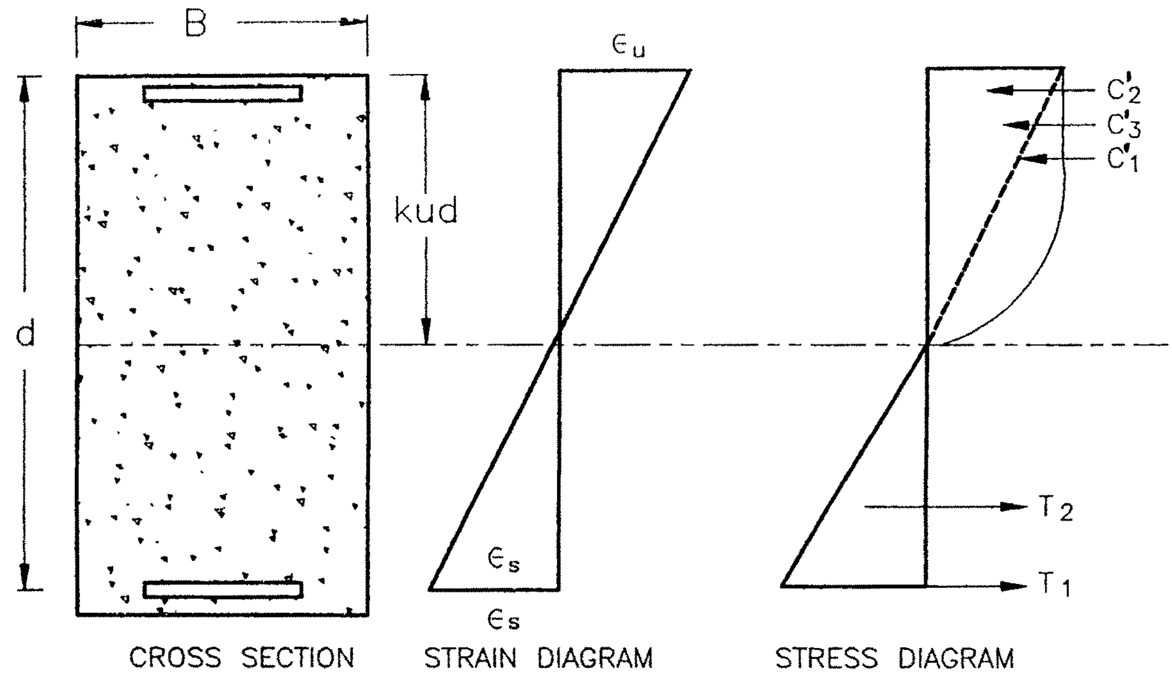


Fig. 5.5 DERIVATION OF ULTIMATE FLEXURAL MOMENT EQUATIONS

(a) FULL DEPTH FIBROUS CONCRETE SECTION



(b) LOWER HALF DEPTH FIBROUS CONCRETE SECTION

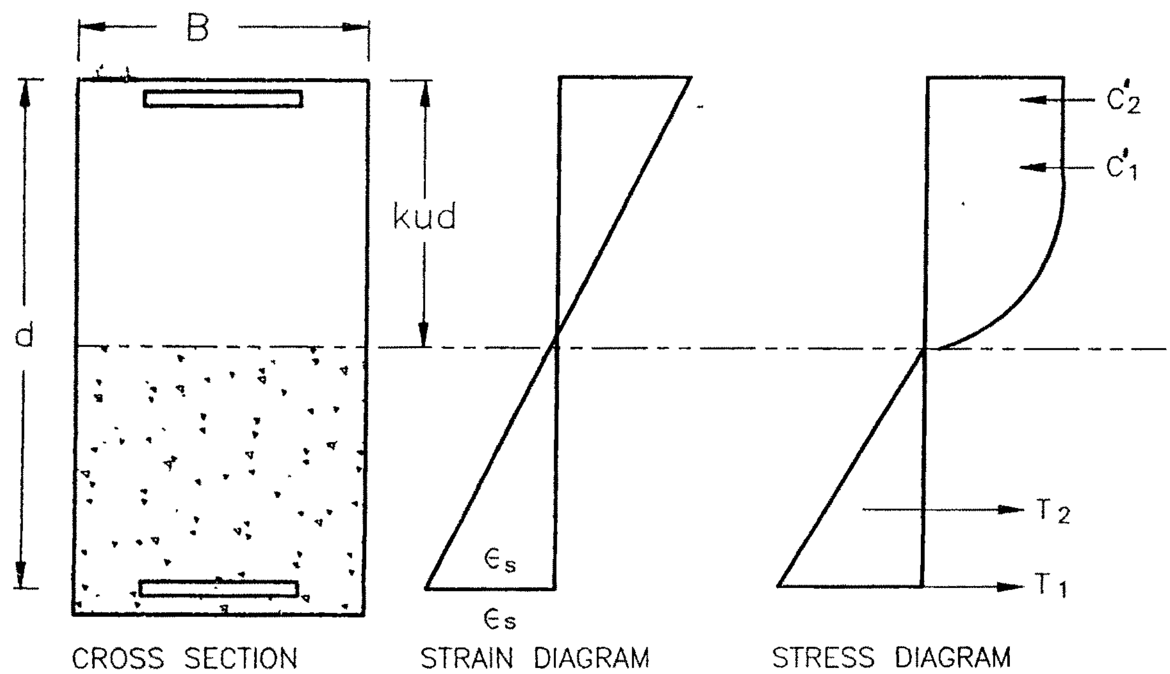


Fig. 5.6 GEOMETRY OF A FLEXURAL CROSS SECTION

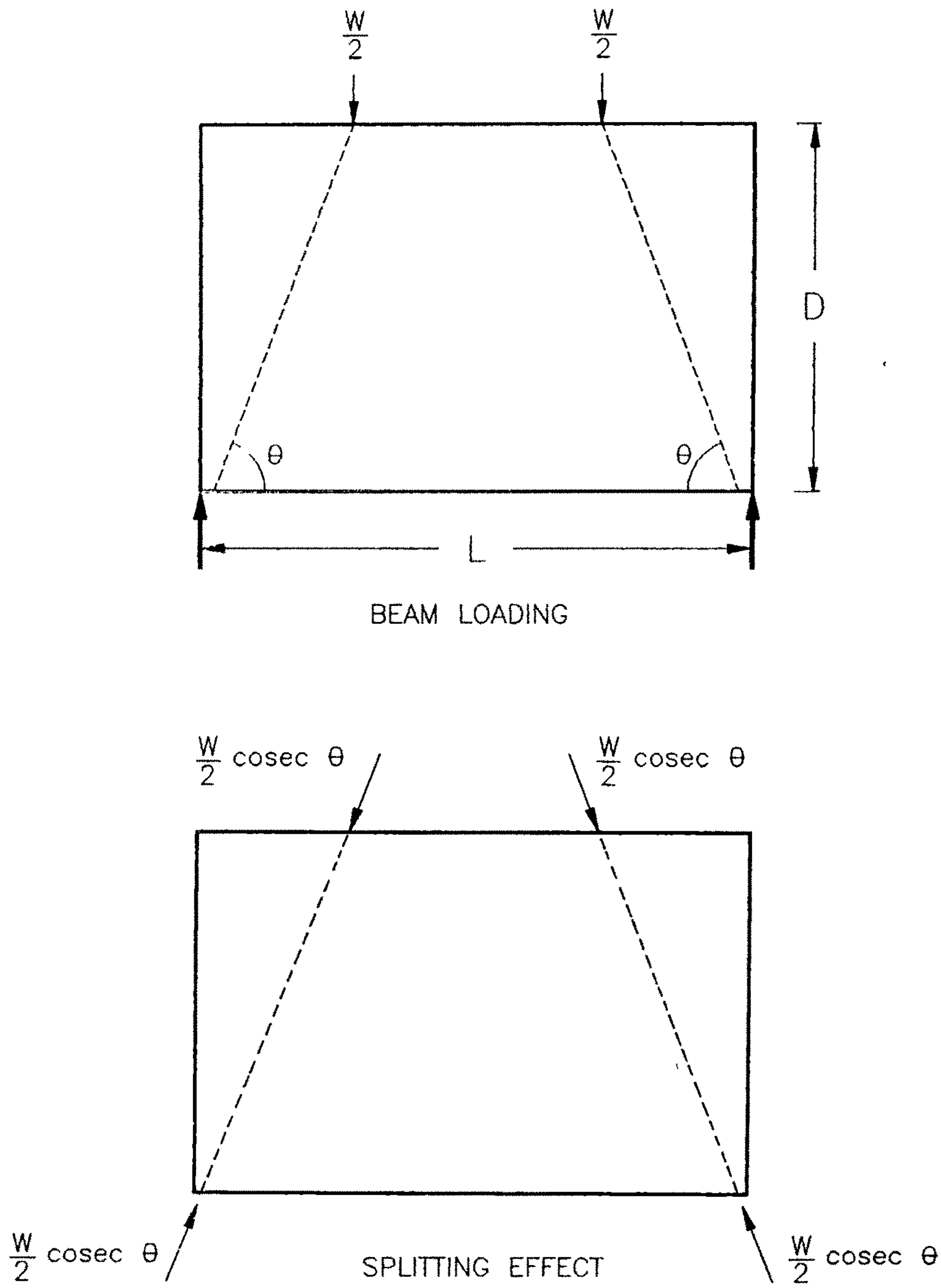


Fig. 5.7 SPLITTING ANALOGY FOR A DEEP BEAM

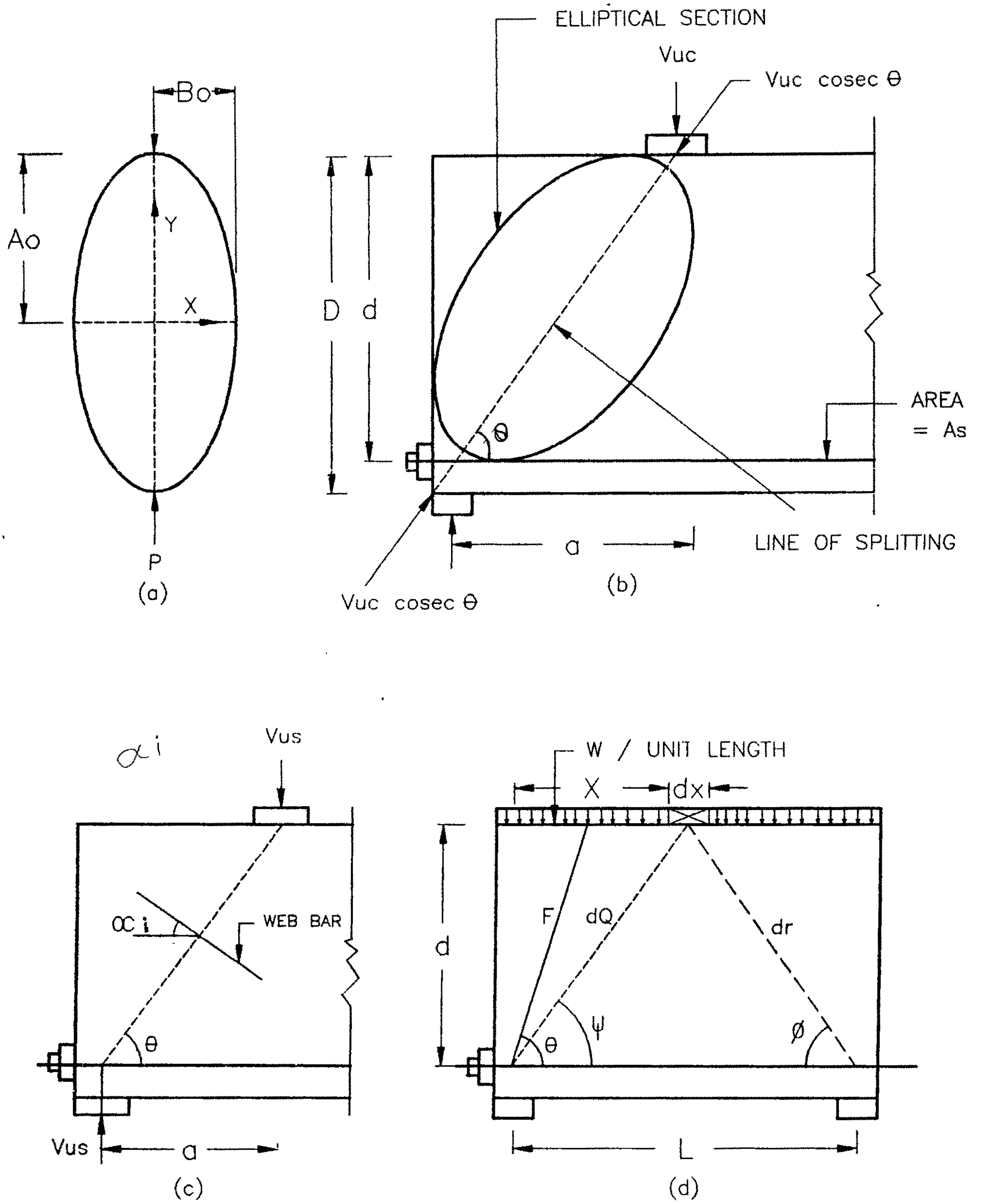


Fig. 5.8 DERIVATION OF SHEAR STRENGTH EQUATIONS.

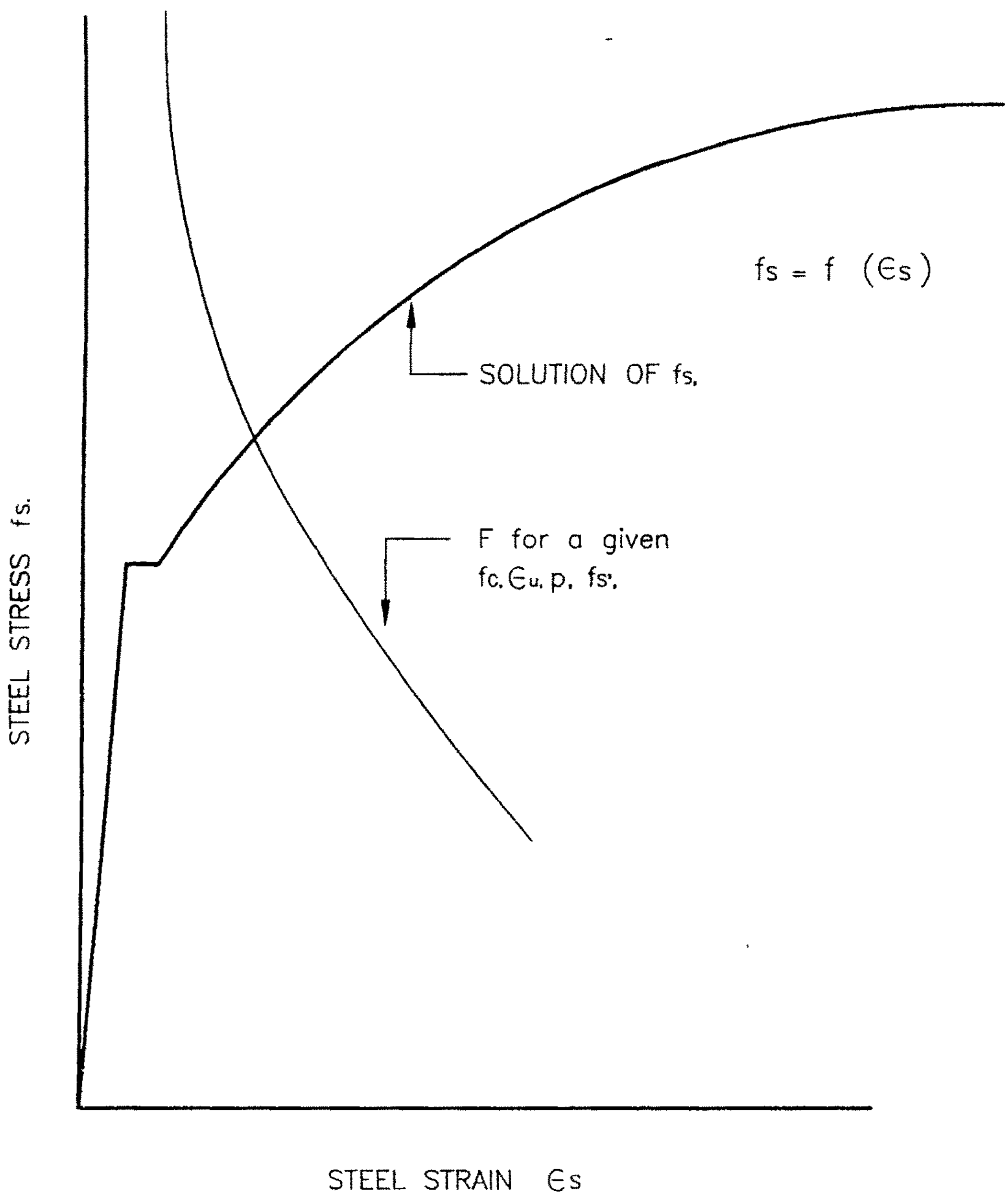


Fig. 5.9 GENERAL GRAPHICAL SOLUTION FOR fs.

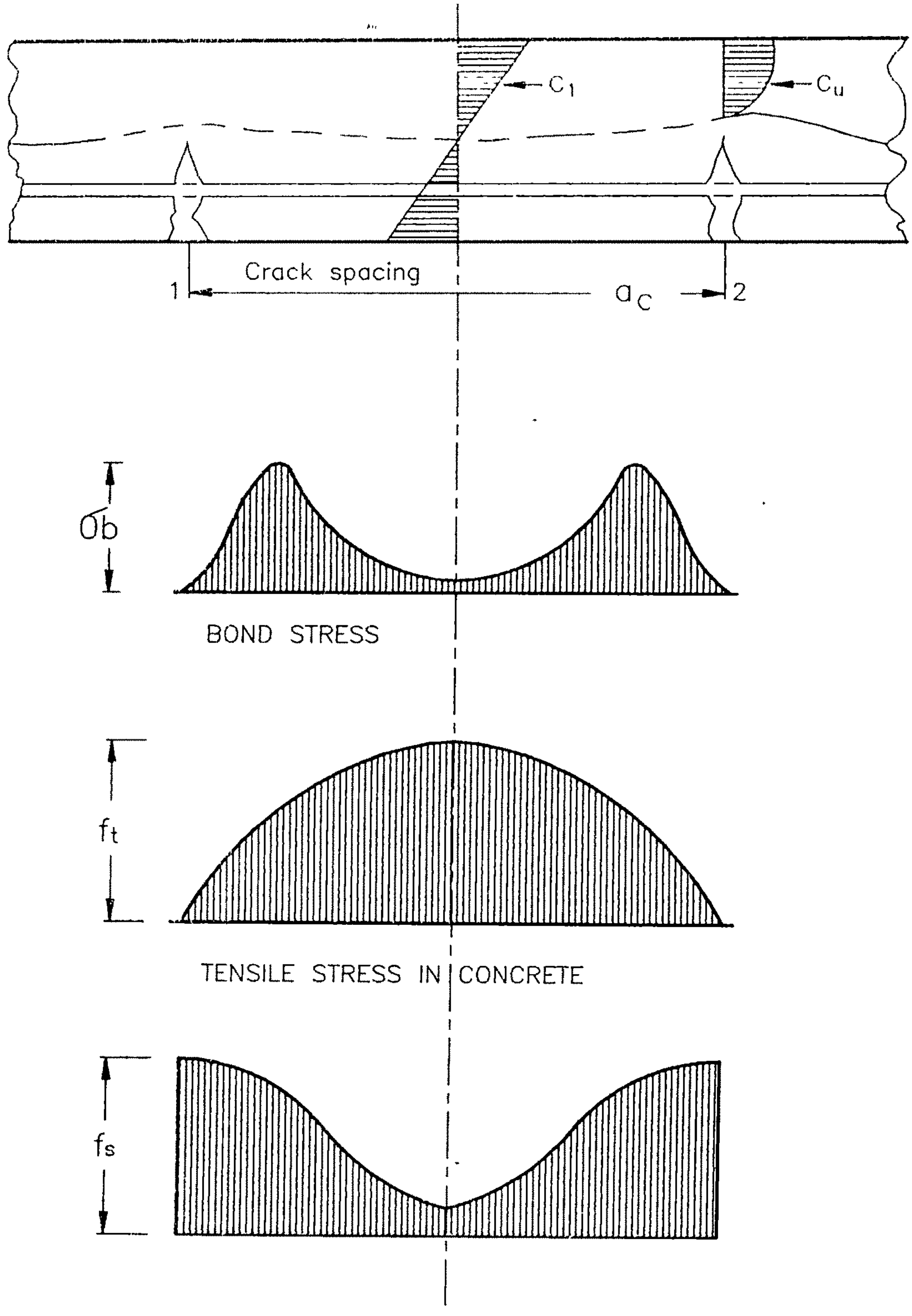


Fig. 5.10 STRESS DISTRIBUTION BETWEEN ADJACENT FLEXURAL CRACKS.(116)