APPENDIX-E

Design Example : 1 Deep Beam

Design deep beam to support a working load of 200 kN each applied at one third of the span of beam. The beam has a span of 1.5 m. Using M15 mix and Mild steel as a reinforcement. Assume width of beam of 100 mm.

$R_1 = R_2$

= 200 kN at each support

working moment looking to beam moment diagram

M_{max} = 200 x 0.5

= 100 kN-m

Design moment $M_u = r_f \times M_{max}$

= 1.5 x 100

= 150 kN-m

where r_f = 1.5 as per IS:456

partial safety factor for loading.

Design of Section :

The cross sectional dimension and quantity of tension steel is to calculated on the basis of recommendations given in the code IS:456-1978.

Caluse No.28.1 L/D should be between 1.0 and 2.0.

Considering L/D = 1.5 for general consideration

$$\frac{1500}{1.5} = D \cong D = 1000mm$$

(i) Considering Without Fibre Condition

Adopting Cross Section of 100 mm x 1000 mm

Second stage of the design is the calculations of main longitudinal steel area. This is found by using Eq.(5.19).

$$M_{FL} = A_s f_{s1} d\left[1 - \frac{0.6 f_{s1} p}{f_{c1}}\right] + 2.65 \times 10^{-3} f_{c1} b d^2 \cdot \times$$

$$\left[0.9 - \frac{1.6 f_{s1} p}{f_{c1}}\right]\left[15.9 - \frac{1.6 f_{s1} p}{f_{c1}}\right]$$

properties of material to be taken into account

$$f_c = 15 N/mm^2, f_t = 1.5 N/mm^2, f_s = 250 N/mm^2$$

As per caluse 35.4.2 taking partial safety factor r_m for material.

r_m should be taken as 1.5 for concrete and 1.15 for steel

$$f_{s1} = \frac{f_s}{1.15} = \frac{250}{1.15} = 217.4 N/mm^2$$

$$f_{c1} = \frac{f_c}{1.5} = \frac{15}{1.5} = 10 N/mm^2$$

$$150 \times 10^6 = A_s \times 217.4 \times 900\left[1 - \frac{0.6 \times 217.4 \times A_s}{1000 \times 100 \times 900}\right]$$

$$+2.65 \times 10^{-3} \times 1000 \times 100 \times 900^2$$

$$\times \left[0.9 - \frac{0.6 \times 217.4 \times A_s}{1000 \times 100 \times 900}\right]\left[15.9 - \frac{0.6 \times 217.4 \times A_s}{1000 \times 100 \times 900}\right]$$

$A_s = 740 mm^2$ (main longitudinal steel)

= 800 mm^2 provided 4-16 ϕ bars.

These main longitudinal bars must be anchord in the support zone.

Next step is to check its shear capacity because such beams fails by general shear only.

Total actual maximum shear force in the section can be obtain from loading diagram.

Maximum shear force = 200 kN

Design shear force = r_f x 200

= 1.5 x 200

V_u = 300 kN

Shear strength contribution of the concrete and longitudinal main steel is calculated using Eq. (5.40).

$$W_u = 2V_u = \frac{3.0 f_{t1} bd}{\sqrt{1+0.75\frac{a^2}{d^2}}} + \frac{1.4 f_{y1}}{\sqrt{1+\frac{a^2}{d^2}}} \Sigma \frac{y_i}{d} \cdot A_s \sin\theta + 0$$

Where $f_{t1} = \frac{f_t}{1.5} = \frac{1.5}{1.5} = 1 N/mm^2$

$\frac{a}{d} = \frac{0.5}{0.9} = 0.55$ $\sin\theta = \frac{0.9}{1.02} = 0.88$

$W_u = 220 + 164 = 384 > 300 kN$

Design is O.K. No addition steel is required

(ii) Considering With Full Depth Fibre Condition

Due addition of fibres f_c value remain same where f_t will increase by 100 to 150% for 1% volume of fibres

f_c = 15 N/mm^2 change i.e. f_c = 15 N/mm^2 = 10 N/mm^2

f_t = 1.5 N/mm^2 change f_{ts} = 3.0 N/mm^2 as 100% = 2.0 N/mm^2

Full depth

$$W_u = 2V_u = \frac{3.0 f_{t1} bd}{\sqrt{1+0.75\frac{a^2}{d^2}}} + \frac{1.4 f_{y1}}{\sqrt{1+\frac{a^2}{d^2}}} \Sigma \frac{y_i}{d} \cdot A_s \sin\theta + 2K_b \sigma_b d\sqrt{V_f \cdot L_f}$$

where V_f = 1% volume

L_f = 50 mm as aspect ratio is 100 i.e $\frac{L_f}{d_f} = 100$

$$= \frac{3.0\times2\times100\times900}{\sqrt{1+0.75(0.55)^2}} + \frac{1.4\times217.4}{\sqrt{1+(0.55)^2}} \times 1 \times A_s \sin 61^\circ$$
$$+2\times0.3\times2.54\times900\times\sqrt{1\times50}$$

$$= 440 + 164 + 48.4$$

$$= 652.4kN$$

(iii) Considering Lower Half Depth Fibre Condition

Due to addition of fibres f_c value remain same where f_t value will increase by about 50% to 70% for 1% half depth condition.

$$W_u = 2V_u = \frac{3.0 f_{t1} bd}{\sqrt{1+0.75\frac{a^2}{d^2}}} + \frac{1.4 f_{y1}}{\sqrt{1+\frac{a^2}{d^2}}} \Sigma \frac{y_i}{d} \cdot A_s si\theta + 2K_b \sigma_b d\sqrt{V_f \cdot L_f}$$

$$= 308 + 164 + 30.43$$

$$= 475.43kN$$

Design Example : 2 Moderate Deep Beam

Design moderate deep beam to support a working moment of 60 kN-m. The beam has a span of 3.0 m. Using M25 and Fe415 as reinforcement considering width of beam of 100 mm.Material properties

f_{ck} = 25 N/mm^2
f_{sy} = 415 N/mm^2

As per IS:456-1978 caluse 28 for partial safety factor r_f

M_w working moment = 60 kN-m

M_u design moment = $r_f \times M_w$

= 1.5 x 60

= 90 kN-m

Design of Section

The cross sectional dimension and quantity of tension steel is to be calculated on the basis of recommendations given in the code IS:456-1978.

Caluse 28.1 For Moderate Beam L/D ratio should be between 4.0 to 6.0.

Let $\frac{L}{D} = 5$ $\frac{3000}{5} = 600$ i..e. D is a depth of the section

Taking cover of 50 mm d = 550 mm

Let $\frac{L}{D} = 6$ $\frac{3000}{6} = 500$ i.e. D is a depth of the section

Taking cover of 50 mm d = 450 mm

(i) Considering Without Fibre Condition

Limit state method consideration

Appendix of IS:456

$$0.149 f_{ck} b d^2 .... for F_e 250$$

$$0.138 f_{ck} b d^2 .. for F_e 415$$

$$0.133 f_{ck} b d^2 .. for F_e 500$$

According to f_{ck} and F_e

$$M_u = 0.138 \times f_{ck} \times bd^2$$

$$90 \times 10^6 = 0.138 \times 25 \times 100 \times d^2$$

$$d = 510mm$$

Taking b = 150 mm and over all depth D = 600.

Let D = 600 with an effective depth of 550 mm is decided, so that the beam will be under reinforced since the adopted value of the effective depth is greater than that required for a balanced section.

For under reinforced section

$$M_u = 0.87 f_{sy} A_{st} d\left[1 - \frac{A_{st} \times f_{sy}}{f_{ck} \cdot b \cdot d}\right]$$

$$90 \times 10^6 = 0.87 \times 415 \times A_{st} \times 550\left[1 - \frac{415 A_{st}}{25 \times 100 \times 550}\right]$$

$$A_{st} = 609mm^2$$

$= 628mm^2$ (Providing to 2 bars of 20mm diameter)

By using Eq. (5.19)

$$M_{FL} = \left(A_s f_s - A_s' f_s'\right) d\left[1 - \frac{0.6}{f_c}\left(f_{sp} - f_s' p'\right)\right]$$

$$+ A_s' f_s' d' + 2.65 \times 10^{-3} f_c b d^2$$

$$\times\left[0.9 - \frac{1.6}{f_c}\left(f_s p - f_s' p'\right)\right] \times \left[15.0 - \frac{1.6}{f_c}\left(f_s p - f_s' p'\right)\right]$$

$$M_{FL} = A_s f_s d\left[1 - \frac{0.6}{f_c} f_s p\right] + 2.65 \times 10^{-3} f_c b d^2$$

$$\times\left[0.9 - \frac{1.6}{f_c} f_s p\right] \times \left[15.9 - \frac{1.6}{f_c} f_s p\right]$$

$$= 494mm^2$$

Modular ratio according IS:456

$$m = \frac{280}{3 \times \text{persible stress in concrete}}$$

$$= \frac{280}{3 \times 8.5} = 11.0$$

Taking moment of equivalent area

$$\frac{b \times x^2}{2} = m \cdot A_{st} \cdot (d - x)$$

$$\frac{100 \times x^2}{2} = 11 \times 628(550 - x)$$

x = 138 mm

Actual lever arm = $d - \frac{x}{3}$

= 504.00

Stress in steel at service load

$$f_s = \frac{60 \times 10^6}{628 \times 504} = 189.55 N/mm^2$$

Strain in steel $\in_s = \frac{f_s}{E_s} = \frac{189\ 55}{2} \times 10^{-5} = 94 \times 10^{-5}$

$$f_s = 0.57 \frac{k_u}{p} f_c + \frac{p'}{p} f'_s - 0.075 \frac{f_c}{p}(1 - k_u)$$

where $k_u = 1.56 \frac{f_s}{f_c} p + 0.12$

$$f_s = 216.4 N/m^2$$

Now crack width calculations

$$W_b = 10.8 \times 10^{-6} \times f_y \times \frac{h_1}{h_2} \times \sqrt[3]{A_{ct} d_c}$$

$$= 10.8 \times 10^{-6} \times 360 \times 1.1\overset{3}{1} \cdot \sqrt[3]{461 \times 100 \times 50}$$

= 0.411 mm

= 0.411 > 0.3 permisible crack width

Hence steel fibres may be added in tension zone to limit the crack width.

Design of Fibre Parameters

i.e. Aspect ratio and volume fraction

a_f and V_f

since the crack width is inversly proportional to area of steel equivalent area of steel required can be obtained as under

For equivalent area for fibres = $\frac{0.411}{0.3} \times A_{st}$ provide i.e. 628

= 860 mm^2

Area of fibre = 860 - 628 = 232 mm^2

(ii) Let fibres are provided in 1/4 bottom depth

$$A_{ef} = 150 \times 100 \times \frac{V_f}{100}$$

$$V_f = \frac{232 \times 100}{15000} = 1.54\%$$

Taking fibre efficency parameter of 0.6

$$V_f = \frac{1.54}{0.6} \Rightarrow 2.56\%$$

The above volume percentage of fibres is not practically admissible as well as not desirable from the point of mixing and compaction

(iii) Let fibre provided in lower half depth of the beam

Fibres may be added in half depth to reduce the $V_f\%$.

$$V_f = \frac{232 \times 100}{300 \times 100} = 0.77\%$$

Taking fibre efficency parameter of 0.6

$$V_f = \frac{0.77}{0.6} = 1.28\%$$

Hence volume of 1.3% fibres can be adopted

Using ACI modified Eq. (5.46)

$$W_b = 10.8 \times 10^6 f_y \frac{h_1}{h_2} \cdot \sqrt[3]{A_{ct} dc}\left[A\frac{L}{D} - B\right][1 - \alpha_f V_f]$$

$$= 0.411[0.09 \times 6 + 0.366][1 - 0.175 \times 1.3]$$

$$= 0.411(0.90)(0.78)$$

$$W_b = 0.288 mm$$

Hence addition of 1.3% volume of plain round steel fibres having aspect ratio of 100 over the half depth is recommended.