

## CHAPTER 7

# **RESULTS AND DISCUSSION FOR EFFECTIVE INTERFACIAL AREAS**

## CHAPTER - 7

## RESULTS AND DISCUSSION FOR INTERFACIAL AREAS

7.1.0 RESULTS AND DISCUSSION FOR WETTED SURFACE AREA ( $a_w$ ) :

## 7.1.1 Critical analysis of data and mathematical modelling :

A data bank consisting of 100 data points covering 21 systems/variations reported in Table (5.1), could be used for analysing the effect of different variables on the values of  $a_w$ .

The parameters that affect the values of  $a_w$  are liquid flow rate ( $L$ ), physical properties of solvent/absorption media like liquid density ( $\rho_L$ ), viscosity ( $\mu_L$ ), surface tension ( $\sigma$ ), packing size ( $d_p$ ) and its dry area ( $a_t$ ) and the affinity of the packing material for wetting by liquid which can be characterised by the critical surface tension ( $\sigma_c$ ).

It is observed that the values of  $a_w$  increase with an increase in  $L$  and decrease with a decrease in  $a_t$  i.e. with an increase in  $d_p$ . With an increase in  $\mu_L$  the values of  $a_w$  do decrease, however the effect appears to be quite small.

The data of Fujita et al. (10) and Hikita et al. (11) indicate that under otherwise identical conditions, by increasing the values of  $\mu_L$  by two times or so, the value of  $a_w$  decreases marginally. With an increase in the value of  $\rho_L$  or  $\sigma_L$ , the values

of  $a_w$  also decrease. This is expected in view of the lesser spreading of the liquid of higher surface tension on the packing surfaces than that of the liquid of lower surface tension. The data of Hikita et al. (11) and Onda et al. (12) confirm these observations. Further, with a decrease in the value of  $\sigma / \sigma_c$ , the values of  $a_w$  increase appreciably as could be seen from the data of Shulman et al. (8) and Mayo et al. (9).

Thus in order to correlate the data for  $a_w$  in terms of system parameters and hydrodynamic parameters, one can propose the following equation :-

$$a_w/a_t = f(L, a_t^{-1}, \mu_L^{-1}, \rho_L^{-1}, \sigma^{-1}, \sigma_c) \quad (7.1)$$

With the help of dimensional analysis, these parameters can be grouped in terms of dimensionless numbers as per the following equation :-

$$a_w/a_t = C (Re)^\alpha (We)^\beta (Fr)^\gamma (\sigma / \sigma_c)^\delta \quad (7.2)$$

It seems obvious that all the indices except that of  $(\sigma / \sigma_c)$  must be positive but of varying magnitudes to fulfil the parametric dependencies discussed earlier. Hence, the relevant dimensionless groups [calculated using the data reported in Table (5.1)] required for processing the data for  $a_w$  are tabulated in Table (7.1.1).

#### 7.1.2 Statistical analysis for different correlations :

Processing of the data reported in Table (7.1.1) was done by three methodologies namely (i) Multiple Linear Regression. (ii)

Table - ( 7.1.1 )

Results inclusive of processing parameters for  
wettted surface area (  $a_w$  )

| No. | $a_w$<br>$\frac{z_w}{m/m^3}$ | Re     | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_w/a_t$<br>exp. | $a_w/a_t$<br>pred. | % err  |
|-----|------------------------------|--------|---------------------|---------------------|-------------------|-------------------|--------------------|--------|
| 1   | 79.1                         | 40.48  | 369.480             | 81.027              | 1.510             | 0.460             | 0.469              | -1.96  |
| 2   | 80.8                         | 72.86  | 1197.153            | 262.535             | 1.510             | 0.470             | 0.571              | -21.54 |
| 3   | 110.4                        | 80.95  | 1477.919            | 324.107             | 1.510             | 0.642             | 0.592              | 7.82   |
| 4   | 103.2                        | 3.76   | 6.868               | 6.970               | 1.510             | 0.279             | 0.241              | 13.61  |
| 5   | 135.8                        | 9.41   | 42.927              | 43.563              | 1.510             | 0.367             | 0.328              | 10.70  |
| 6   | 166.9                        | 18.82  | 171.758             | 174.302             | 1.510             | 0.451             | 0.414              | 8.33   |
| 7   | 36.8                         | 12.11  | 22.098              | 2.166               | 1.510             | 0.320             | 0.292              | 8.73   |
| 8   | 46.0                         | 30.27  | 138.112             | 13.540              | 1.510             | 0.400             | 0.397              | 0.71   |
| 9   | 57.5                         | 60.54  | 552.613             | 54.175              | 1.510             | 0.500             | 0.501              | -0.22  |
| 10  | 64.4                         | 121.08 | 2210.452            | 216.700             | 1.510             | 0.560             | 0.632              | -12.91 |
| 11  | 107.2                        | 2.85   | 6.383               | 10.791              | 1.230             | 0.228             | 0.261              | -14.44 |
| 12  | 165.0                        | 5.70   | 25.532              | 43.163              | 1.230             | 0.351             | 0.329              | 6.21   |
| 13  | 164.6                        | 12.79  | 67.368              | 67.962              | 1.000             | 0.445             | 0.424              | 4.68   |
| 14  | 188.3                        | 17.63  | 127.970             | 129.099             | 1.000             | 0.509             | 0.472              | 7.20   |
| 15  | 253.4                        | 32.80  | 443.231             | 447.141             | 1.000             | 0.685             | 0.582              | 15.07  |
| 16  | 266.4                        | 39.54  | 644.149             | 649.832             | 1.000             | 0.720             | 0.619              | 13.97  |
| 17  | 88.8                         | 1.64   | 3.258               | 2.733               | 0.833             | 0.240             | 0.276              | -15.14 |
| 18  | 146.5                        | 6.56   | 52.124              | 43.732              | 0.833             | 0.396             | 0.440              | -11.09 |
| 19  | 168.4                        | 9.85   | 117.279             | 98.398              | 0.833             | 0.455             | 0.504              | -10.77 |
| 20  | 185.0                        | 13.13  | 208.497             | 174.929             | 0.833             | 0.500             | 0.555              | -11.01 |
| 21  | 55.5                         | 1.79   | 2.096               | 0.509               | 0.833             | 0.279             | 0.256              | 8.20   |
| 22  | 68.1                         | 4.27   | 11.872              | 2.881               | 0.833             | 0.342             | 0.343              | -0.16  |
| 23  | 87.8                         | 8.15   | 43.200              | 10.484              | 0.833             | 0.441             | 0.425              | 3.53   |
| 24  | 119.8                        | 25.84  | 434.606             | 105.478             | 0.833             | 0.602             | 0.627              | -4.08  |
| 25  | 69.3                         | 0.47   | 0.175               | 0.067               | 0.472             | 0.210             | 0.217              | -3.49  |
| 26  | 92.4                         | 0.95   | 0.702               | 0.266               | 0.472             | 0.280             | 0.274              | 2.07   |
| 27  | 115.5                        | 1.86   | 2.707               | 1.027               | 0.472             | 0.350             | 0.344              | 1.74   |
| 28  | 135.3                        | 3.76   | 11.025              | 4.182               | 0.472             | 0.410             | 0.435              | -6.15  |
| 29  | 148.5                        | 4.68   | 17.041              | 6.464               | 0.472             | 0.450             | 0.468              | -4.04  |
| 30  | 198.0                        | 9.42   | 69.157              | 26.234              | 0.472             | 0.600             | 0.592              | 1.31   |
| 31  | 35.5                         | 1.64   | 0.628               | 0.151               | 0.903             | 0.187             | 0.202              | -8.07  |
| 32  | 41.2                         | 3.24   | 2.457               | 0.590               | 0.903             | 0.217             | 0.254              | -17.08 |
| 33  | 55.1                         | 6.53   | 10.007              | 2.403               | 0.903             | 0.290             | 0.322              | -10.87 |
| 34  | 71.3                         | 16.37  | 62.766              | 15.074              | 0.903             | 0.375             | 0.437              | -16.66 |
| 35  | 93.1                         | 32.38  | 245.676             | 59.001              | 0.903             | 0.490             | 0.550              | -12.24 |
| 36  | 34.2                         | 0.82   | 0.172               | 0.038               | 0.833             | 0.180             | 0.169              | 6.36   |
| 37  | 38.0                         | 1.64   | 0.680               | 0.151               | 0.833             | 0.200             | 0.212              | -6.08  |
| 38  | 47.5                         | 3.24   | 2.661               | 0.590               | 0.833             | 0.250             | 0.267              | -6.68  |
| 39  | 57.0                         | 6.53   | 10.840              | 2.403               | 0.833             | 0.300             | 0.338              | -12.51 |
| 40  | 66.5                         | 8.12   | 16.756              | 3.714               | 0.833             | 0.350             | 0.363              | -3.75  |
| 41  | 79.8                         | 16.37  | 67.997              | 15.074              | 0.833             | 0.420             | 0.459              | -9.35  |
| 42  | 38.0                         | 0.82   | 0.207               | 0.038               | 0.694             | 0.200             | 0.188              | 5.86   |
| 43  | 47.5                         | 1.64   | 0.816               | 0.151               | 0.694             | 0.250             | 0.237              | 5.21   |
| 44  | 58.9                         | 3.24   | 3.194               | 0.590               | 0.694             | 0.310             | 0.298              | 3.90   |
| 45  | 72.2                         | 6.53   | 13.009              | 2.403               | 0.694             | 0.380             | 0.377              | 0.78   |
| 46  | 76.0                         | 8.12   | 20.107              | 3.714               | 0.694             | 0.400             | 0.406              | -1.40  |
| 47  | 98.8                         | 16.37  | 81.596              | 15.074              | 0.694             | 0.520             | 0.513              | 1.35   |
| 48  | 43.7                         | 0.82   | 0.272               | 0.038               | 0.528             | 0.230             | 0.222              | 3.30   |
| 49  | 57.0                         | 1.64   | 1.074               | 0.151               | 0.528             | 0.300             | 0.280              | 6.69   |

Table - 7.1.1 ( contd.)

| No. | $a_v$<br>$\frac{m^2}{m^3}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_v/a_t$<br>exp. | $a_v/a_t$<br>pred. | % err  |
|-----|----------------------------|-------|---------------------|---------------------|-------------------|-------------------|--------------------|--------|
| 50  | 72.2                       | 3.24  | 4.202               | 0.590               | 0.528             | 0.380             | 0.352              | 7.39   |
| 51  | 91.2                       | 6.53  | 17.116              | 2.403               | 0.528             | 0.480             | 0.445              | 7.21   |
| 52  | 98.8                       | 8.12  | 26.456              | 3.714               | 0.528             | 0.520             | 0.479              | 7.86   |
| 53  | 117.8                      | 16.37 | 107.364             | 15.074              | 0.528             | 0.620             | 0.606              | 2.26   |
| 54  | 28.5                       | 0.44  | 0.154               | 0.033               | 0.861             | 0.150             | 0.163              | -8.58  |
| 55  | 34.2                       | 0.88  | 0.616               | 0.130               | 0.861             | 0.180             | 0.205              | -14.16 |
| 56  | 50.9                       | 1.72  | 2.378               | 0.503               | 0.861             | 0.268             | 0.258              | 3.84   |
| 57  | 57.0                       | 4.39  | 15.406              | 3.258               | 0.861             | 0.300             | 0.353              | -17.52 |
| 58  | 76.0                       | 8.71  | 60.746              | 12.846              | 0.861             | 0.400             | 0.444              | -10.94 |
| 59  | 85.1                       | 13.00 | 135.371             | 28.627              | 0.861             | 0.448             | 0.508              | -13.30 |
| 60  | 30.4                       | 0.19  | 0.137               | 0.029               | 0.917             | 0.160             | 0.155              | 3.00   |
| 61  | 35.2                       | 0.39  | 0.548               | 0.117               | 0.917             | 0.185             | 0.196              | -5.85  |
| 62  | 52.3                       | 0.78  | 2.194               | 0.468               | 0.917             | 0.275             | 0.247              | 10.16  |
| 63  | 58.9                       | 1.94  | 13.711              | 2.924               | 0.917             | 0.310             | 0.336              | -8.37  |
| 64  | 65.6                       | 2.31  | 19.509              | 4.161               | 0.917             | 0.345             | 0.356              | -3.31  |
| 65  | 72.2                       | 2.91  | 30.849              | 6.579               | 0.917             | 0.380             | 0.385              | -1.29  |
| 66  | 77.9                       | 3.85  | 54.061              | 11.529              | 0.917             | 0.410             | 0.423              | -3.14  |
| 67  | 85.5                       | 5.75  | 120.474             | 25.693              | 0.917             | 0.450             | 0.484              | -7.49  |
| 68  | 26.4                       | 0.28  | 0.215               | 0.020               | 0.833             | 0.200             | 0.174              | 12.83  |
| 69  | 33.0                       | 0.56  | 0.861               | 0.080               | 0.833             | 0.250             | 0.220              | 12.02  |
| 70  | 52.8                       | 2.79  | 21.520              | 1.996               | 0.833             | 0.400             | 0.377              | 5.65   |
| 71  | 54.1                       | 5.54  | 84.853              | 7.871               | 0.833             | 0.410             | 0.475              | -15.86 |
| 72  | 114.1                      | 0.86  | 0.802               | 1.880               | 1.098             | 0.195             | 0.194              | 0.64   |
| 73  | 155.0                      | 1.79  | 3.440               | 8.065               | 1.098             | 0.265             | 0.247              | 6.66   |
| 74  | 175.5                      | 2.59  | 7.215               | 16.916              | 1.098             | 0.300             | 0.280              | 6.64   |
| 75  | 189.5                      | 3.45  | 12.827              | 30.072              | 1.098             | 0.324             | 0.308              | 4.80   |
| 76  | 200.7                      | 4.28  | 19.757              | 46.319              | 1.098             | 0.343             | 0.332              | 3.32   |
| 77  | 136.9                      | 0.95  | 1.168               | 1.880               | 0.754             | 0.234             | 0.243              | -4.05  |
| 78  | 181.4                      | 1.96  | 5.010               | 8.065               | 0.754             | 0.310             | 0.311              | -0.27  |
| 79  | 204.8                      | 2.84  | 10.509              | 16.916              | 0.754             | 0.350             | 0.352              | -0.55  |
| 80  | 265.0                      | 4.71  | 28.777              | 46.319              | 0.754             | 0.453             | 0.417              | 8.01   |
| 81  | 169.6                      | 0.47  | 1.228               | 1.990               | 0.738             | 0.290             | 0.248              | 14.59  |
| 82  | 201.8                      | 0.98  | 5.270               | 8.537               | 0.738             | 0.345             | 0.316              | 8.34   |
| 83  | 231.1                      | 1.41  | 11.053              | 17.906              | 0.738             | 0.395             | 0.358              | 9.36   |
| 84  | 250.4                      | 1.89  | 19.650              | 31.833              | 0.738             | 0.428             | 0.394              | 7.87   |
| 85  | 264.4                      | 2.34  | 30.266              | 49.032              | 0.738             | 0.452             | 0.424              | 6.21   |
| 86  | 105.3                      | 0.38  | 0.668               | 1.635               | 1.230             | 0.180             | 0.179              | 0.77   |
| 87  | 140.4                      | 0.79  | 2.866               | 7.016               | 1.230             | 0.240             | 0.228              | 4.99   |
| 88  | 162.6                      | 1.15  | 6.012               | 14.716              | 1.230             | 0.278             | 0.258              | 7.13   |
| 89  | 187.2                      | 1.53  | 10.688              | 26.161              | 1.230             | 0.320             | 0.284              | 11.14  |
| 90  | 186.0                      | 1.90  | 16.462              | 40.295              | 1.230             | 0.318             | 0.306              | 3.87   |
| 91  | 96.5                       | 0.09  | 0.622               | 1.416               | 1.230             | 0.165             | 0.176              | -6.72  |
| 92  | 114.1                      | 0.18  | 2.667               | 6.074               | 1.230             | 0.195             | 0.225              | -15.28 |
| 93  | 143.3                      | 0.27  | 5.594               | 12.740              | 1.230             | 0.245             | 0.255              | -3.89  |
| 94  | 158.0                      | 0.35  | 9.944               | 22.648              | 1.230             | 0.270             | 0.280              | -3.82  |
| 95  | 166.7                      | 0.44  | 15.317              | 34.884              | 1.230             | 0.285             | 0.301              | -5.75  |
| 96  | 99.5                       | 0.26  | 0.619               | 1.440               | 1.246             | 0.170             | 0.175              | -3.06  |
| 97  | 134.6                      | 0.53  | 2.655               | 6.181               | 1.246             | 0.230             | 0.224              | 2.75   |
| 98  | 152.1                      | 0.77  | 5.569               | 12.964              | 1.246             | 0.260             | 0.253              | 2.60   |
| 99  | 163.8                      | 1.02  | 9.900               | 23.047              | 1.246             | 0.280             | 0.279              | 0.39   |
| 100 | 175.5                      | 1.27  | 15.248              | 35.498              | 1.246             | 0.300             | 0.300              | 0.00   |

Optimization by the DSC - Powell algorithm and (iii) Optimization by the modified simplex algorithm of Nelder and Mead. The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were calculated for each methodology and also for different regression steps in each methodology. The results obtained by these three methodologies are reported in Table (7.1.2) and are discussed herewith.

CASE I : Multiple linear regression using the numerical methods of Gauss Jordan and Gauss Seidel, gave practically comparable statistical analysis as could be seen from the regression steps (1) to (3), wherein %  $E_{avg}$  was in the range of + 0.78 to -1.289 and %  $E_{abs}$  and %  $S_{dev}$  were of the order of 7 and 3.4 respectively.

CASE II : It was observed that using the DSC - Powell algorithm of optimization by changing the direction of search sequence, or by prefixing the values of indices of different numbers (except, that of 'We') did not alter the statistical analysis appreciably as could be seen from the regression steps (4) to (9), exception being the regression step (6). The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  remained in the ranges of (-1.0 to -1.99), (8.18 to 8.38) and (3.81 to 3.89) respectively. (This aspect has been discussed in detail elsewhere wherein statistical analysis does get affected considerably).

CASE III : Using optimization technique - the modified simplex algorithm of Nelder and Mead, by performing regressions as per steps (10 to 18) with relevant details mentioned therein, yielded the following equation :-

$$a_w/a_t = 1.431 (Re)^{0.0014} (We)^{0.165} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442} \quad (7.3)$$

Table - ( 7.1.2 )

Statistical analysis for different correlations of  $a_w$

Correlation Structure :  $a_w / a_t = C \cdot Re^\alpha \cdot We^\beta \cdot Fr^\gamma \cdot (\sigma / \sigma_c)^\delta$ .

| No. | C | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|

Case 1 : - Multiple Linear Regression

Numerical Method : Gauss Jordan algorithm.

|   |       |          |       |       |        |        |       |      |
|---|-------|----------|-------|-------|--------|--------|-------|------|
| 1 | 1.298 | 5.48E-05 | 0.145 | 0.011 | -0.464 | -1.289 | 7.114 | 3.43 |
|---|-------|----------|-------|-------|--------|--------|-------|------|

Numerical Method : Gauss Seidel algorithm.

Convergence criteria  $\epsilon = 0.001$

|   |      |       |       |        |        |      |       |      |
|---|------|-------|-------|--------|--------|------|-------|------|
| 2 | 1.22 | 0.009 | 0.139 | 0.0124 | -0.474 | 1.11 | 7.088 | 3.44 |
|---|------|-------|-------|--------|--------|------|-------|------|

Convergence criteria  $\epsilon = 0.0001$

|   |      |       |       |       |        |      |       |      |
|---|------|-------|-------|-------|--------|------|-------|------|
| 3 | 1.27 | 0.003 | 0.142 | 0.013 | -0.464 | 0.78 | 7.058 | 3.42 |
|---|------|-------|-------|-------|--------|------|-------|------|

Case 2 :- Optimisation by the D S C - Powell algorithm.

Initialisation were based on arbitrary guesses of indices,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and C were fed as 0.002, 0.18, 0.002, 0.4, 1.6. ( Clue for guesses were taken from the values obtained in previous steps )

Direction of search sequence  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\gamma$ , C. respectively.

|   |        |        |       |       |        |       |      |      |
|---|--------|--------|-------|-------|--------|-------|------|------|
| 4 | 1.4176 | 0.0014 | 0.165 | 0.001 | -0.589 | -1.56 | 8.25 | 3.84 |
|---|--------|--------|-------|-------|--------|-------|------|------|

Initialisation based on values of indices obtained in step no. 4. However  $\alpha$ , value fed as 0.0014.

|   |        |        |       |        |        |       |      |       |
|---|--------|--------|-------|--------|--------|-------|------|-------|
| 5 | 1.4176 | 0.0014 | 0.165 | 0.0015 | -0.588 | -1.09 | 8.16 | 3.82. |
|---|--------|--------|-------|--------|--------|-------|------|-------|

$\beta$  value arbitrarily fed as 0.14 and statistics of error recorded.

|   |        |        |      |        |        |       |       |       |
|---|--------|--------|------|--------|--------|-------|-------|-------|
| 6 | 1.4176 | 0.0014 | 0.14 | 0.0015 | -0.588 | 27.44 | 27.66 | 4.78. |
|---|--------|--------|------|--------|--------|-------|-------|-------|

Direction of search sequence changed to C,  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no 5.

|   |       |        |        |       |        |       |      |       |
|---|-------|--------|--------|-------|--------|-------|------|-------|
| 7 | 1.433 | 0.0014 | 0.1653 | 0.002 | -0.585 | -1.59 | 8.18 | 3.81. |
|---|-------|--------|--------|-------|--------|-------|------|-------|

Table - 7.1.2 ( contd.)

| No.                                                                                                                                      | C     | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|------------------------------------------------------------------------------------------------------------------------------------------|-------|----------|---------|----------|----------|--------------------|--------------------|--------------------|
| Direction of search sequence changed to $\delta$ , C, $\beta$ , $\alpha$ , $\gamma$ . Initialisation as in step no. 5.                   |       |          |         |          |          |                    |                    |                    |
| 8                                                                                                                                        | 1.431 | 0.0014   | 0.1653  | 0.002    | -0.595   | -1.92              | 8.38               | 3.89.              |
| Direction of search sequence changed to $\beta$ , C, $\delta$ , $\alpha$ , $\gamma$ . Initialisation as in step no. 5.                   |       |          |         |          |          |                    |                    |                    |
| 9                                                                                                                                        | 1.432 | 0.0014   | 0.165   | 0.002    | -0.5947  | -1.99              | 8.38               | 3.87.              |
| Case 3 : - Optimisation by the Modified Simplex algorithm of Nelder and Mead                                                             |       |          |         |          |          |                    |                    |                    |
| All parameters are iterated, base values as obtained from step no 3. Each indice value perturbed 50 % at a time.                         |       |          |         |          |          |                    |                    |                    |
| 10                                                                                                                                       | 1.425 | 0.001    | 0.1637  | 0.0031   | -0.60    | 1.368              | 8.27               | 3.88.              |
| $\alpha$ value fixed as 0.0014. Other parameters iterated as in step 10.                                                                 |       |          |         |          |          |                    |                    |                    |
| 11                                                                                                                                       | 1.406 | 0.0014   | 0.163   | 0.0024   | -0.589   | 1.51               | 8.197              | 3.836.             |
| $\alpha$ and $\delta$ values fixed as 0.0014 and -0.442. Other parameters iterated as in step 11.( Discussed in details later )          |       |          |         |          |          |                    |                    |                    |
| 12 <sup>a</sup>                                                                                                                          | 1.431 | 0.0014   | 0.165   | 0.002    | -0.442   | -0.45              | 7.04               | 3.29.              |
| C fixed as 1.08 as obtained in the case of effective interfacial areas for physical absorpyion. Other parameters iterated as in step 10. |       |          |         |          |          |                    |                    |                    |
| 13                                                                                                                                       | 1.08  | 0.024    | 0.134   | 0.0057   | -0.434   | 0.966              | 7.11               | 3.51.              |
| C fixed as 1.081, $\delta$ fixed as -0.442. Other parameters iterated as in step no. 10.                                                 |       |          |         |          |          |                    |                    |                    |
| 14                                                                                                                                       | 1.08  | 0.010    | 0.1346  | 0.0039   | -0.442   | 1.94               | 7.39               | 3.64.              |
| C fixed as 1.08 and $\gamma$ fixed as 0.002. Other parameters iterated as in step no. 10.                                                |       |          |         |          |          |                    |                    |                    |
| 15                                                                                                                                       | 1.08  | 0.023    | 0.1375  | 0.002    | -0.445   | 0.44               | 7.23               | 3.56.              |
| C, $\gamma$ , $\delta$ fixed as 1.08, 0.002, -0.442 respectively. Other parameters iterated as in step no 10.                            |       |          |         |          |          |                    |                    |                    |
| 16                                                                                                                                       | 1.08  | 0.019    | 0.1386  | 0.002    | -0.442   | 1.94               | 7.39               | 3.64.              |

Table - 7.1.2 ( contd.)

| No.                                                                                                                           | C     | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-------------------------------------------------------------------------------------------------------------------------------|-------|----------|---------|----------|----------|--------------------|--------------------|--------------------|
| C, $\alpha$ , $\gamma$ , $\delta$ fixed as 1.08, 0.099, 0.002, -0.442 respectively. Only $\beta$ iterated as in step no. 10.  |       |          |         |          |          |                    |                    |                    |
| 17                                                                                                                            | 1.081 | 0.099    | 0.144   | 0.002    | -0.442   | 2.95               | 11.78              | 6.04.              |
| C, $\beta$ , $\gamma$ , $\delta$ fixed as 1.08, 0.22, 0.002, -0.442 respectively. Only $\alpha$ is iterated as in step no 10. |       |          |         |          |          |                    |                    |                    |
| 18                                                                                                                            | 1.08  | 0.198    | 0.22    | 0.002    | -0.442   | 37.01              | 42.2               | 10.0.              |

Explanatory note :-

1 % E<sub>avg</sub> is the percentage average error defined as follows :-

$$\% E_{avg} = \frac{1}{n} \left[ \sum \frac{a_{exp} - a_{pred}}{a_{exp}} \right] * 100$$

2. % E<sub>abs</sub> is the percentage absolute error defined as follows :-

$$\% E_{abs} = \frac{1}{n} \left[ \sum \left| \frac{a_{exp} - a_{pred}}{a_{exp}} \right| \right] * 100$$

3. % S<sub>dev</sub> is the percentage standard deviation of errors defined as follows

$$\% S_{dev} = \left[ \sqrt{\frac{S_t}{n-1}} \right] * 100$$

where  $S_t$  is the sum of residuals between the individual error and the mean of the error i.e ,  $S_t = \sum (\varepsilon_i - \bar{\varepsilon})^2$  and  $\bar{\varepsilon} = (\sum \varepsilon_i) / n$ .

Note : The values of indices in the finalised generalised correlation are as per regression step marked by ( \* )

For obtaining the equation (7.3), regression was performed as per step (12) and the values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were -0.45, 7.04 and 3.29 respectively. In equation - (7.3) the values of indices of all the dimensionless numbers inclusive of proportionality constant are such that the correlation under consideration predicts the values of  $a_w$  with the least amount of error.

In the regression step (17), all the indices except that of Weber number ( $We$ ) were prefixed to  $C = 1.08$ ,  $\alpha = 0.099$ ,  $\gamma = 0.002$  and  $\delta = -0.442$  (These indices were obtained for a generalised correlation which had been developed for predicting the values of  $a_p/a_t$  at a later stage) and regression was performed to optimize the power of Weber number ( $We$ ). This methodology resulted in the following correlation having the statistical analysis as %  $E_{avg} = 2.95$ , %  $E_{abs} = 11.78$  and %  $S_{dev} = 6.04$  :-

$$a_w/a_t = 1.081 (Re)^{0.099} (We)^{0.144} (Fr)^{0.002} (\sigma/\sigma_c)^{-0.442} \quad (7.4)$$

The data fit of  $a_w$  by equation (7.4) is expected to be to some extent inferior than the data fit of  $a_w$  by equation (7.3) as could be seen from the statistical analysis of both the equations. However, this equation could also be considered as a satisfactory correlation for predicting the values of  $a_w$ . It is interesting to observe that all the indices of different groups in equation (7.4) which correlates data on  $a_w$  and that equation (7.7) (developed later on) which correlates data on  $a_p$  are identical except the index of Weber number. This aspect has been discussed in detail elsewhere.

### 7.1.3 Comparison between experimental values (based on data banks) and predicted values of $a_w$ .

The values of  $(a_w/a_t)$  obtained from the data bank and the values of  $(a_w/a_t)$  predicted by using the correlation [Equation (7.3)] are tabulated in Table (7.1.3) and are plotted in Figure (7.1).

The detailed statistical analysis of the correlation [Equation (7.3)] can be described as follows :

- (i) The total number of data points used in the correlation is 100 with 21 different systems/variations included.
- (ii) The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were -0.45, 7.04, and 3.29 respectively.
- (iii) From the 100 data points, 92 % of the predicted values of  $a_w$  were within  $\pm 15$  % of the experimental data values, 73 % were within  $\pm 10$  % and 40 % were within  $\pm 5$  %.

Figure (7.1) shows a parity plot of equation (7.3) wherein the values of  $(a_w/a_t)_{pred.}$  have been plotted versus the values of  $(a_w/a_t)_{exp.(Lit)}$ . The satisfactory correlation fit in Figure (7.1) clearly reflects that equation (7.3) can correlate all the data obtained from various sources covering a wide range of variables as mentioned earlier.

Most of the existing correlation for  $a_w$  already surveyed in

Table - ( 7.1.3 )

Comparison between experimental values based on data bank  
and predicted values of  $a_w$ .

| No. | $a_w$<br>$\frac{2}{m}/m^3$ | $a_w/a_t$<br>exp | $a_w/a_t$<br>This work | % err  | $a_w/a_t$<br>Onda et al. | % err. |
|-----|----------------------------|------------------|------------------------|--------|--------------------------|--------|
| 1   | 79.1                       | 0.460            | 0.469                  | -1.96  | 0.512                    | -11.32 |
| 2   | 80.8                       | 0.470            | 0.571                  | -21.54 | 0.597                    | -26.93 |
| 3   | 110.4                      | 0.642            | 0.592                  | 7.82   | 0.612                    | 4.67   |
| 4   | 103.2                      | 0.279            | 0.241                  | 13.61  | 0.250                    | 10.24  |
| 5   | 135.8                      | 0.367            | 0.328                  | 10.70  | 0.340                    | 7.29   |
| 6   | 166.9                      | 0.451            | 0.414                  | 8.33   | 0.422                    | 6.37   |
| 7   | 36.8                       | 0.320            | 0.292                  | 8.73   | 0.352                    | -10.02 |
| 8   | 46.0                       | 0.400            | 0.397                  | 0.71   | 0.465                    | -16.32 |
| 9   | 57.5                       | 0.500            | 0.501                  | -0.22  | 0.562                    | -12.45 |
| 10  | 64.4                       | 0.560            | 0.632                  | -12.91 | 0.664                    | -18.54 |
| 11  | 107.2                      | 0.228            | 0.261                  | -14.44 | 0.271                    | -18.65 |
| 12  | 165.0                      | 0.351            | 0.329                  | 6.21   | 0.340                    | 3.00   |
| 13  | 164.6                      | 0.445            | 0.424                  | 4.68   | 0.465                    | -4.48  |
| 14  | 188.3                      | 0.509            | 0.472                  | 7.20   | 0.509                    | 0.00   |
| 15  | 253.4                      | 0.685            | 0.582                  | 15.07  | 0.598                    | 12.69  |
| 16  | 266.4                      | 0.720            | 0.619                  | 13.97  | 0.626                    | 13.12  |
| 17  | 88.8                       | 0.240            | 0.276                  | -15.14 | 0.312                    | -30.04 |
| 18  | 146.5                      | 0.396            | 0.440                  | -11.09 | 0.479                    | -20.88 |
| 19  | 168.4                      | 0.455            | 0.504                  | -10.77 | 0.535                    | -17.62 |
| 20  | 185.0                      | 0.500            | 0.555                  | -11.01 | 0.577                    | -15.33 |
| 21  | 55.5                       | 0.279            | 0.256                  | 8.20   | 0.313                    | -12.31 |
| 22  | 68.1                       | 0.342            | 0.343                  | -0.16  | 0.412                    | -20.60 |
| 23  | 87.8                       | 0.441            | 0.425                  | 3.53   | 0.498                    | -12.85 |
| 24  | 119.8                      | 0.602            | 0.627                  | -4.08  | 0.665                    | -10.41 |
| 25  | 69.3                       | 0.210            | 0.217                  | -3.49  | 0.288                    | -37.13 |
| 26  | 92.4                       | 0.280            | 0.274                  | 2.07   | 0.361                    | -29.00 |
| 27  | 115.5                      | 0.350            | 0.344                  | 1.74   | 0.444                    | -26.88 |
| 28  | 135.3                      | 0.410            | 0.435                  | -6.15  | 0.540                    | -31.82 |
| 29  | 148.5                      | 0.450            | 0.468                  | -4.04  | 0.572                    | -27.08 |
| 30  | 198.0                      | 0.600            | 0.592                  | 1.31   | 0.675                    | -12.42 |
| 31  | 35.5                       | 0.187            | 0.202                  | -8.07  | 0.254                    | -35.78 |
| 32  | 41.2                       | 0.217            | 0.254                  | -17.08 | 0.319                    | -47.20 |
| 33  | 55.1                       | 0.290            | 0.322                  | -10.87 | 0.399                    | -37.68 |
| 34  | 71.3                       | 0.375            | 0.437                  | -16.66 | 0.521                    | -38.89 |
| 35  | 93.1                       | 0.490            | 0.550                  | -12.24 | 0.620                    | -26.45 |
| 36  | 34.2                       | 0.180            | 0.169                  | 6.36   | 0.214                    | -18.63 |
| 37  | 38.0                       | 0.200            | 0.212                  | -6.08  | 0.271                    | -35.48 |
| 38  | 47.5                       | 0.250            | 0.267                  | -6.68  | 0.340                    | -35.92 |
| 39  | 57.0                       | 0.300            | 0.338                  | -12.51 | 0.423                    | -40.99 |
| 40  | 66.5                       | 0.350            | 0.363                  | -3.75  | 0.451                    | -28.89 |
| 41  | 79.8                       | 0.420            | 0.459                  | -9.35  | 0.548                    | -30.45 |
| 42  | 38.0                       | 0.200            | 0.188                  | 5.86   | 0.248                    | -24.23 |
| 43  | 47.5                       | 0.250            | 0.237                  | 5.21   | 0.313                    | -25.30 |
| 44  | 58.9                       | 0.310            | 0.298                  | 3.90   | 0.390                    | -25.69 |
| 45  | 72.2                       | 0.380            | 0.377                  | 0.78   | 0.480                    | -26.30 |
| 46  | 76.0                       | 0.400            | 0.406                  | -1.40  | 0.510                    | -27.50 |
| 47  | 98.8                       | 0.520            | 0.513                  | 1.35   | 0.611                    | -17.48 |
| 48  | 43.7                       | 0.230            | 0.222                  | 3.30   | 0.310                    | -34.67 |

Table - 7.1.3 ( contd.)

| No. | $a_w$<br>$m^2/m^3$ | $a_w/a_t$<br>exp | $a_w/a_t$<br>This work | % err  | $a_w/a_t$<br>Onda et al. | % err. |
|-----|--------------------|------------------|------------------------|--------|--------------------------|--------|
| 49  | 57.0               | 0.300            | 0.280                  | 6.69   | 0.386                    | -28.66 |
| 50  | 72.2               | 0.380            | 0.352                  | 7.39   | 0.473                    | -24.51 |
| 51  | 91.2               | 0.480            | 0.445                  | 7.21   | 0.572                    | -19.16 |
| 52  | 98.8               | 0.520            | 0.479                  | 7.86   | 0.604                    | -16.11 |
| 53  | 117.8              | 0.620            | 0.606                  | 2.26   | 0.706                    | -13.92 |
| 54  | 28.5               | 0.150            | 0.163                  | -8.58  | 0.195                    | -29.97 |
| 55  | 34.2               | 0.180            | 0.205                  | -14.16 | 0.249                    | -38.25 |
| 56  | 50.9               | 0.268            | 0.258                  | 3.84   | 0.313                    | -16.65 |
| 57  | 57.0               | 0.300            | 0.353                  | -17.52 | 0.420                    | -40.00 |
| 58  | 76.0               | 0.400            | 0.444                  | -10.94 | 0.512                    | -27.91 |
| 59  | 85.1               | 0.448            | 0.508                  | -13.30 | 0.569                    | -26.97 |
| 60  | 30.4               | 0.160            | 0.155                  | 3.00   | 0.171                    | -6.76  |
| 61  | 35.2               | 0.185            | 0.196                  | -5.85  | 0.219                    | -18.37 |
| 62  | 52.3               | 0.275            | 0.247                  | 10.16  | 0.278                    | -1.20  |
| 63  | 58.9               | 0.310            | 0.336                  | -8.37  | 0.375                    | -21.07 |
| 64  | 65.6               | 0.345            | 0.356                  | -3.31  | 0.396                    | -14.91 |
| 65  | 72.2               | 0.380            | 0.385                  | -1.29  | 0.425                    | -11.84 |
| 66  | 77.9               | 0.410            | 0.423                  | -3.14  | 0.462                    | -12.58 |
| 67  | 85.5               | 0.450            | 0.484                  | -7.49  | 0.516                    | -14.78 |
| 68  | 26.4               | 0.200            | 0.174                  | 12.83  | 0.208                    | -3.82  |
| 69  | 33.0               | 0.250            | 0.220                  | 12.02  | 0.264                    | -5.77  |
| 70  | 52.8               | 0.400            | 0.377                  | 5.65   | 0.443                    | -10.67 |
| 71  | 54.1               | 0.410            | 0.475                  | -15.86 | 0.537                    | -30.88 |
| 72  | 114.1              | 0.195            | 0.194                  | 0.64   | 0.197                    | -1.08  |
| 73  | 155.0              | 0.265            | 0.247                  | 6.66   | 0.255                    | 3.95   |
| 74  | 175.5              | 0.300            | 0.280                  | 6.64   | 0.289                    | 3.77   |
| 75  | 189.5              | 0.324            | 0.308                  | 4.80   | 0.318                    | 1.96   |
| 76  | 200.7              | 0.343            | 0.332                  | 3.32   | 0.341                    | 0.65   |
| 77  | 136.9              | 0.234            | 0.243                  | -4.05  | 0.271                    | -16.02 |
| 78  | 181.4              | 0.310            | 0.311                  | -0.27  | 0.345                    | -11.45 |
| 79  | 204.8              | 0.350            | 0.352                  | -0.55  | 0.388                    | -10.95 |
| 80  | 265.0              | 0.453            | 0.417                  | 8.01   | 0.452                    | 0.25   |
| 81  | 169.6              | 0.290            | 0.248                  | 14.59  | 0.261                    | 9.99   |
| 82  | 201.8              | 0.345            | 0.316                  | 8.34   | 0.333                    | 3.51   |
| 83  | 231.1              | 0.395            | 0.358                  | 9.36   | 0.375                    | 5.15   |
| 84  | 250.4              | 0.428            | 0.394                  | 7.87   | 0.409                    | 4.34   |
| 85  | 264.4              | 0.452            | 0.424                  | 6.21   | 0.437                    | 3.35   |
| 86  | 105.3              | 0.180            | 0.179                  | 0.77   | 0.165                    | 8.22   |
| 87  | 140.4              | 0.240            | 0.228                  | 4.99   | 0.215                    | 10.56  |
| 88  | 162.6              | 0.278            | 0.258                  | 7.13   | 0.244                    | 12.09  |
| 89  | 187.2              | 0.320            | 0.284                  | 11.14  | 0.270                    | 15.70  |
| 90  | 186.0              | 0.318            | 0.306                  | 3.87   | 0.290                    | 8.74   |
| 91  | 96.5               | 0.165            | 0.176                  | -6.72  | 0.143                    | 13.04  |
| 92  | 114.1              | 0.195            | 0.225                  | -15.28 | 0.187                    | 4.01   |
| 93  | 143.3              | 0.245            | 0.255                  | -3.89  | 0.214                    | 12.80  |
| 94  | 158.0              | 0.270            | 0.280                  | -3.82  | 0.236                    | 12.46  |
| 95  | 166.7              | 0.285            | 0.301                  | -5.75  | 0.255                    | 10.63  |
| 96  | 99.5               | 0.170            | 0.175                  | -3.06  | 0.156                    | 7.96   |
| 97  | 134.6              | 0.230            | 0.224                  | 2.75   | 0.204                    | 11.47  |
| 98  | 152.1              | 0.260            | 0.253                  | 2.60   | 0.232                    | 10.75  |
| 99  | 163.8              | 0.280            | 0.279                  | 0.39   | 0.256                    | 8.43   |
| 100 | 175.5              | 0.300            | 0.300                  | 0.00   | 0.276                    | 8.00   |

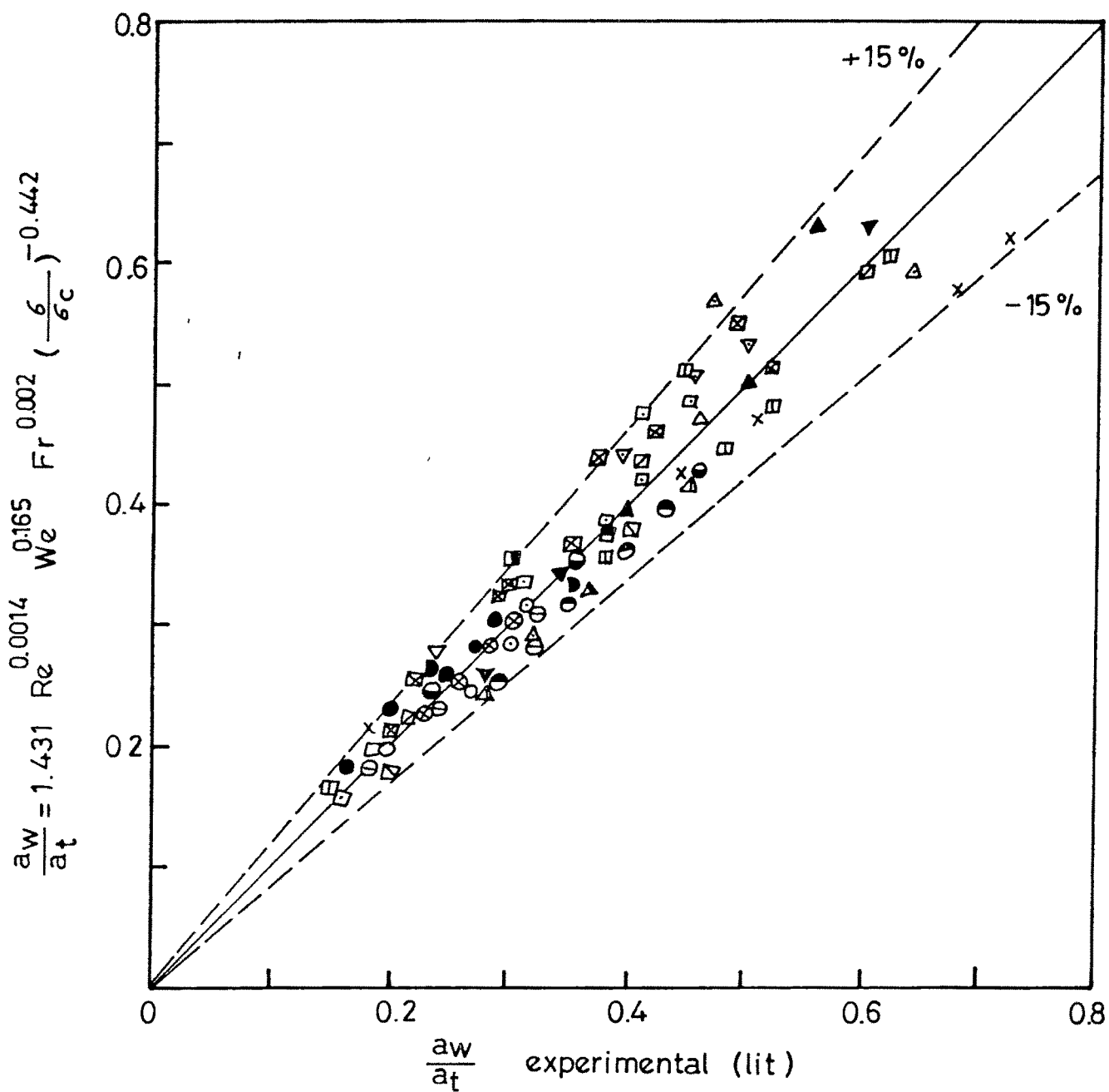


Fig.71 WETTED SURFACE AREA.

COMPARISON OF PREDICTED vs. EXPERIMENTAL  
(LIT.) DATA.

## LEGENDS FOR FIG. 7.1

| Data No. | Relevant Information                                                                                                                                                                                                           | Symbol                                                             |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| 1-10     | Shulman (6,7) Air-Water, Napthalene R.R.,<br>dp = 25mm, 13mm, 38mm.                                                                                                                                                            | $\Delta$ $\triangle$ $\blacktriangle$                              |
| 11-12    | Grimley (8), only water flow, C.R.R., 9.5mm.                                                                                                                                                                                   | $\bullet$                                                          |
| 13-16    | Mayo (9), only water flow, Paper R.R., 13 mm.                                                                                                                                                                                  | X                                                                  |
| 17-24    | Fujita (10), only water flow, Paper R.R.,<br>dp = 13mm, 25mm.                                                                                                                                                                  | $\nabla$ $\blacktriangledown$                                      |
| 25-30    | Hikita (11), water + organic, Paper R.R.,<br>dp = 15mm, $\sigma$ = 34 mN/m.                                                                                                                                                    | $\boxtimes$                                                        |
| 31-67    | Hikita (11), water + org. solvents, Paper R.R.,<br>dp = 25mm, $\sigma$ = 60 to 65 mN/m, 50 mN/m, 38 mN/m.<br>$\sigma$ = 62 mN/m, $\mu_L$ = 1.62 mNs/m <sup>2</sup> .<br>$\sigma$ = 66 mN/m, $\mu_L$ = 3.8 mNs/m <sup>2</sup> . | $\boxtimes$ $\blacksquare$ $\square$<br>$\boxplus$<br>$\square$    |
| 68-71    | Hikita (11), water + org. solvents, Paper RR,<br>dp = 35mm, $\mu_L$ = 3.8 mNs/m <sup>2</sup> .                                                                                                                                 | $\boxtimes$                                                        |
| 72-100   | Onda (12), water + org. solvents, C.R.R., 8mm,<br>$\sigma$ = 67mN/m, 45 mN/m.<br>$\sigma$ = 45mN/m, $\mu_L$ = 2.03mNs/m <sup>2</sup> .<br>$\sigma$ = 75 mN/m.<br>$\mu_L$ = 10.8mNs/m <sup>2</sup> , 3.7 mNs/m <sup>2</sup>     | $\circ$ $\ominus$<br>$\bullet$<br>$\ominus$<br>$\bullet$ $\otimes$ |

Chapter - (2), are in terms of dimensional quantities. Further these correlations can be utilised to correlate the experimental data of that particular investigator only. The correlation proposed by Onda et al. (97) [Equation (2.5)] in terms of dimensionless numbers perhaps happens to be the correlation which is generalised in nature and which can predict the value of  $a_w$  satisfactorily.

The values of  $(a_w/a_t)$  obtained by using the generalised correlation proposed by Onda et al. are also reported in Table (7.1.3). The detailed statistical analysis of the correlation of Onda et al. can be described as follows :

(i) The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were -12.05, 17.04 and 4.29 respectively.

(ii) From the 100 data points only 54 % of the predicted values of  $a_w$  were within  $\pm 15$  % of the experimental data values, 26 % were within  $\pm 10$  % and 16 % were within  $\pm 5$  %.

The comparison of the detailed statistical analysis for both these correlations under consideration reveals that the values of  $E_{abs}$  for the correlation of Onda and for the correlation developed in this investigation are 17 % and 7 % respectively. Hence, it can be concluded that the generalised correlation developed in this investigation is expected to be superior in comparison to the existing generalised correlation of Onda and coworkers.

## 7.2.0 RESULTS AND DISCUSSION FOR EFFECTIVE INTERFACIAL AREA DURING PHYSICAL ABSORPTION ( $a_p$ ) :

### 7.2.1 Critical analysis of data and mathematical modelling :

A data bank consisting of 100 data points covering 20 variations including systems ammonia-water,  $\text{CO}_2$ -water,  $\text{CO}_2$ -methanol, methanol vapour - water etc, reported in Table (5.2) could be used conveniently for analysing the effect of different variables/parameters -  $L$ ,  $a_t$ ,  $\rho_L$ ,  $\mu_L$ ,  $\sigma$  and  $\sigma_c$  on the values of  $a_p$ .

It is observed that the values of  $a_p$  increase with an increase in ' $L$ ' and  $a_t$ . Changes in packed height do not affect the values of  $a_p$  as could be seen from the data of Yoshida and Koyanagi (14,16). The values of  $a_p$  also increase with a decrease in  $\rho_L$ ,  $\sigma$  and  $(\sigma/\sigma_c)$  as observed from the data of Yoshida et al. (14) and Hikita et al. (15). With an increase in  $\mu_L$ , the values of  $a_p$  do decrease, however the effect appears to be quite small as could be seen from the data of Hikita et al. (15).

On the whole, it is observed that the parameters affecting the values of  $a_p$  and that affecting the values of  $a_w$  are identical. Further the effect of the parameters on the values of  $a_p$  and that of  $a_w$  also appears to be similar in nature but different in magnitude. Hence with the help of dimensional analysis, an equation similar to that proposed for  $(a_w/a_t)$  can also be proposed for the present case. However, it is expected that the values of the indices of the various dimensionless numbers/quantities will be significantly

different than that obtained for the correlation of  $(a_w/a_t)$ . Hence the various dimensionless groups [calculated using the data of  $a_p$  reported in Table (5.2)] required for processing of the data for  $a_p$  are tabulated in Table (7.2.1).

### 7.2.2 Statistical analysis for different correlations :

Processing of data reported in Table (7.2.1) was also done by three methodologies namely (i) Multiple linear regression, (ii) Optimization by the DSC-Powell algorithm and (iii) Optimization by the modified simplex algorithm of Nelder and Mead.

The results obtained by use of three methodologies are reported in Table (7.2.2) and are discussed herewith.

CASE I : Using the algorithms of Gauss Jordan and Gauss Seidel yielded the following equations (7.5) and (7.6) respectively :-

$$(a_p/a_t) = 1.832 (\text{Re})^{-0.031} (\text{We})^{0.501} (\text{Fr})^{-0.226} (\sigma/\sigma_c)^{-0.195} \quad (7.5)$$

$$(a_p/a_t) = 0.2979 (\text{Re})^{0.31} (\text{We})^{0.14} (\text{Fr})^{-0.0226} (\sigma/\sigma_c)^{-0.339} \quad (7.6)$$

The normal equations obtained by multiple linear regression may not be diagonally dominant. Hence the Gauss Seidel method initially perhaps did not converge for convergence criterias lower than 0.01 as could be seen from regression steps (2) and (3). Further the magnitude of different indices of dimensionless number obtained by Gauss Jordan and Gauss Seidel algorithms differ considerably as could be seen from the comparison of equations (7.5) and (7.6). The effect of different parameters like  $L$ ,  $a_t$ ,  $\rho_L$ ,  $\mu_L$ ,  $\sigma$  and  $\sigma_c$  on the

Table - ( 7.2.1 )

Results inclusive of processing parameters for  
effective interfacial area during physical absorption ( $a_p$ )

| No. | $a_p$<br>$\frac{m^2}{m^3}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_p/a_t$<br>exp | $a_p/a_t$<br>pred | % err  |
|-----|----------------------------|-------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 1   | 26.1                       | 3.42  | 3.200               | 0.905               | 1.193             | 0.134            | 0.113             | 15.57  |
| 2   | 47.6                       | 10.27 | 28.804              | 8.145               | 1.193             | 0.244            | 0.205             | 15.81  |
| 3   | 59.7                       | 17.12 | 80.010              | 22.626              | 1.193             | 0.306            | 0.271             | 11.43  |
| 4   | 71.9                       | 30.82 | 259.232             | 73.307              | 1.193             | 0.369            | 0.373             | -1.10  |
| 5   | 86.8                       | 51.37 | 720.088             | 203.631             | 1.193             | 0.445            | 0.492             | -10.63 |
| 6   | 21.4                       | 4.98  | 4.657               | 0.622               | 1.193             | 0.159            | 0.127             | 20.03  |
| 7   | 43.8                       | 14.95 | 41.916              | 5.597               | 1.193             | 0.326            | 0.231             | 29.13  |
| 8   | 57.8                       | 24.92 | 116.432             | 15.548              | 1.193             | 0.431            | 0.305             | 29.21  |
| 9   | 65.6                       | 44.86 | 377.240             | 50.375              | 1.193             | 0.490            | 0.420             | 14.18  |
| 10  | 75.3                       | 74.76 | 1047.889            | 139.931             | 1.193             | 0.562            | 0.554             | 1.33   |
| 11  | 40.0                       | 19.64 | 55.066              | 4.261               | 1.193             | 0.392            | 0.252             | 35.71  |
| 12  | 64.3                       | 58.93 | 495.590             | 38.345              | 1.193             | 0.630            | 0.458             | 27.33  |
| 13  | 52.8                       | 9.77  | 27.398              | 8.563               | 1.193             | 0.257            | 0.202             | 21.45  |
| 14  | 73.3                       | 29.32 | 246.586             | 77.067              | 1.193             | 0.358            | 0.367             | -2.69  |
| 15  | 40.8                       | 13.91 | 39.005              | 6.015               | 1.193             | 0.284            | 0.226             | 20.28  |
| 16  | 62.4                       | 41.74 | 351.043             | 54.135              | 1.193             | 0.434            | 0.411             | 5.29   |
| 17  | 11.6                       | 1.04  | 0.295               | 0.083               | 1.193             | 0.060            | 0.058             | 3.32   |
| 18  | 14.5                       | 1.59  | 0.686               | 0.194               | 1.193             | 0.074            | 0.073             | 2.29   |
| 19  | 21.0                       | 2.24  | 1.368               | 0.387               | 1.193             | 0.108            | 0.088             | 18.41  |
| 20  | 19.4                       | 3.16  | 2.729               | 0.772               | 1.193             | 0.099            | 0.106             | -6.64  |
| 21  | 30.8                       | 7.08  | 13.679              | 3.868               | 1.193             | 0.158            | 0.165             | -4.36  |
| 22  | 36.5                       | 12.88 | 45.291              | 12.808              | 1.193             | 0.187            | 0.229             | -22.30 |
| 23  | 52.2                       | 23.99 | 157.052             | 44.412              | 1.193             | 0.268            | 0.321             | -20.10 |
| 24  | 72.0                       | 40.43 | 446.031             | 126.131             | 1.193             | 0.369            | 0.428             | -15.76 |
| 25  | 19.7                       | 1.82  | 0.904               | 0.256               | 1.193             | 0.101            | 0.078             | 22.56  |
| 26  | 33.5                       | 6.31  | 10.865              | 3.073               | 1.193             | 0.172            | 0.155             | 9.95   |
| 27  | 45.4                       | 10.39 | 29.469              | 8.333               | 1.193             | 0.233            | 0.203             | 12.71  |
| 28  | 57.2                       | 15.85 | 68.553              | 19.386              | 1.193             | 0.293            | 0.256             | 12.70  |
| 29  | 70.9                       | 31.87 | 277.133             | 78.369              | 1.193             | 0.364            | 0.375             | -3.21  |
| 30  | 22.3                       | 1.67  | 1.352               | 1.218               | 1.193             | 0.064            | 0.085             | -32.83 |
| 31  | 32.5                       | 2.07  | 2.079               | 1.872               | 1.193             | 0.093            | 0.096             | -2.59  |
| 32  | 30.1                       | 2.64  | 3.397               | 3.059               | 1.193             | 0.087            | 0.110             | -26.68 |
| 33  | 13.9                       | 0.58  | 0.163               | 0.146               | 1.193             | 0.040            | 0.048             | -19.45 |
| 34  | 27.7                       | 1.51  | 1.108               | 0.998               | 1.193             | 0.080            | 0.081             | -1.45  |
| 35  | 32.3                       | 2.41  | 2.826               | 2.545               | 1.193             | 0.093            | 0.104             | -12.42 |
| 36  | 43.9                       | 5.82  | 16.508              | 14.868              | 1.193             | 0.126            | 0.169             | -33.97 |
| 37  | 21.3                       | 1.83  | 1.601               | 0.180               | 0.377             | 0.109            | 0.148             | -35.34 |
| 38  | 33.5                       | 3.80  | 6.883               | 0.772               | 0.377             | 0.172            | 0.221             | -28.38 |
| 39  | 41.8                       | 5.58  | 14.829              | 1.663               | 0.377             | 0.214            | 0.272             | -27.10 |
| 40  | 21.8                       | 0.74  | 0.324               | 0.265               | 1.205             | 0.066            | 0.057             | 14.08  |
| 41  | 29.6                       | 1.46  | 1.248               | 1.021               | 1.205             | 0.090            | 0.082             | 8.41   |
| 42  | 43.7                       | 2.20  | 2.843               | 2.324               | 1.205             | 0.132            | 0.103             | 22.16  |
| 43  | 46.8                       | 2.97  | 5.177               | 4.232               | 1.205             | 0.142            | 0.121             | 14.41  |
| 44  | 54.6                       | 4.42  | 11.510              | 9.410               | 1.205             | 0.165            | 0.151             | 8.72   |
| 45  | 58.7                       | 5.88  | 20.339              | 16.629              | 1.205             | 0.178            | 0.176             | 0.70   |
| 46  | 64.0                       | 7.36  | 31.895              | 26.076              | 1.205             | 0.194            | 0.200             | -2.98  |
| 47  | 87.4                       | 14.57 | 124.841             | 102.066             | 1.205             | 0.265            | 0.290             | -9.50  |
| 48  | 70.4                       | 9.37  | 32.425              | 10.431              | 1.205             | 0.340            | 0.205             | 39.77  |

Table - 7.2.1 ( contd.)

| No. | $a$<br>$\frac{2P}{m} \frac{a}{m}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_p/a_t$<br>exp | $a_p/a_t$<br>pred | % err  |
|-----|-----------------------------------|-------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 49  | 76.6                              | 11.74 | 50.847              | 16.357              | 1.205             | 0.370            | 0.232             | 37.40  |
| 50  | 104.5                             | 23.23 | 199.022             | 64.023              | 1.205             | 0.505            | 0.336             | 33.39  |
| 51  | 54.6                              | 1.62  | 1.377               | 0.425               | 0.359             | 0.165            | 0.145             | 12.44  |
| 52  | 65.5                              | 3.19  | 5.314               | 1.642               | 0.359             | 0.199            | 0.210             | -5.56  |
| 53  | 71.8                              | 4.81  | 12.101              | 3.739               | 0.359             | 0.217            | 0.262             | -20.71 |
| 54  | 93.6                              | 6.49  | 22.035              | 6.808               | 0.359             | 0.284            | 0.309             | -9.03  |
| 55  | 109.2                             | 9.68  | 48.991              | 15.136              | 0.359             | 0.331            | 0.385             | -16.28 |
| 56  | 124.8                             | 12.86 | 86.574              | 26.747              | 0.359             | 0.378            | 0.450             | -18.89 |
| 57  | 131.0                             | 16.11 | 135.760             | 41.943              | 0.359             | 0.397            | 0.508             | -28.05 |
| 58  | 188.8                             | 31.87 | 531.382             | 164.169             | 0.359             | 0.572            | 0.739             | -29.11 |
| 59  | 64.2                              | 5.08  | 8.471               | 1.030               | 0.359             | 0.310            | 0.243             | 21.55  |
| 60  | 82.8                              | 7.67  | 19.292              | 2.345               | 0.359             | 0.400            | 0.305             | 23.85  |
| 61  | 89.0                              | 10.35 | 35.129              | 4.270               | 0.359             | 0.430            | 0.359             | 16.55  |
| 62  | 105.6                             | 15.43 | 78.101              | 9.494               | 0.359             | 0.510            | 0.447             | 12.48  |
| 63  | 124.2                             | 20.51 | 138.016             | 16.778              | 0.359             | 0.600            | 0.522             | 13.04  |
| 64  | 140.8                             | 25.68 | 216.428             | 26.309              | 0.359             | 0.680            | 0.590             | 13.23  |
| 65  | 109.2                             | 5.90  | 87.321              | 32.561              | 0.492             | 0.331            | 0.363             | -9.71  |
| 66  | 120.1                             | 9.41  | 139.207             | 20.425              | 0.492             | 0.580            | 0.421             | 27.37  |
| 67  | 90.5                              | 5.27  | 61.436              | 28.654              | 0.656             | 0.274            | 0.292             | -6.60  |
| 68  | 104.5                             | 8.39  | 97.941              | 17.974              | 0.656             | 0.505            | 0.339             | 32.82  |
| 69  | 78.0                              | 7.11  | 43.405              | 27.042              | 0.902             | 0.236            | 0.242             | -2.43  |
| 70  | 86.9                              | 11.33 | 69.197              | 16.963              | 0.902             | 0.420            | 0.281             | 33.11  |
| 71  | 49.9                              | 1.20  | 7.604               | 5.377               | 1.148             | 0.151            | 0.124             | 18.29  |
| 72  | 65.5                              | 2.41  | 30.415              | 21.508              | 1.148             | 0.199            | 0.181             | 9.05   |
| 73  | 90.5                              | 4.81  | 121.659             | 86.030              | 1.148             | 0.274            | 0.264             | 3.77   |
| 74  | 74.9                              | 3.64  | 69.421              | 49.091              | 1.148             | 0.227            | 0.226             | 0.26   |
| 75  | 49.9                              | 1.13  | 7.553               | 5.006               | 1.115             | 0.151            | 0.124             | 17.89  |
| 76  | 65.5                              | 2.26  | 30.211              | 20.025              | 1.115             | 0.199            | 0.181             | 8.60   |
| 77  | 74.9                              | 3.41  | 68.956              | 45.706              | 1.115             | 0.227            | 0.227             | 0.00   |
| 78  | 90.5                              | 4.52  | 120.843             | 80.099              | 1.115             | 0.274            | 0.265             | 3.30   |
| 79  | 21.9                              | 0.65  | 0.287               | 0.262               | 1.208             | 0.063            | 0.055             | 13.14  |
| 80  | 20.6                              | 0.67  | 0.307               | 0.280               | 1.210             | 0.059            | 0.056             | 6.14   |
| 81  | 28.2                              | 1.62  | 1.752               | 1.597               | 1.208             | 0.081            | 0.090             | -10.74 |
| 82  | 31.1                              | 1.63  | 1.808               | 1.649               | 1.209             | 0.089            | 0.090             | -1.10  |
| 83  | 34.9                              | 2.90  | 5.970               | 5.455               | 1.211             | 0.100            | 0.125             | -24.30 |
| 84  | 40.5                              | 3.38  | 7.361               | 6.703               | 1.207             | 0.116            | 0.133             | -14.29 |
| 85  | 20.3                              | 0.65  | 0.284               | 0.259               | 1.208             | 0.058            | 0.054             | 6.51   |
| 86  | 21.1                              | 0.66  | 0.290               | 0.264               | 1.208             | 0.061            | 0.055             | 9.54   |
| 87  | 19.6                              | 0.75  | 0.331               | 0.300               | 1.202             | 0.056            | 0.057             | -1.68  |
| 88  | 27.9                              | 1.80  | 1.822               | 1.649               | 1.200             | 0.080            | 0.092             | -14.27 |
| 89  | 28.3                              | 1.70  | 1.912               | 1.742               | 1.208             | 0.081            | 0.092             | -12.80 |
| 90  | 45.4                              | 3.50  | 7.637               | 6.943               | 1.205             | 0.131            | 0.135             | -3.13  |
| 91  | 23.9                              | 3.02  | 3.160               | 0.902               | 1.205             | 0.122            | 0.109             | 11.27  |
| 92  | 23.1                              | 2.93  | 3.229               | 0.924               | 1.208             | 0.118            | 0.109             | 8.20   |
| 93  | 27.2                              | 3.17  | 3.504               | 0.999               | 1.203             | 0.140            | 0.112             | 19.91  |
| 94  | 43.7                              | 5.37  | 12.232              | 3.521               | 1.215             | 0.224            | 0.155             | 30.80  |
| 95  | 75.9                              | 16.78 | 114.196             | 32.771              | 1.211             | 0.389            | 0.287             | 26.31  |
| 96  | 27.2                              | 3.24  | 3.278               | 0.931               | 1.199             | 0.139            | 0.111             | 20.62  |
| 97  | 26.1                              | 3.08  | 3.441               | 0.984               | 1.207             | 0.134            | 0.111             | 17.26  |
| 98  | 24.6                              | 3.18  | 3.858               | 1.105               | 1.209             | 0.126            | 0.114             | 9.59   |
| 99  | 41.6                              | 6.85  | 15.668              | 4.460               | 1.202             | 0.213            | 0.169             | 20.78  |
| 100 | 84.6                              | 18.89 | 129.663             | 37.068              | 1.207             | 0.434            | 0.299             | 31.05  |

Table - ( 7.2.2 )

Statistical analysis for different correlations of  $a_p$

Correlation Structure :  $a_p / a_t = C \cdot Re^\alpha \cdot We^\beta \cdot Fr^\gamma \cdot (\sigma / \sigma_c)^\delta$ .

| No. | C | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|

Case 1 : - Multiple Linear Regression

Numerical Method : Gauss Jordan algorithm.

|   |       |         |        |        |        |       |      |      |
|---|-------|---------|--------|--------|--------|-------|------|------|
| 1 | 1.832 | -0.0312 | 0.5013 | -0.226 | -0.195 | -1.26 | 12.5 | 4.57 |
|---|-------|---------|--------|--------|--------|-------|------|------|

Numerical Method : Gauss Seidel algorithm.

|   |                                         |  |  |  |  |          |  |  |
|---|-----------------------------------------|--|--|--|--|----------|--|--|
| 2 | Convergence criteria $\epsilon = 0.001$ |  |  |  |  | Overflow |  |  |
|---|-----------------------------------------|--|--|--|--|----------|--|--|

|   |                                         |  |  |  |  |          |  |  |
|---|-----------------------------------------|--|--|--|--|----------|--|--|
| 3 | Convergence criteria $\epsilon = 0.005$ |  |  |  |  | Overflow |  |  |
|---|-----------------------------------------|--|--|--|--|----------|--|--|

|   |                                        |  |  |  |  |  |  |  |
|---|----------------------------------------|--|--|--|--|--|--|--|
| 4 | Convergence criteria $\epsilon = 0.01$ |  |  |  |  |  |  |  |
|---|----------------------------------------|--|--|--|--|--|--|--|

|  |        |        |      |         |        |       |       |      |
|--|--------|--------|------|---------|--------|-------|-------|------|
|  | 0.2979 | 0.3129 | 0.14 | -0.0226 | -0.339 | 2.268 | 16.01 | 5.76 |
|--|--------|--------|------|---------|--------|-------|-------|------|

Case 2 :- Optimisation by the D S C - Powell algorithm.

Initialisation were based on arbitrary guesses of indices,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and C were fed as 0.02, 0.8, 0.002, 0.05, 3.0.

Direction of search sequence  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\gamma$ , C. respectively.

|   |        |        |       |       |        |      |       |       |
|---|--------|--------|-------|-------|--------|------|-------|-------|
| 5 | 0.9442 | 0.1037 | 0.188 | 0.024 | -0.482 | 9.04 | 17.47 | 6.42. |
|---|--------|--------|-------|-------|--------|------|-------|-------|

Direction of search sequence as in step 5, initialisation based on values of indices of step no. 5. However  $\alpha$ ,  $\gamma$  values fed as 0.001.

|   |       |        |        |       |       |      |       |       |
|---|-------|--------|--------|-------|-------|------|-------|-------|
| 6 | 1.005 | 0.0906 | 0.2168 | 0.002 | -0.46 | 8.68 | 16.84 | 6.18. |
|---|-------|--------|--------|-------|-------|------|-------|-------|

Direction of search sequence changed to C,  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no. 6.

|   |       |        |      |       |        |     |       |       |
|---|-------|--------|------|-------|--------|-----|-------|-------|
| 7 | 1.056 | 0.1017 | 0.22 | 0.002 | -0.448 | 5.1 | 16.07 | 6.00. |
|---|-------|--------|------|-------|--------|-----|-------|-------|

Direction of search sequence changed to  $\delta$ , C,  $\beta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no. 6.

|   |       |       |       |       |        |      |       |       |
|---|-------|-------|-------|-------|--------|------|-------|-------|
| 8 | 1.031 | 0.108 | 0.219 | 0.002 | -0.439 | 5.65 | 16.04 | 5.95. |
|---|-------|-------|-------|-------|--------|------|-------|-------|

Table - 7.2.2 ( contd.)

| No. | C | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|----------|--------------------|--------------------|--------------------|

Direction of search sequence as in step no. 8, initialisation done by values of step no. 7.

|   |       |        |       |       |        |      |       |       |
|---|-------|--------|-------|-------|--------|------|-------|-------|
| 9 | 1.054 | 0.1043 | 0.221 | 0.002 | -0.442 | 5.89 | 16.08 | 5.97. |
|---|-------|--------|-------|-------|--------|------|-------|-------|

Direction of search sequence changed to  $\beta$ , C,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no. 6.

|     |      |        |       |       |        |      |       |       |
|-----|------|--------|-------|-------|--------|------|-------|-------|
| 10* | 1.08 | 0.0992 | 0.222 | 0.002 | -0.442 | 3.45 | 15.66 | 5.97. |
|-----|------|--------|-------|-------|--------|------|-------|-------|

Case 3 : - Optimisation by the Modified Simplex algorithm of Nelder and Mead

All parameters are iterated, base values as obtained from step no 10. Each indice value perturbed 50 % at a time.

|    |       |       |       |         |        |     |       |       |
|----|-------|-------|-------|---------|--------|-----|-------|-------|
| 11 | 1.092 | 0.102 | 0.226 | 0.00239 | -0.445 | 2.4 | 16.41 | 5.99. |
|----|-------|-------|-------|---------|--------|-----|-------|-------|

$\gamma$  value fixed as 0.0024. Other parameters iterated as in step 11.

|    |        |        |       |        |        |      |       |       |
|----|--------|--------|-------|--------|--------|------|-------|-------|
| 12 | 1.1034 | 0.1014 | 0.226 | 0.0024 | -0.431 | 6.58 | 16.12 | 5.94. |
|----|--------|--------|-------|--------|--------|------|-------|-------|

$\delta$  value fixed as 0.589 (as obtained in wetted area). Other parameters iterated as in step 11.

|    |       |       |       |        |        |     |       |       |
|----|-------|-------|-------|--------|--------|-----|-------|-------|
| 13 | 1.299 | 0.079 | 0.237 | 0.0024 | -0.589 | 2.3 | 17.76 | 7.18. |
|----|-------|-------|-------|--------|--------|-----|-------|-------|

$\gamma$  fixed as 0.002 as obtained in steps 6 to 10.

|    |       |       |      |       |        |      |       |       |
|----|-------|-------|------|-------|--------|------|-------|-------|
| 14 | 1.158 | 0.096 | 0.23 | 0.002 | -0.441 | 5.76 | 16.14 | 5.98. |
|----|-------|-------|------|-------|--------|------|-------|-------|

C fixed as 1.431 as obtained in wetted areas. Other parameters iterated as in step 11.

|    |       |       |       |        |        |     |       |       |
|----|-------|-------|-------|--------|--------|-----|-------|-------|
| 15 | 1.431 | 0.053 | 0.238 | 0.0626 | -0.295 | 4.0 | 15.55 | 6.06. |
|----|-------|-------|-------|--------|--------|-----|-------|-------|

C fixed as 1.431,  $\delta$  fixed as -0.442. Other parameters iterated as in step no. 11.

|    |       |        |       |       |        |      |       |       |
|----|-------|--------|-------|-------|--------|------|-------|-------|
| 16 | 1.431 | 0.0528 | 0.241 | 0.002 | -0.442 | 1.59 | 16.27 | 6.09. |
|----|-------|--------|-------|-------|--------|------|-------|-------|

C fixed as 1.431 and  $\gamma$  fixed as 0.002. Other parameters iterated as in step no. 11.

|    |       |        |        |       |         |      |       |       |
|----|-------|--------|--------|-------|---------|------|-------|-------|
| 17 | 1.431 | 0.0497 | 0.2434 | 0.002 | -0.2691 | 5.22 | 15.38 | 6.15. |
|----|-------|--------|--------|-------|---------|------|-------|-------|

Table - 7.2.2 ( contd.)

| No.                                                                                                                             | C     | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|---------------------------------------------------------------------------------------------------------------------------------|-------|----------|---------|----------|----------|--------------------|--------------------|--------------------|
| C, $\gamma$ , $\delta$ fixed as 1.431, 0.002, -0.442 respectively. Other parameters iterated as in step no 11.                  |       |          |         |          |          |                    |                    |                    |
| 18                                                                                                                              | 1.431 | 0.027    | 0.2427  | 0.002    | -0.442   | 7.01               | 16.58              | 6.35.              |
| C, $\alpha$ , $\gamma$ , $\delta$ fixed as 1.431, 0.0014, 0.002, -0.442 respectively. Only $\beta$ iterated as in step no. 11.  |       |          |         |          |          |                    |                    |                    |
| 19                                                                                                                              | 1.431 | 0.0014   | 0.237   | 0.002    | -0.442   | 5.99               | 17.38              | 6.73.              |
| C, $\beta$ , $\gamma$ , $\delta$ fixed as 1.431, 0.165, 0.002, -0.442 respectively. Only $\alpha$ is iterated as in step no 11. |       |          |         |          |          |                    |                    |                    |
| 20                                                                                                                              | 1.431 | -0.282   | 0.165   | 0.002    | -0.442   | 28.7               | 60.2               | 13.8.              |

Note : The values of indices in the finalised generalised correlation are as per regression step marked by ( \* )

value of  $a_p$  reflected in these two equations appears to be contradictory. For example equation (7.5) indicates that with an increase in  $\mu_L$  the value of  $(a_p/a_t)$  is expected to increase (Re index being negative), while equation (7.6) indicates that with an increase in  $\mu_L$  the value of  $(a_p/a_t)$  is expected to decrease considerably (Re index being positive and of high magnitude). Similarly, other contradictory observations can be made from the indices of other dimensionless numbers like We, Fr and  $(\sigma / \sigma_c)$ . Further, the correlation (7.6) obtained by Gauss Seidel methodology appears to be to some extent inferior in comparison to the correlations (7.5) obtained by Gauss Jordan methodology as could be seen from the values  $E_{abs}$  being 16.01 % and 12.5 % respectively.

Thus it could be observed that by using different numerical methods sometimes misleading results can be obtained/inconsistent results are also likely to be obtained. Also correlations having inferior statistical analysis are likely to be developed. Hence while using different optimization techniques one must be very careful in order to obtain a generalized correlation by mathematical modelling.

CASE II : It was observed that using methodology ; optimization by the DSC-Powell algorithm by changing the direction of search sequences, some decrease in the values of %  $E_{abs}$  was obtained. The values of %  $E_{abs}$  for regression steps (5) and (10) were 17.47 and 15.66 respectively.

By performing regression as per step (10), yielded the following equation :-

$$(a_p/a_t) = 1.08 (Re)^{0.099} (We)^{0.222} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442} \quad (7.7)$$

Though the data fit by this correlation (equation 7.7) is expected to be to some extent inferior than the correlation (7.5), the interpretation of the experimental data on  $a_p$  and their parametric dependencies involves no problems. The values of different indices obtained in equation (7.7) and the effect of different parameters on the values of  $a_p$  thus involves no ambiguity.

CASE III : It was observed that using the modified simplex algorithm of Nelder and Mead and performing regressions as per steps (11) to (20) by prefixing indices of some number did not alter the statistical analysis appreciably, exception being the regression step (20). The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  remained in the ranges of (2 to 7), (15.5 to 17.5) and (6 to 7) respectively.

In regression step (19), all the indices except that of Weber number (We) were prefixed to  $C = 1.431$ ,  $\alpha = 0.0014$ ,  $\gamma = 0.002$  and  $\delta = -0.442$ . These indices were obtained from a generalised correlation which had been already developed for predicting the values  $(a_w/a_t)$  by equation (7.3). The regression of the data was performed to optimise for power of Weber number (We). This methodology resulted in the following correlation having the statistical analysis as

%  $E_{avg} = 5.99$ , %  $E_{abs} = 17.38$  and %  $S_{dev} = 6.731$  :-

$$(a_p/a_t) = 1.431 (Re)^{0.0014} (We)^{0.237} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442} \quad (7.8)$$

In regression step (20) all the indices except that of Reynolds number (Re) were prefixed to  $C = 1.431$ ,  $\beta = 0.165$ ,  $\gamma = 0.002$  and  $\delta = -0.442$ . [These indices were obtained from a generalised correlation which was developed for predicting the values of  $(a_w/a_t)$  by equation (7.3)]. The regression of the data was performed to optimise the power of Re. This methodology resulted in the following correlation having the statistical analysis of %  $E_{avg} = 28.7$ , %  $E_{abs} = 60.22$  and %  $S_{dev} = 13.8$ .

$$(a_p/a_t) = 1.431 (Re)^{-0.282} (We)^{0.165} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442} \quad (7.9)$$

Thus, under these conditions the algorithm failed to minimize the error by altering the index appropriately. Hence Weber number appears to be the most dominating parameter affecting the values of  $a_p$  in comparison to all other parameters.

The data fit of  $a_p$  by equation (7.8) is expected to be to some extent inferior than the data fit of  $a_p$  by equation (7.7). However, this equation could also be considered as a satisfactory correlation for predicting the values of  $(a_p)$ . It is interesting to observe that all the indices of different groups in equation (7.8) which correlates data on  $a_p$  and that in equation (7.3) which correlates data on  $a_w$ , are identical except the index of Weber number.

### 7.2.3 Comparison between experimental values (based on data banks) and predicted values of $a_p$ :

The values of  $(a_p/a_t)$  obtained from the data bank of  $a_p$  and

the values of  $(a_p/a_t)$  predicted by using equation (7.7) are tabulated in Table (7.2.3) and are plotted in Figure (7.2). The detailed statistical analysis of the correlation [ Equation (7.7)] can be described as follows :

- (i) The total number of data points used in the correlation is 100 with 20 different systems/variations included.
- (ii) The values of %  $E_{avg}$ , %  $E_{abs}$ , and %  $S_{dev}$  were 3.45, 15.66 and 5.97 respectively.
- (iii) Out of 100 data points 77% of the predicted values of  $a_p$  were within  $\pm 25$  % of the experimental data values, 63 % were within  $\pm 20$ % and 51% were within  $\pm 15$  % error.
- (iv) From 100 data points only 13 points had deviations above  $\pm 30$ %, and the value of maximum deviation observed was + 39.77 %.

Figure (7.2) shows a parity plot of equation (7.7) where in the values of  $(a_p/a_t)_{pred}$  have been plotted versus  $(a_p/a_t)_{exp.(Lit)}$ . The satisfactory fit obtained for majority of data points indicate that the correlation can satisfactorily correlate the data of  $a_p$  obtained from various sources covering a wide range of variables mentioned earlier.

The existing correlations for predicting the values of  $a_p$  surveyed in Chapter - (2) : Literature Survey were mostly developed by their respective investigators using limited data bank. However the correlation proposed by Zech and Mersmann (104) [Equation - (2.8)] based on theoretical considerations, was developed using a

Table - ( 7.2.3 )

Comparison between experimental values based on data bank  
and predicted values of  $a_p$

| No. | $a_p$<br>$\frac{m^2}{m^3}$ | $a_p/a_t$<br>exp | $a_p/a_t$<br>This work | % err  | $a_p/a_t$<br>Mersmann et al. | % err | $a_p/a_t$<br>Billet et al. | % err   |
|-----|----------------------------|------------------|------------------------|--------|------------------------------|-------|----------------------------|---------|
| 1   | 26.1                       | 0.134            | 0.113                  | 15.57  | 0.095                        | 28.78 | 0.160                      | -19.15  |
| 2   | 47.6                       | 0.244            | 0.205                  | 15.81  | 0.165                        | 32.26 | 0.248                      | -1.55   |
| 3   | 59.7                       | 0.306            | 0.271                  | 11.43  | 0.213                        | 30.29 | 0.304                      | 0.70    |
| 4   | 71.9                       | 0.369            | 0.373                  | -1.10  | 0.286                        | 22.41 | 0.385                      | -4.21   |
| 5   | 86.8                       | 0.445            | 0.492                  | -10.63 | 0.370                        | 16.94 | 0.472                      | -6.00   |
| 6   | 21.4                       | 0.159            | 0.127                  | 20.03  | 0.113                        | 28.83 | 0.223                      | -39.79  |
| 7   | 43.8                       | 0.326            | 0.231                  | 29.13  | 0.196                        | 39.85 | 0.346                      | -5.87   |
| 8   | 57.8                       | 0.431            | 0.305                  | 29.21  | 0.254                        | 41.22 | 0.424                      | 1.70    |
| 9   | 65.6                       | 0.490            | 0.420                  | 14.18  | 0.340                        | 30.51 | 0.536                      | -9.57   |
| 10  | 75.3                       | 0.562            | 0.554                  | 1.33   | 0.439                        | 21.85 | 0.658                      | -17.09  |
| 11  | 40.0                       | 0.392            | 0.252                  | 35.71  | 0.222                        | 43.51 | 0.474                      | -20.71  |
| 12  | 64.3                       | 0.630            | 0.458                  | 27.33  | 0.384                        | 39.10 | 0.735                      | -16.69  |
| 13  | 52.8                       | 0.257            | 0.202                  | 21.45  | 0.157                        | 38.92 | 0.226                      | 12.23   |
| 14  | 73.3                       | 0.358            | 0.367                  | -2.69  | 0.272                        | 23.84 | 0.351                      | 1.94    |
| 15  | 40.8                       | 0.284            | 0.226                  | 20.28  | 0.183                        | 35.58 | 0.329                      | -15.86  |
| 16  | 62.4                       | 0.434            | 0.411                  | 5.29   | 0.317                        | 27.00 | 0.510                      | -17.62  |
| 17  | 11.6                       | 0.060            | 0.058                  | 3.32   | 0.053                        | 11.94 | 0.099                      | -66.00  |
| 18  | 14.5                       | 0.074            | 0.073                  | 2.29   | 0.065                        | 12.76 | 0.117                      | -57.66  |
| 19  | 21.0                       | 0.108            | 0.088                  | 18.41  | 0.077                        | 28.32 | 0.135                      | -25.14  |
| 20  | 19.4                       | 0.099            | 0.106                  | -6.64  | 0.092                        | 7.82  | 0.155                      | -55.45  |
| 21  | 30.8                       | 0.158            | 0.165                  | -4.36  | 0.137                        | 13.15 | 0.213                      | -35.14  |
| 22  | 36.5                       | 0.187            | 0.229                  | -22.30 | 0.185                        | 1.04  | 0.271                      | -45.03  |
| 23  | 52.2                       | 0.268            | 0.321                  | -20.10 | 0.253                        | 5.62  | 0.348                      | -29.98  |
| 24  | 72.0                       | 0.369            | 0.428                  | -15.76 | 0.328                        | 11.24 | 0.429                      | -16.03  |
| 25  | 19.7                       | 0.101            | 0.078                  | 22.56  | 0.070                        | 31.30 | 0.124                      | -22.45  |
| 26  | 33.5                       | 0.172            | 0.155                  | 9.95   | 0.130                        | 24.65 | 0.204                      | -18.60  |
| 27  | 45.4                       | 0.233            | 0.203                  | 12.71  | 0.166                        | 28.65 | 0.249                      | -6.83   |
| 28  | 57.2                       | 0.293            | 0.256                  | 12.70  | 0.205                        | 30.05 | 0.295                      | -0.41   |
| 29  | 70.9                       | 0.364            | 0.375                  | -3.21  | 0.291                        | 19.96 | 0.390                      | -7.14   |
| 30  | 22.3                       | 0.064            | 0.085                  | -32.83 | 0.041                        | 36.73 | 0.081                      | -26.26  |
| 31  | 32.5                       | 0.093            | 0.096                  | -2.59  | 0.045                        | 51.62 | 0.088                      | 5.52    |
| 32  | 30.1                       | 0.087            | 0.110                  | -26.68 | 0.051                        | 40.95 | 0.097                      | -12.53  |
| 33  | 13.9                       | 0.040            | 0.048                  | -19.45 | 0.024                        | 40.20 | 0.053                      | -32.66  |
| 34  | 27.7                       | 0.080            | 0.081                  | -1.45  | 0.039                        | 51.45 | 0.078                      | 2.15    |
| 35  | 32.3                       | 0.093            | 0.104                  | -12.42 | 0.049                        | 47.37 | 0.094                      | -1.22   |
| 36  | 43.9                       | 0.126            | 0.169                  | -33.97 | 0.076                        | 39.83 | 0.134                      | -5.95   |
| 37  | 21.3                       | 0.109            | 0.148                  | -35.34 | 0.106                        | 3.28  | 0.223                      | -103.70 |
| 38  | 33.5                       | 0.172            | 0.221                  | -28.38 | 0.152                        | 11.35 | 0.298                      | -73.58  |
| 39  | 41.8                       | 0.214            | 0.272                  | -27.10 | 0.185                        | 13.80 | 0.348                      | -62.43  |
| 40  | 21.8                       | 0.066            | 0.057                  | 14.08  | 0.028                        | 58.13 | 0.065                      | 2.06    |
| 41  | 29.6                       | 0.090            | 0.082                  | 8.41   | 0.039                        | 56.76 | 0.085                      | 5.46    |
| 42  | 43.7                       | 0.132            | 0.103                  | 22.16  | 0.048                        | 63.96 | 0.100                      | 24.37   |
| 43  | 46.8                       | 0.142            | 0.121                  | 14.41  | 0.055                        | 60.92 | 0.113                      | 20.43   |
| 44  | 54.6                       | 0.165            | 0.151                  | 8.72   | 0.068                        | 59.10 | 0.132                      | 19.97   |
| 45  | 58.7                       | 0.178            | 0.176                  | 0.70   | 0.078                        | 56.10 | 0.148                      | 16.52   |
| 46  | 64.0                       | 0.194            | 0.200                  | -2.98  | 0.087                        | 54.95 | 0.162                      | 16.24   |
| 47  | 87.4                       | 0.265            | 0.290                  | -9.50  | 0.123                        | 53.61 | 0.213                      | 19.43   |
| 48  | 70.4                       | 0.340            | 0.205                  | 39.77  | 0.153                        | 55.12 | 0.247                      | 27.43   |
| 49  | 76.6                       | 0.370            | 0.232                  | 37.40  | 0.171                        | 53.85 | 0.270                      | 27.03   |

Table - 7.2.3 ( contd.)

| No. | $a_{2p}^a$<br>$m/m$ | $a_p/a_t$<br>exp | $a_p/a_t$<br>This work | % err  | $a_p/a_t$<br>Mersmann et al. | % err | $a_p/a_t$<br>Billet et al. | % err  |
|-----|---------------------|------------------|------------------------|--------|------------------------------|-------|----------------------------|--------|
| 50  | 104.5               | 0.505            | 0.336                  | 33.39  | 0.240                        | 52.44 | 0.355                      | 29.76  |
| 51  | 54.6                | 0.165            | 0.145                  | 12.44  | 0.064                        | 61.62 | 0.133                      | 19.85  |
| 52  | 65.5                | 0.199            | 0.210                  | -5.56  | 0.089                        | 55.17 | 0.174                      | 12.50  |
| 53  | 71.8                | 0.217            | 0.262                  | -20.71 | 0.109                        | 49.72 | 0.205                      | 5.81   |
| 54  | 93.6                | 0.284            | 0.309                  | -9.03  | 0.127                        | 55.22 | 0.231                      | 18.59  |
| 55  | 109.2               | 0.331            | 0.385                  | -16.28 | 0.155                        | 53.13 | 0.271                      | 18.13  |
| 56  | 124.8               | 0.378            | 0.450                  | -18.89 | 0.179                        | 52.72 | 0.304                      | 19.73  |
| 57  | 131.0               | 0.397            | 0.508                  | -28.05 | 0.200                        | 49.61 | 0.332                      | 16.35  |
| 58  | 188.8               | 0.572            | 0.739                  | -29.11 | 0.281                        | 50.80 | 0.436                      | 23.71  |
| 59  | 64.2                | 0.310            | 0.243                  | 21.55  | 0.174                        | 43.85 | 0.289                      | 6.80   |
| 60  | 82.8                | 0.400            | 0.305                  | 23.85  | 0.214                        | 46.55 | 0.341                      | 14.84  |
| 61  | 89.0                | 0.430            | 0.359                  | 16.55  | 0.248                        | 42.24 | 0.384                      | 10.70  |
| 62  | 105.6               | 0.510            | 0.447                  | 12.48  | 0.303                        | 40.55 | 0.451                      | 11.69  |
| 63  | 124.2               | 0.600            | 0.522                  | 13.04  | 0.350                        | 41.72 | 0.505                      | 15.85  |
| 64  | 140.8               | 0.680            | 0.590                  | 13.23  | 0.391                        | 42.45 | 0.552                      | 18.76  |
| 65  | 109.2               | 0.331            | 0.363                  | -9.71  | 0.111                        | 66.36 | 0.327                      | 1.24   |
| 66  | 120.1               | 0.580            | 0.421                  | 27.37  | 0.218                        | 62.47 | 0.543                      | 6.30   |
| 67  | 90.5                | 0.274            | 0.292                  | -6.60  | 0.095                        | 65.33 | 0.272                      | 0.77   |
| 68  | 104.5               | 0.505            | 0.339                  | 32.82  | 0.186                        | 63.18 | 0.452                      | 10.37  |
| 69  | 78.0                | 0.236            | 0.242                  | -2.43  | 0.097                        | 58.98 | 0.203                      | 14.26  |
| 70  | 86.9                | 0.420            | 0.281                  | 33.11  | 0.190                        | 54.86 | 0.337                      | 19.76  |
| 71  | 49.9                | 0.151            | 0.124                  | 18.29  | 0.038                        | 75.09 | 0.162                      | -7.04  |
| 72  | 65.5                | 0.199            | 0.181                  | 9.05   | 0.053                        | 73.16 | 0.214                      | -7.61  |
| 73  | 90.5                | 0.274            | 0.264                  | 3.77   | 0.075                        | 72.51 | 0.282                      | -2.83  |
| 74  | 74.9                | 0.227            | 0.226                  | 0.26   | 0.065                        | 71.13 | 0.252                      | -11.06 |
| 75  | 49.9                | 0.151            | 0.124                  | 17.89  | 0.038                        | 75.16 | 0.169                      | -11.39 |
| 76  | 65.5                | 0.199            | 0.181                  | 8.60   | 0.053                        | 73.23 | 0.222                      | -11.99 |
| 77  | 74.9                | 0.227            | 0.227                  | 0.00   | 0.065                        | 71.21 | 0.262                      | -15.57 |
| 78  | 90.5                | 0.274            | 0.265                  | 3.30   | 0.075                        | 72.59 | 0.293                      | -7.00  |
| 79  | 21.9                | 0.063            | 0.055                  | 13.14  | 0.025                        | 59.73 | 0.061                      | 2.95   |
| 80  | 20.6                | 0.059            | 0.056                  | 6.14   | 0.026                        | 56.78 | 0.062                      | -4.75  |
| 81  | 28.2                | 0.081            | 0.090                  | -10.74 | 0.040                        | 50.73 | 0.088                      | -8.22  |
| 82  | 31.1                | 0.089            | 0.090                  | -1.10  | 0.040                        | 55.26 | 0.088                      | 1.09   |
| 83  | 34.9                | 0.100            | 0.125                  | -24.30 | 0.053                        | 46.96 | 0.113                      | -12.07 |
| 84  | 40.5                | 0.116            | 0.133                  | -14.29 | 0.058                        | 50.55 | 0.116                      | 0.00   |
| 85  | 20.3                | 0.058            | 0.054                  | 6.51   | 0.025                        | 56.65 | 0.061                      | -4.54  |
| 86  | 21.1                | 0.061            | 0.055                  | 9.54   | 0.025                        | 58.02 | 0.061                      | 0.00   |
| 87  | 19.6                | 0.056            | 0.057                  | -1.68  | 0.027                        | 51.67 | 0.062                      | -10.10 |
| 88  | 27.9                | 0.080            | 0.092                  | -14.27 | 0.042                        | 47.47 | 0.087                      | -8.61  |
| 89  | 28.3                | 0.081            | 0.092                  | -12.80 | 0.041                        | 49.88 | 0.089                      | -9.48  |
| 90  | 45.4                | 0.131            | 0.135                  | -3.13  | 0.059                        | 55.10 | 0.117                      | 10.41  |
| 91  | 23.9                | 0.122            | 0.109                  | 11.27  | 0.089                        | 27.16 | 0.162                      | -32.72 |
| 92  | 23.1                | 0.118            | 0.109                  | 8.20   | 0.088                        | 25.82 | 0.164                      | -38.67 |
| 93  | 27.2                | 0.140            | 0.112                  | 19.91  | 0.092                        | 34.42 | 0.166                      | -18.92 |
| 94  | 43.7                | 0.224            | 0.155                  | 30.80  | 0.119                        | 47.14 | 0.216                      | 3.45   |
| 95  | 75.9                | 0.389            | 0.287                  | 26.31  | 0.210                        | 46.09 | 0.337                      | 13.35  |
| 96  | 27.2                | 0.139            | 0.111                  | 20.62  | 0.093                        | 33.48 | 0.162                      | -16.54 |
| 97  | 26.1                | 0.134            | 0.111                  | 17.26  | 0.090                        | 32.76 | 0.166                      | -23.72 |
| 98  | 24.6                | 0.126            | 0.114                  | 9.59   | 0.091                        | 27.49 | 0.170                      | -35.10 |
| 99  | 41.6                | 0.213            | 0.169                  | 20.78  | 0.135                        | 36.87 | 0.223                      | -4.72  |
| 100 | 84.6                | 0.434            | 0.299                  | 31.05  | 0.223                        | 48.59 | 0.343                      | 20.98  |

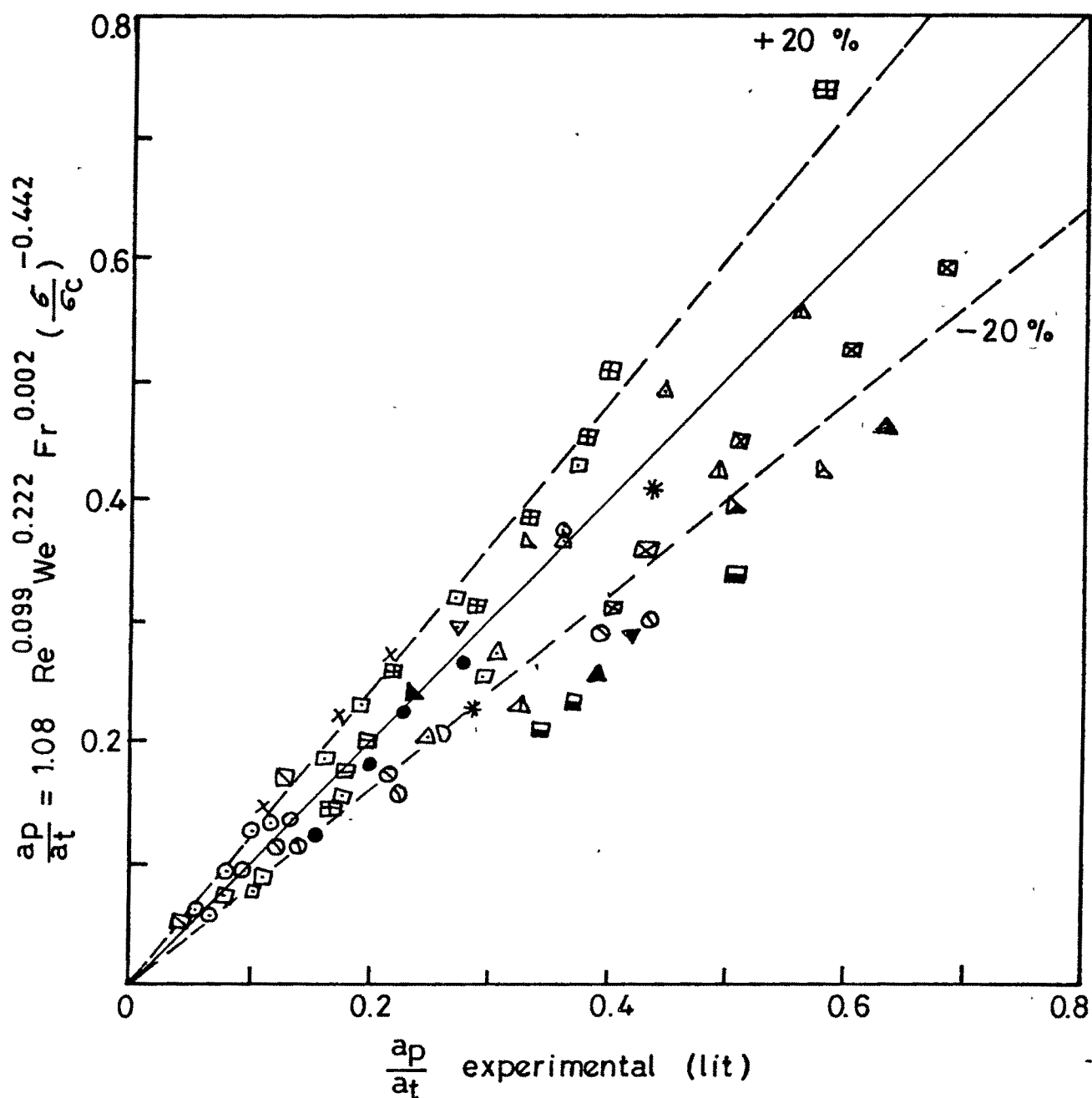


Fig. 7.2 EFFECTIVE INTERFACIAL AREA DURING PHYSICAL ABSORPTION.  
COMPARISON OF PREDICTED vs. EXPERIMENTAL (LIT.) DATA.

## LEGENDS FOR FIG. 7.2

| Data No. | Relevant Details                                                                                     | Symbol                            |
|----------|------------------------------------------------------------------------------------------------------|-----------------------------------|
| 1-12     | Fellinger's data (14), $\text{NH}_3$ - Water,<br>C.R.R., 25mm, 38mm, 50mm.                           | $\Delta \triangle \blacktriangle$ |
| 13-16    | Fellinger's data (14), $\text{NH}_3$ - Water,<br>Ceramic Berl Saddle, 25mm, 38mm.                    | D ✕                               |
| 17-36    | Yoshida (15), $\text{CO}_2$ - Water,<br>C.R.R., 25mm, 15mm.                                          | $\square \boxtimes$               |
| 37-39    | Yoshida (15), $\text{CO}_2$ - Methanol,<br>C.R.R., 25mm, $\sigma = 23 \text{ mN/m}$ .                | X                                 |
| 40-50    | Hikita (16), $\text{CO}_2$ - Water,<br>C.R.R., 15mm, 25mm.                                           | $\boxplus \boxtimes$              |
| 51-64    | Hikita (16), $\text{CO}_2$ - Methanol,<br>C.R.R., 15mm, 25mm.                                        | $\boxplus \boxtimes$              |
| 65-66    | Hikita (16), $\text{CO}_2$ - 60 % Methanol,<br>$\sigma = 30 \text{ mN/m}$ , C.R.R., 15mm, 25mm.      | $\triangleleft \triangle$         |
| 67-68    | Hikita (16), $\text{CO}_2$ - 30 % Methanol,<br>$\sigma = 40 \text{ mN/m}$ , C.R.R., 15mm, 25mm.      | $\nabla \blacktriangle$           |
| 69-70    | Hikita (16), $\text{CO}_2$ - 10 % Methanol,<br>$\sigma = 55 \text{ mN/m}$ , C.R.R., 15mm, 25mm.      | $\blacktriangle \nabla$           |
| 71-78    | Hikita (16), $\text{CO}_2$ - 5 to 10 % cane sugar,<br>$\mu_L = 3.5 \text{ mNs/m}^2$ ., C.R.R., 15mm. | ●                                 |
| 79-100   | Yoshida (17), Methanol vapour - water,<br>C.R.R., 25mm, 25 mm.                                       | O O                               |

wide range of systems/variations. Therefore, the values of  $(a_p/a_t)$  obtained by using the correlations of Zech and Mersmann are also reported in Table (7.2.3) for comparison purpose. The detailed statistical analysis of  $(a_p/a_t)$  values obtained by this correlation can be described as follows :

(i) The values of %  $E_{avg}$ , %  $E_{abs}$ , %  $S_{dev}$  were 42.9, 42.9, and 7.83 respectively.

(ii) Out of 100 data points only 24 % of the predicted values of  $a_p$  were within  $\pm 30$  % of the experimental data values.

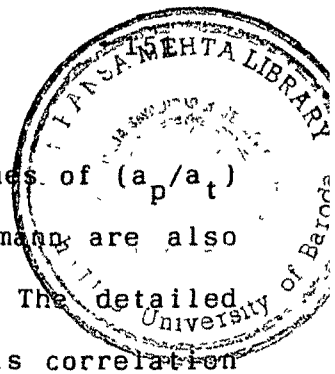
(iii) From 100 data points, 76 points had deviations above  $\pm 30$ % and the maximum deviation observed was 75 %.

The recently proposed correlation by Billet and Schultes (105) [Equation (2.9)] also appears to be a generalised correlation. The values of  $(a_p/a_t)$  obtained by the correlation of Billet and Schultes are also reported in Table (7.2.3). The detailed statistical analysis of  $(a_p/a_t)$  values obtained by the correlation of Billet and Schultes can be described as follows :

(i) The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were - 6.3, 17.4 and 5.25 respectively.

(ii) Out of 100 data points 79 % of predicted values of  $a_p$  were within  $\pm 25$  % of the experimental data values and 65% were within  $\pm 15$  %.

(iii) From 100 data points, 13 points had deviations greater than  $\pm 30$  %, and 6 points had deviations greater than  $\pm 50$  %. Further the maximum deviation observed was -103.7 %.



The comparison of detailed statistical analysis for the three correlations under consideration reveals that the values of  $\% E_{abs}$ , for the correlation of Zech & Mersmann, Billet and Schultes and for the correlation developed in this investigation are 42.9, 17.4 and 15.66 respectively. Hence it can be concluded that the generalised correlation developed in this investigation is expected to be superior in comparison to the existing generalised correlations of Zech and Mersmann. The statistical analysis for the correlation of Billet and Schultes and that for the correlation developed in this investigation is comparable, exception being the value of maximum deviation observed in both this correlations. Since the range of maximum deviation values for the correlation developed in this investigation is of the order  $\pm 30\%$  to  $\pm 40\%$  and that of Billet's correlation is  $\pm 50$  to  $-103.7\%$ , the correlation proposed in this investigation [Equation (7.7)] appears to be better than that proposed by Billet and Schultes.

### 7.3.0 RESULTS AND DISCUSSION FOR EFFECTIVE INTERFACIAL AREA DURING CHEMICAL ABSORPTION ( $a_c$ ) :

#### 7.3.1 Critical analysis of data and mathematical modelling :

A data bank consisting of 167 data points covering 32 different variations inclusive of system variations like  $\text{CO}_2$ -NaOH  $\text{CO}_2$ -potassium buffer solutions, sulphite oxidation, dithionite oxidation etc. reported in Table - (5.3) could be conveniently used for analysing the effect of different variables on the values of  $a_c$ .

The parameters that affect  $a_c$  are the liquid flow rate ( $L$ ) the physical properties of the absorption media like density ( $\rho_L$ ) viscosity ( $\mu_L$ ), surface tension ( $\sigma$ ), surface area of packing ( $a_t$ ) and the critical surface tension of the packing material ( $\sigma_c$ ).

The values of  $a_c$  increase with an increase in  $L$  and decrease with an increase in  $\rho_L$ ,  $\mu_L$ , and  $\sigma$ . The data of Danckwerts et al. (19) and Richards et al. (21) indicate that under otherwise similar conditions with an increase in the viscosity by 30 %, the value of  $a_c$  decreases by approximately by 10 %. Similar observations can be made from the data of Rizzuti and coworkers (37). The data of Jhaveri (25,26) indicates only a marginal decrease in the values of  $a_c$  with a two fold increase in the value of viscosity. It appears that  $(\sigma / \sigma_c)$  is the dominant parameter affecting the values of  $a_c$ . The data of Sahay (31), Andrieu and Claudel (32), Andrieu (34), and Linek (33) indicate that with a two fold increase in  $(\sigma / \sigma_c)$  value, the values of  $a_c$  decrease appreciably.

On the whole, it is thus observed that the parameters affecting the value of  $a_c$  and those affecting the values of  $a_w$  and  $a_p$  are identical. Further, the effect of these parameters on the values of  $a_c$  and that on the values of  $a_w$  and  $a_p$  also appears to be similar in nature but different in magnitude.

Hence, with the help of dimensional analysis, an equation similar to that for  $(a_w/a_t)$  can be proposed. However, it is expected that the indices of the various dimensionless numbers will be significantly different than those obtained for the correlations

of  $(a_w/a_t)$  and  $(a_p/a_t)$ . The relevant dimensionless groups [calculated using the data of  $a_c$  reported in Table (5.3)] required for processing the data of  $a_c$  are tabulated in Table (7.3.1).

### 7.3.2 Statistical analysis for different correlations :

Processing of the data reported in Table (7.3.1) was also done by three methodologies namely (i) Multiple linear regression, (ii) Optimization by the DSC - Powell algorithm and (iii) Optimization by the modified simplex algorithm of Nelder and Mead. The results obtained by using these three methodologies are reported in Table (7.3.2) and are discussed herewith.

CASE I : Multiple linear regression using the Gauss Jordan and Gauss Seidel methods gave practically comparable statistics as could be seen from the regression steps (1) to (3), wherein the values of  $E_{avg}$ ,  $E_{abs}$  and  $S_{dev}$  were of the order -0.5 %, 8.8 %, 4.8 % respectively.

CASE II : It was observed that using the methodology of optimization by the DSC - Powell algorithm, by changing the direction of search sequence some decrease in the values %  $E_{abs}$  and %  $E_{avg}$  was obtained. The values of  $E_{abs}$  and  $E_{avg}$  for the regression steps (7) and (6) were (10.4 % and - 0.33 %) and (9.7 % and 0.086 %) respectively.

CASE III : Using the modified simplex algorithm of Nelder and Mead by performing regression as per step (9) to (13) which also included

Table - ( 7.3.1 )

Results inclusive of processing parameters for  
effective interfacial area during chemical absorption (  $a_c$  )

| No. | $a_c$<br>$\frac{m^2}{m^3}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_c/a_t$<br>exp | $a_c/a_t$<br>pred | % err  |
|-----|----------------------------|-------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 1   | 106.0                      | 3.09  | 8.387               | 7.988               | 1.230             | 0.286            | 0.266             | 7.02   |
| 2   | 134.0                      | 4.45  | 6.821               | 6.806               | 1.148             | 0.362            | 0.308             | 14.84  |
| 3   | 141.0                      | 8.66  | 25.860              | 25.804              | 1.148             | 0.381            | 0.389             | -1.98  |
| 4   | 154.0                      | 13.04 | 58.732              | 58.603              | 1.148             | 0.416            | 0.448             | -7.66  |
| 5   | 82.0                       | 1.92  | 1.969               | 1.957               | 1.213             | 0.222            | 0.222             | 0.00   |
| 6   | 97.0                       | 3.86  | 7.986               | 7.938               | 1.213             | 0.262            | 0.284             | -8.19  |
| 7   | 120.0                      | 5.75  | 17.722              | 17.615              | 1.213             | 0.324            | 0.326             | -0.42  |
| 8   | 127.5                      | 7.73  | 31.944              | 31.752              | 1.213             | 0.345            | 0.361             | -4.69  |
| 9   | 144.5                      | 9.65  | 49.781              | 49.481              | 1.213             | 0.391            | 0.390             | 0.24   |
| 10  | 63.0                       | 29.14 | 207.594             | 25.450              | 1.230             | 0.485            | 0.535             | -10.46 |
| 11  | 78.0                       | 37.98 | 352.787             | 43.250              | 1.230             | 0.600            | 0.587             | 2.19   |
| 12  | 92.7                       | 60.37 | 891.107             | 109.244             | 1.230             | 0.713            | 0.689             | 3.35   |
| 13  | 110.0                      | 75.96 | 1411.149            | 172.998             | 1.230             | 0.846            | 0.746             | 11.78  |
| 14  | 119.0                      | 87.14 | 1856.774            | 227.629             | 1.230             | 0.915            | 0.783             | 14.48  |
| 15  | 128.0                      | 0.62  | 15.881              | 12.123              | 0.902             | 0.272            | 0.270             | 0.73   |
| 16  | 156.0                      | 1.11  | 51.020              | 38.948              | 0.902             | 0.332            | 0.331             | 0.27   |
| 17  | 172.0                      | 1.61  | 108.211             | 82.608              | 0.902             | 0.366            | 0.377             | -3.06  |
| 18  | 126.0                      | 2.58  | 6.204               | 9.530               | 1.200             | 0.268            | 0.258             | 3.71   |
| 19  | 142.0                      | 4.02  | 15.112              | 23.216              | 1.200             | 0.302            | 0.301             | 0.29   |
| 20  | 154.0                      | 4.90  | 22.448              | 34.486              | 1.200             | 0.328            | 0.323             | 1.52   |
| 21  | 168.0                      | 6.16  | 35.465              | 54.485              | 1.200             | 0.357            | 0.349             | 2.27   |
| 22  | 179.0                      | 7.35  | 50.437              | 77.487              | 1.200             | 0.381            | 0.371             | 2.50   |
| 23  | 187.0                      | 8.61  | 69.293              | 106.455             | 1.200             | 0.398            | 0.392             | 1.38   |
| 24  | 190.0                      | 9.97  | 92.936              | 142.777             | 1.200             | 0.404            | 0.413             | -2.13  |
| 25  | 141.0                      | 2.41  | 5.434               | 8.348               | 1.200             | 0.300            | 0.252             | 15.91  |
| 26  | 178.0                      | 4.81  | 21.599              | 33.182              | 1.200             | 0.379            | 0.321             | 15.37  |
| 27  | 206.0                      | 7.28  | 49.461              | 75.986              | 1.200             | 0.438            | 0.370             | 15.57  |
| 28  | 143.0                      | 2.54  | 6.033               | 9.268               | 1.200             | 0.304            | 0.257             | 15.57  |
| 29  | 181.0                      | 5.00  | 23.393              | 35.938              | 1.200             | 0.385            | 0.325             | 15.61  |
| 30  | 208.0                      | 7.54  | 53.112              | 81.596              | 1.200             | 0.443            | 0.375             | 15.34  |
| 31  | 220.0                      | 9.76  | 89.013              | 136.751             | 1.200             | 0.468            | 0.410             | 12.45  |
| 32  | 74.5                       | 11.80 | 52.599              | 13.206              | 1.200             | 0.392            | 0.413             | -5.34  |
| 33  | 101.0                      | 24.46 | 225.987             | 56.738              | 1.200             | 0.532            | 0.532             | 0.00   |
| 34  | 127.0                      | 2.36  | 6.791               | 9.788               | 1.049             | 0.270            | 0.295             | -9.17  |
| 35  | 134.0                      | 2.80  | 9.598               | 13.833              | 1.049             | 0.285            | 0.313             | -9.87  |
| 36  | 156.0                      | 4.24  | 21.970              | 31.663              | 1.049             | 0.332            | 0.362             | -8.96  |
| 37  | 168.0                      | 5.66  | 39.141              | 56.409              | 1.049             | 0.357            | 0.400             | -11.84 |
| 38  | 180.0                      | 6.63  | 53.671              | 77.349              | 1.049             | 0.383            | 0.422             | -10.26 |
| 39  | 131.0                      | 1.67  | 7.838               | 10.083              | 1.216             | 0.279            | 0.234             | 16.17  |
| 40  | 160.0                      | 3.40  | 32.440              | 41.736              | 1.216             | 0.340            | 0.299             | 12.18  |
| 41  | 183.0                      | 4.78  | 64.200              | 82.598              | 1.216             | 0.389            | 0.337             | 13.56  |
| 42  | 185.0                      | 6.11  | 104.711             | 134.717             | 1.216             | 0.394            | 0.366             | 6.92   |
| 43  | 171.0                      | 4.22  | 52.421              | 67.443              | 1.216             | 0.364            | 0.323             | 11.19  |
| 44  | 164.0                      | 3.32  | 32.440              | 41.736              | 1.216             | 0.349            | 0.297             | 14.80  |
| 45  | 142.0                      | 1.97  | 11.416              | 14.688              | 1.216             | 0.302            | 0.248             | 17.91  |
| 46  | 136.0                      | 5.15  | 13.258              | 20.147              | 1.180             | 0.289            | 0.322             | -11.25 |

Table - 7.3.1 ( contd.)

| No. | $a_c$<br>$\frac{m^2}{m^3}$ | Re     | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_c/a_t$<br>exp | $a_c/a_t$<br>pred | % err  |
|-----|----------------------------|--------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 47  | 180.0                      | 12.18  | 74.243              | 112.821             | 1.180             | 0.383            | 0.434             | -13.34 |
| 48  | 140.0                      | 4.26   | 13.374              | 19.184              | 1.148             | 0.298            | 0.318             | -6.82  |
| 49  | 152.0                      | 5.87   | 25.469              | 36.533              | 1.148             | 0.323            | 0.356             | -10.02 |
| 50  | 180.0                      | 8.98   | 59.542              | 85.408              | 1.148             | 0.383            | 0.412             | -7.66  |
| 51  | 147.0                      | 5.13   | 13.945              | 22.159              | 1.167             | 0.313            | 0.327             | -4.46  |
| 52  | 160.0                      | 6.52   | 22.489              | 35.736              | 1.167             | 0.340            | 0.355             | -4.27  |
| 53  | 175.0                      | 9.26   | 45.418              | 72.170              | 1.167             | 0.372            | 0.401             | -7.70  |
| 54  | 181.0                      | 11.22  | 66.671              | 105.942             | 1.167             | 0.385            | 0.429             | -11.30 |
| 55  | 138.0                      | 3.78   | 9.131               | 14.688              | 1.189             | 0.294            | 0.291             | 0.77   |
| 56  | 147.0                      | 5.62   | 20.147              | 32.406              | 1.189             | 0.313            | 0.334             | -6.86  |
| 57  | 173.0                      | 7.45   | 35.384              | 56.914              | 1.189             | 0.368            | 0.369             | -0.12  |
| 58  | 182.0                      | 11.78  | 88.556              | 142.440             | 1.189             | 0.387            | 0.432             | -11.59 |
| 59  | 64.0                       | 34.19  | 283.004             | 33.224              | 1.213             | 0.492            | 0.574             | -16.57 |
| 60  | 82.0                       | 44.43  | 478.007             | 56.117              | 1.213             | 0.631            | 0.629             | 0.36   |
| 61  | 105.0                      | 64.92  | 1020.460            | 119.801             | 1.213             | 0.808            | 0.717             | 11.24  |
| 62  | 127.0                      | 95.65  | 2215.260            | 260.068             | 1.213             | 0.977            | 0.820             | 16.05  |
| 63  | 145.0                      | 146.87 | 5222.913            | 613.163             | 1.213             | 1.115            | 0.952             | 14.68  |
| 64  | 79.2                       | 1.48   | 1.075               | 0.817               | 1.200             | 0.240            | 0.205             | 14.77  |
| 65  | 105.6                      | 4.52   | 10.041              | 7.633               | 1.200             | 0.320            | 0.301             | 5.80   |
| 66  | 125.4                      | 7.77   | 29.686              | 22.567              | 1.200             | 0.380            | 0.364             | 4.26   |
| 67  | 155.1                      | 15.57  | 119.173             | 90.595              | 1.200             | 0.470            | 0.463             | 1.49   |
| 68  | 178.2                      | 28.62  | 402.524             | 305.999             | 1.200             | 0.540            | 0.572             | -5.90  |
| 69  | 198.0                      | 46.58  | 1066.123            | 810.469             | 1.200             | 0.600            | 0.677             | -12.86 |
| 70  | 121.8                      | 50.68  | 776.314             | 223.321             | 1.200             | 0.600            | 0.676             | -12.66 |
| 71  | 148.1                      | 101.35 | 3105.254            | 893.286             | 1.200             | 0.730            | 0.860             | -17.84 |
| 72  | 76.0                       | 1.11   | 0.869               | 0.878               | 1.230             | 0.205            | 0.184             | 10.31  |
| 73  | 114.0                      | 2.39   | 4.053               | 4.095               | 1.230             | 0.308            | 0.241             | 21.89  |
| 74  | 133.0                      | 4.94   | 17.281              | 17.459              | 1.230             | 0.359            | 0.310             | 13.89  |
| 75  | 148.2                      | 6.76   | 32.326              | 32.660              | 1.230             | 0.401            | 0.345             | 13.85  |
| 76  | 171.0                      | 9.88   | 69.124              | 69.837              | 1.230             | 0.462            | 0.394             | 14.82  |
| 77  | 114.0                      | 2.40   | 4.629               | 4.431               | 1.213             | 0.308            | 0.246             | 20.07  |
| 78  | 125.4                      | 3.89   | 12.208              | 11.685              | 1.213             | 0.339            | 0.291             | 14.02  |
| 79  | 144.4                      | 6.86   | 37.952              | 36.326              | 1.213             | 0.390            | 0.355             | 9.10   |
| 80  | 163.4                      | 11.47  | 106.052             | 101.509             | 1.213             | 0.442            | 0.424             | 3.99   |
| 81  | 182.4                      | 19.86  | 318.190             | 304.559             | 1.213             | 0.493            | 0.513             | -4.07  |
| 82  | 190.0                      | 24.57  | 486.973             | 466.112             | 1.213             | 0.514            | 0.552             | -7.56  |
| 83  | 66.0                       | 10.07  | 24.080              | 6.280               | 1.189             | 0.347            | 0.384             | -10.63 |
| 84  | 93.0                       | 17.57  | 73.277              | 19.111              | 1.189             | 0.489            | 0.466             | 4.77   |
| 85  | 100.0                      | 26.15  | 150.664             | 39.294              | 1.189             | 0.526            | 0.533             | -1.21  |
| 86  | 108.0                      | 31.36  | 216.800             | 56.543              | 1.189             | 0.568            | 0.567             | 0.18   |
| 87  | 132.0                      | 38.50  | 364.254             | 95.000              | 1.189             | 0.695            | 0.613             | 11.73  |
| 88  | 130.0                      | 57.74  | 819.572             | 213.750             | 1.189             | 0.684            | 0.706             | -3.16  |
| 89  | 130.0                      | 44.82  | 493.698             | 128.760             | 1.189             | 0.684            | 0.646             | 5.52   |
| 90  | 27.0                       | 8.35   | 17.979              | 4.657               | 2.599             | 0.142            | 0.153             | -7.34  |
| 91  | 36.0                       | 15.84  | 64.677              | 16.752              | 2.599             | 0.189            | 0.190             | -0.53  |
| 92  | 169.0                      | 2.47   | 6.555               | 9.738               | 1.027             | 0.367            | 0.305             | 17.10  |
| 93  | 206.0                      | 3.72   | 14.811              | 22.004              | 1.027             | 0.448            | 0.351             | 21.66  |
| 94  | 227.0                      | 4.95   | 26.219              | 38.953              | 1.027             | 0.493            | 0.387             | 21.50  |
| 95  | 258.0                      | 6.18   | 40.863              | 60.709              | 1.027             | 0.561            | 0.418             | 25.41  |
| 96  | 268.0                      | 7.42   | 58.992              | 87.644              | 1.027             | 0.583            | 0.446             | 23.47  |
| 97  | 284.0                      | 9.89   | 104.875             | 155.811             | 1.027             | 0.617            | 0.493             | 20.20  |

Table - 7.3.1 ( contd.)

| No. | $a_c$<br>$\frac{2}{m} \frac{c}{m^3}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_c/a_t$<br>exp | $a_c/a_t$<br>pred | % err  |
|-----|--------------------------------------|-------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 98  | 72.6                                 | 1.31  | 1.084               | 0.842               | 1.215             | 0.220            | 0.196             | 10.77  |
| 99  | 107.3                                | 4.18  | 11.097              | 8.620               | 1.215             | 0.325            | 0.294             | 9.57   |
| 100 | 136.0                                | 8.36  | 44.389              | 34.482              | 1.215             | 0.412            | 0.374             | 9.30   |
| 101 | 208.0                                | 16.24 | 167.698             | 130.269             | 1.215             | 0.630            | 0.471             | 25.31  |
| 102 | 24.0                                 | 1.31  | 1.084               | 0.842               | 2.675             | 0.073            | 0.082             | -12.90 |
| 103 | 43.0                                 | 4.18  | 11.097              | 8.620               | 2.675             | 0.130            | 0.123             | 5.65   |
| 104 | 49.5                                 | 8.36  | 44.389              | 34.482              | 2.675             | 0.150            | 0.156             | -4.25  |
| 105 | 65.4                                 | 16.24 | 167.698             | 130.269             | 2.675             | 0.198            | 0.197             | 0.63   |
| 106 | 33.0                                 | 1.31  | 1.084               | 0.842               | 2.245             | 0.100            | 0.100             | 0.00   |
| 107 | 46.5                                 | 4.18  | 11.097              | 8.620               | 2.245             | 0.141            | 0.149             | -5.85  |
| 108 | 59.6                                 | 8.36  | 44.389              | 34.482              | 2.245             | 0.181            | 0.190             | -5.04  |
| 109 | 81.0                                 | 16.24 | 167.698             | 130.269             | 2.245             | 0.245            | 0.239             | 2.66   |
| 110 | 107.5                                | 32.64 | 677.320             | 526.148             | 2.245             | 0.326            | 0.304             | 6.56   |
| 111 | 92.7                                 | 3.68  | 14.536              | 21.596              | 1.641             | 0.201            | 0.209             | -3.49  |
| 112 | 107.4                                | 4.91  | 25.803              | 38.335              | 1.641             | 0.234            | 0.230             | 1.37   |
| 113 | 121.9                                | 6.17  | 40.825              | 60.653              | 1.641             | 0.265            | 0.249             | 5.86   |
| 114 | 131.8                                | 7.46  | 59.560              | 88.487              | 1.641             | 0.286            | 0.266             | 7.05   |
| 115 | 60.7                                 | 1.43  | 1.230               | 0.860               | 1.215             | 0.194            | 0.202             | -4.04  |
| 116 | 92.0                                 | 4.29  | 11.074              | 7.739               | 1.215             | 0.294            | 0.295             | -0.51  |
| 117 | 112.7                                | 8.57  | 44.295              | 30.955              | 1.215             | 0.360            | 0.376             | -4.40  |
| 118 | 102.1                                | 8.88  | 45.909              | 29.867              | 1.215             | 0.338            | 0.380             | -12.34 |
| 119 | 140.4                                | 15.99 | 148.745             | 96.770              | 1.215             | 0.465            | 0.466             | -0.12  |
| 120 | 153.4                                | 23.26 | 314.700             | 204.737             | 1.215             | 0.508            | 0.530             | -4.38  |
| 121 | 35.5                                 | 2.76  | 4.735               | 3.524               | 2.675             | 0.110            | 0.106             | 3.28   |
| 122 | 43.6                                 | 4.85  | 14.606              | 10.870              | 2.675             | 0.135            | 0.129             | 4.25   |
| 123 | 33.6                                 | 1.53  | 1.319               | 0.802               | 2.245             | 0.115            | 0.104             | 9.22   |
| 124 | 44.3                                 | 4.59  | 11.870              | 7.220               | 2.245             | 0.152            | 0.153             | -0.81  |
| 125 | 57.8                                 | 9.19  | 47.481              | 28.878              | 2.245             | 0.198            | 0.195             | 1.73   |
| 126 | 28.2                                 | 1.54  | 1.323               | 0.799               | 2.245             | 0.097            | 0.105             | -7.90  |
| 127 | 50.6                                 | 9.22  | 47.644              | 28.779              | 2.245             | 0.174            | 0.195             | -11.98 |
| 128 | 59.3                                 | 16.60 | 154.367             | 93.246              | 2.245             | 0.204            | 0.239             | -17.17 |
| 129 | 81.5                                 | 24.14 | 326.596             | 197.280             | 2.245             | 0.280            | 0.272             | 2.91   |
| 130 | 28.0                                 | 1.44  | 1.242               | 0.852               | 2.245             | 0.090            | 0.103             | -13.71 |
| 131 | 56.0                                 | 8.65  | 44.724              | 30.659              | 2.245             | 0.181            | 0.191             | -5.88  |
| 132 | 77.5                                 | 15.58 | 144.906             | 99.334              | 2.245             | 0.250            | 0.235             | 6.19   |
| 133 | 93.0                                 | 22.66 | 306.579             | 210.161             | 2.245             | 0.300            | 0.267             | 10.97  |
| 134 | 23.6                                 | 1.55  | 1.337               | 0.791               | 2.245             | 0.082            | 0.105             | -27.98 |
| 135 | 53.0                                 | 9.32  | 48.141              | 28.483              | 2.245             | 0.184            | 0.195             | -6.12  |
| 136 | 67.0                                 | 16.77 | 155.975             | 92.284              | 2.245             | 0.233            | 0.239             | -2.94  |
| 137 | 51.4                                 | 1.53  | 1.319               | 0.802               | 1.372             | 0.176            | 0.180             | -2.21  |
| 138 | 77.1                                 | 4.59  | 11.870              | 7.220               | 1.372             | 0.264            | 0.263             | 0.21   |
| 139 | 97.8                                 | 9.19  | 47.481              | 28.878              | 1.372             | 0.335            | 0.335             | 0.00   |
| 140 | 44.5                                 | 1.54  | 1.323               | 0.799               | 1.372             | 0.153            | 0.180             | -17.77 |
| 141 | 137.9                                | 16.60 | 154.367             | 93.246              | 1.372             | 0.474            | 0.411             | 13.22  |
| 142 | 116.4                                | 24.14 | 326.596             | 197.280             | 1.372             | 0.400            | 0.468             | -17.09 |
| 143 | 114.0                                | 8.65  | 44.724              | 30.659              | 1.372             | 0.368            | 0.329             | 10.42  |
| 144 | 137.6                                | 15.58 | 144.906             | 99.334              | 1.372             | 0.444            | 0.404             | 8.99   |
| 145 | 140.7                                | 22.69 | 307.289             | 210.647             | 1.372             | 0.454            | 0.460             | -1.40  |
| 146 | 95.0                                 | 9.32  | 48.141              | 28.483              | 1.372             | 0.330            | 0.336             | -1.97  |
| 147 | 105.7                                | 16.77 | 155.975             | 92.284              | 1.372             | 0.367            | 0.412             | -12.38 |

Table - 7.3.1 ( contd.)

| No. | $a_c$<br>$\frac{m^2}{m^3}$ | Re    | We<br>$\times 10^5$ | Fr<br>$\times 10^5$ | $\sigma/\sigma_c$ | $a_c/a_t$<br>exp | $a_c/a_t$<br>pred | % err  |
|-----|----------------------------|-------|---------------------|---------------------|-------------------|------------------|-------------------|--------|
| 148 | 109.7                      | 8.44  | 43.599              | 31.450              | 1.372             | 0.345            | 0.327             | 5.19   |
| 149 | 135.0                      | 15.19 | 141.261             | 101.897             | 1.372             | 0.425            | 0.401             | 5.53   |
| 150 | 150.0                      | 22.09 | 298.866             | 215.584             | 1.372             | 0.472            | 0.457             | 3.17   |
| 151 | 43.7                       | 3.91  | 9.975               | 5.593               | 2.290             | 0.151            | 0.143             | 5.22   |
| 152 | 52.8                       | 7.70  | 38.704              | 21.703              | 2.290             | 0.182            | 0.181             | 0.91   |
| 153 | 61.4                       | 11.99 | 93.961              | 52.689              | 2.290             | 0.212            | 0.211             | 0.57   |
| 154 | 71.2                       | 14.99 | 146.814             | 82.327              | 2.290             | 0.246            | 0.228             | 7.33   |
| 155 | 199.0                      | 16.16 | 259.931             | 267.577             | 1.068             | 0.510            | 0.557             | -9.07  |
| 156 | 175.5                      | 10.84 | 116.943             | 120.383             | 1.068             | 0.450            | 0.485             | -7.67  |
| 157 | 144.0                      | 5.91  | 34.793              | 35.816              | 1.068             | 0.369            | 0.393             | -6.33  |
| 158 | 123.0                      | 3.35  | 11.181              | 11.510              | 1.068             | 0.315            | 0.322             | -2.23  |
| 159 | 215.8                      | 14.72 | 312.465             | 315.223             | 1.068             | 0.553            | 0.551             | 0.44   |
| 160 | 201.0                      | 11.75 | 198.856             | 200.611             | 1.068             | 0.515            | 0.509             | 1.17   |
| 161 | 171.0                      | 7.11  | 72.939              | 73.583              | 1.068             | 0.438            | 0.428             | 2.38   |
| 162 | 151.0                      | 4.96  | 35.503              | 35.816              | 1.068             | 0.387            | 0.378             | 2.44   |
| 163 | 132.8                      | 3.14  | 14.241              | 14.366              | 1.068             | 0.341            | 0.322             | 5.33   |
| 164 | 161.0                      | 5.32  | 29.843              | 43.396              | 1.008             | 0.345            | 0.406             | -17.64 |
| 165 | 176.0                      | 6.22  | 40.786              | 59.309              | 1.008             | 0.377            | 0.428             | -13.61 |
| 166 | 188.0                      | 7.12  | 53.297              | 77.503              | 1.008             | 0.403            | 0.449             | -11.41 |
| 167 | 204.0                      | 8.89  | 83.277              | 121.098             | 1.008             | 0.437            | 0.485             | -10.94 |

Table - ( 7.3.2 )

Statistical analysis for different correlations of  $a_c$ Correlation Structure :  $a_c / a_i = C \cdot Re^\alpha \cdot We^\beta \cdot Fr^\gamma \cdot (\sigma / \sigma_c)^\delta$ 

| No. | C | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>av</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|----------|-------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|----------|-------------------|--------------------|--------------------|

Case 1 : - Multiple Linear Regression

Numerical Method : Gauss Jordan algorithm.

|   |      |       |       |          |        |       |      |       |
|---|------|-------|-------|----------|--------|-------|------|-------|
| 1 | 1.57 | 0.194 | 0.075 | 3.39E-06 | -1.159 | -0.47 | 8.11 | 4.88. |
|---|------|-------|-------|----------|--------|-------|------|-------|

Numerical Method : Gauss Seidel algorithm.

Convergence criteria  $\epsilon = 0.001$ 

|   |       |        |        |        |        |        |      |        |
|---|-------|--------|--------|--------|--------|--------|------|--------|
| 2 | 0.589 | 0.1918 | 0.0698 | 0.0087 | -1.148 | -0.645 | 8.83 | 4.938. |
|---|-------|--------|--------|--------|--------|--------|------|--------|

Convergence criteria  $\epsilon = 0.0001$ 

|   |       |        |        |       |        |       |      |       |
|---|-------|--------|--------|-------|--------|-------|------|-------|
| 3 | 0.573 | 0.1935 | 0.0745 | 0.001 | -1.158 | -0.59 | 8.83 | 4.89. |
|---|-------|--------|--------|-------|--------|-------|------|-------|

Case 2 :- Optimisation by the D.S.C. - Powell algorithm.

Initialisation were based on arbitrary guesses of indices,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , and C were fed as 0.2, 0.07, 0.002, -1.1, 0.6. ( Clue for guesses were taken from the values obtained in previous steps )

Direction of search sequence  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\gamma$ , C. respectively.

|   |        |        |       |       |        |      |       |       |
|---|--------|--------|-------|-------|--------|------|-------|-------|
| 4 | 0.5186 | 0.2205 | 0.035 | 0.029 | -1.314 | -0.6 | 10.24 | 5.21. |
|---|--------|--------|-------|-------|--------|------|-------|-------|

Direction of search sequence changed to C,  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no 4.

|   |       |        |        |       |        |       |      |      |
|---|-------|--------|--------|-------|--------|-------|------|------|
| 5 | 0.516 | 0.2079 | 0.0393 | 0.021 | -1.263 | -1.02 | 9.76 | 5.2. |
|---|-------|--------|--------|-------|--------|-------|------|------|

Direction of search sequence changed to C,  $\beta$ ,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no 4.  $\gamma$  value fixed as 0.002.

|   |       |       |       |       |        |       |     |      |
|---|-------|-------|-------|-------|--------|-------|-----|------|
| 6 | 0.400 | 0.244 | 0.036 | 0.002 | -1.293 | 0.086 | 9.7 | 5.04 |
|---|-------|-------|-------|-------|--------|-------|-----|------|

Direction of search sequence changed to  $\delta$ , C,  $\beta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no. 4.

|   |       |        |        |        |        |       |       |       |
|---|-------|--------|--------|--------|--------|-------|-------|-------|
| 7 | 0.517 | 0.2209 | 0.0355 | 0.0276 | -1.333 | -0.33 | 10.41 | 5.22. |
|---|-------|--------|--------|--------|--------|-------|-------|-------|

Table - 7.3.2 ( contd.)

| No. | C | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | % E <sub>av</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|----------|-------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|----------|-------------------|--------------------|--------------------|

Direction of search sequence changed to  $\beta$ , C,  $\delta$ ,  $\alpha$ ,  $\gamma$ . Initialisation as in step no. 4.

8    0.515    0.23    0.042    0.0238    -1.3275    0.74    10.37    5.12.

Case 3 : -      Optimisation by the Modified Simplex algorithm of Nelder and Mead

All parameters are iterated, base values as obtained from step no 3. Each indice value perturbed 50 % at a time.

9    0.6212    0.1807    0.074    0.013    -1.09    1.57    8.51    5.04.

$\beta$  value fixed as 0.002 . Other parameters iterated as in step 9.

10\*    0.455    0.227    0.058    0.002    -1.104    1.38    8.25    4.896.

$\gamma$  and  $\delta$  values fixed as 0.002 and -0.442. Other parameters iterated as in step 9.

11    0.898    0.101    0.133    0.002    -0.442    -3.69    17.79    7.1.

C fixed as 1.43 and  $\gamma$  fixed as 0.002. Other parameters iterated as in step no. 9.

12    1.43    0.049    0.159    0.002    -1.069    1.1    9.138    5.57.

$\beta$  and  $\gamma$  fixed as 0.165 and 0.002 respectively , as in wetted surface area correlations.

13    1.49    0.051    0.165    0.002    -1.063    1.25    9.196    5.49.

Note : The values of indices in the finalised generalised correlation are as per regression step marked by ( \* )

prefixing the values of some indices, yielded the following equation:-

$$a_c/a_t = 0.455 (Re)^{0.227} (We)^{0.058} (Fr)^{0.002} (\sigma / \sigma_c)^{-1.104} \quad (7.10)$$

wherein the values of indices of all the dimensionless numbers inclusive of proportionality constant are such that the correlation predicts the values of  $a_c$  with least amount of error.

For obtaining equation (7.10) regression of the data on  $a_c$  was performed as per step (10) and the values of  $E_{avg}$ ,  $E_{abs}$  and  $S_{dev}$  were 1.38 %, 8.25 % and 4.9 % respectively. Thus, the values of  $E_{abs}$  further decreased to 8.25 % (regression step - 10) in comparison to 10.4 % obtained during the regression step (7). This clearly indicates that the different optimization methodologies sometimes do give different statistical analysis as well as data fit. Hence one has to be careful while selecting a methodology for performing mathematical modelling of data.

Further in the regression step (11), the value of the index of  $(\sigma / \sigma_c)$  was prefixed to -0.442 which was similar to the value obtained for the cases of  $a_w$  and  $a_p$ . However, under these conditions the algorithm failed to minimise the error by altering other indices appropriately. The resulting equation based on the regression step (11), yielded the values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  as -3.69, 17.79 and 7.1 respectively. This statistical analysis for the regression step (11) was very much inferior in comparison to the statistical analysis for the regression step (10). Except for the regression step (11), all the other regression steps in Table (7.3.2)

yielded nearly identical values of the index for the  $(\sigma / \sigma_c)$  parameter. Thus it can be concluded that the  $(\sigma / \sigma_c)$  parameter is the dominant parameter affecting the values of  $a_c$  in comparison to the other parameters. The effect of parameters  $(\sigma / \sigma_c)$  and  $Re$  on the values of  $a_c$  appears to be more pronounced than that of  $a_w$  and  $a_p$ , as could be seen from the indices of these parameters in equations (7.3), (7.7) and (7.10) respectively.

### 7.3.3 Comparisons between experimental values (based on data banks) and predicted values of $a_c$ :

The values of  $(a_c/a_t)$  obtained from the data bank and the values of  $(a_c/a_t)$  predicted by using the correlation [Equation (7.10)] are tabulated in Table (7.3.3) and are plotted in Figure (7.3). The detailed statistical analysis of the correlation [Equation (7.10)] can be described as follows :

- (i) The total number of data points used in the correlation are 167 with 32 different systems/variations included.
- (ii) The values of  $\% E_{avg}$ ,  $\% E_{abs}$  and  $\% S_{dev}$  were 1.38, 8.25 and 4.89 respectively.
- (iii) From the 167 data points 94 % of the predicted values of  $a_c$  were within  $\pm 20$  % of the experimental data values, 85 % were within  $\pm 15$  %, 60 % were within  $\pm 10$  % and 36 % were within  $\pm 5$  %.

Figure (7.3), shows a parity plot of equation (7.10) wherein the values of  $(a_c/a_t)_{pred}$  have been plotted versus  $(a_c/a_t)_{exp}$ . The satisfactory correlation fit in Figure (7.3) clearly reflects that

Table - ( 7.3.3 )

Comparison between experimental values based on data bank  
and predicted values of  $a_c$

| No. | $a_c$<br>$m^2/m^3$ | $a_c/a_t$<br>exp. | $a_c/a_t$<br>This work | % err  | $a_c/a_t$<br>Puranik & Vogelpohl | % err  |
|-----|--------------------|-------------------|------------------------|--------|----------------------------------|--------|
| 1   | 106.0              | 0.286             | 0.266                  | 7.02   | 0.304                            | -6.09  |
| 2   | 134.0              | 0.362             | 0.308                  | 14.84  | 0.304                            | 16.08  |
| 3   | 141.0              | 0.381             | 0.389                  | -1.98  | 0.373                            | 2.14   |
| 4   | 154.0              | 0.416             | 0.448                  | -7.66  | 0.423                            | -1.62  |
| 5   | 82.0               | 0.222             | 0.222                  | 0.00   | 0.246                            | -11.18 |
| 6   | 97.0               | 0.262             | 0.284                  | -8.19  | 0.305                            | -16.53 |
| 7   | 120.0              | 0.324             | 0.326                  | -0.42  | 0.345                            | -6.45  |
| 8   | 127.5              | 0.345             | 0.361                  | -4.69  | 0.378                            | -9.67  |
| 9   | 144.5              | 0.391             | 0.390                  | 0.24   | 0.405                            | -3.59  |
| 10  | 63.0               | 0.485             | 0.535                  | -10.46 | 0.511                            | -5.36  |
| 11  | 78.0               | 0.600             | 0.587                  | 2.19   | 0.554                            | 7.68   |
| 12  | 92.7               | 0.713             | 0.689                  | 3.35   | 0.639                            | 10.45  |
| 13  | 110.0              | 0.846             | 0.746                  | 11.78  | 0.685                            | 19.01  |
| 14  | 119.0              | 0.915             | 0.783                  | 14.48  | 0.715                            | 21.92  |
| 15  | 128.0              | 0.272             | 0.270                  | 0.73   | 0.328                            | -20.33 |
| 16  | 156.0              | 0.332             | 0.331                  | 0.27   | 0.392                            | -18.11 |
| 17  | 172.0              | 0.366             | 0.377                  | -3.06  | 0.440                            | -20.23 |
| 18  | 126.0              | 0.268             | 0.258                  | 3.71   | 0.291                            | -8.59  |
| 19  | 142.0              | 0.302             | 0.301                  | 0.29   | 0.334                            | -10.46 |
| 20  | 154.0              | 0.328             | 0.323                  | 1.52   | 0.355                            | -8.23  |
| 21  | 168.0              | 0.357             | 0.349                  | 2.27   | 0.380                            | -6.43  |
| 22  | 179.0              | 0.381             | 0.371                  | 2.50   | 0.402                            | -5.44  |
| 23  | 187.0              | 0.398             | 0.392                  | 1.38   | 0.422                            | -5.97  |
| 24  | 190.0              | 0.404             | 0.413                  | -2.13  | 0.441                            | -9.11  |
| 25  | 141.0              | 0.300             | 0.252                  | 15.91  | 0.285                            | 4.92   |
| 26  | 178.0              | 0.379             | 0.321                  | 15.37  | 0.353                            | 6.91   |
| 27  | 206.0              | 0.438             | 0.370                  | 15.57  | 0.400                            | 8.66   |
| 28  | 143.0              | 0.304             | 0.257                  | 15.57  | 0.290                            | 4.73   |
| 29  | 181.0              | 0.385             | 0.325                  | 15.61  | 0.357                            | 7.33   |
| 30  | 208.0              | 0.443             | 0.375                  | 15.34  | 0.405                            | 8.54   |
| 31  | 220.0              | 0.468             | 0.410                  | 12.45  | 0.438                            | 6.39   |
| 32  | 74.5               | 0.392             | 0.413                  | -5.34  | 0.412                            | -5.01  |
| 33  | 101.0              | 0.532             | 0.532                  | 0.00   | 0.515                            | 3.12   |
| 34  | 127.0              | 0.270             | 0.295                  | -9.17  | 0.301                            | -11.33 |
| 35  | 134.0              | 0.285             | 0.313                  | -9.87  | 0.317                            | -11.27 |
| 36  | 156.0              | 0.332             | 0.362                  | -8.96  | 0.360                            | -8.53  |
| 37  | 168.0              | 0.357             | 0.400                  | -11.84 | 0.394                            | -10.12 |
| 38  | 180.0              | 0.383             | 0.422                  | -10.26 | 0.413                            | -7.88  |
| 39  | 131.0              | 0.279             | 0.234                  | 16.17  | 0.294                            | -5.58  |
| 40  | 160.0              | 0.340             | 0.299                  | 12.18  | 0.366                            | -7.51  |
| 41  | 183.0              | 0.389             | 0.337                  | 13.56  | 0.406                            | -4.38  |
| 42  | 185.0              | 0.394             | 0.366                  | 6.92   | 0.438                            | -11.30 |
| 43  | 171.0              | 0.364             | 0.323                  | 11.19  | 0.394                            | -8.17  |
| 44  | 164.0              | 0.349             | 0.297                  | 14.80  | 0.366                            | -4.78  |
| 45  | 142.0              | 0.302             | 0.248                  | 17.91  | 0.311                            | -3.09  |
| 46  | 136.0              | 0.289             | 0.322                  | -11.25 | 0.332                            | -14.84 |

Table - 7.3.3 ( contd. )

| No. | $a_c$<br>$m^2/m^3$ | $a_c/a_t$<br>exp. | $a_c/a_t$<br>This work | % err  | $a_c/a_t$<br>Puranik & Vogelpohl | % err   |
|-----|--------------------|-------------------|------------------------|--------|----------------------------------|---------|
| 47  | 180.0              | 0.383             | 0.434                  | -13.34 | 0.433                            | -13.04  |
| 48  | 140.0              | 0.298             | 0.318                  | -6.82  | 0.332                            | -11.39  |
| 49  | 152.0              | 0.323             | 0.356                  | -10.02 | 0.366                            | -13.26  |
| 50  | 180.0              | 0.383             | 0.412                  | -7.66  | 0.417                            | -8.96   |
| 51  | 147.0              | 0.313             | 0.327                  | -4.46  | 0.335                            | -7.17   |
| 52  | 160.0              | 0.340             | 0.355                  | -4.27  | 0.361                            | -5.96   |
| 53  | 175.0              | 0.372             | 0.401                  | -7.70  | 0.402                            | -7.91   |
| 54  | 181.0              | 0.385             | 0.429                  | -11.30 | 0.426                            | -10.67  |
| 55  | 138.0              | 0.294             | 0.291                  | 0.77   | 0.312                            | -6.22   |
| 56  | 147.0              | 0.313             | 0.334                  | -6.86  | 0.352                            | -12.59  |
| 57  | 173.0              | 0.368             | 0.369                  | -0.12  | 0.384                            | -4.31   |
| 58  | 182.0              | 0.387             | 0.432                  | -11.59 | 0.442                            | -14.15  |
| 59  | 64.0               | 0.492             | 0.574                  | -16.57 | 0.537                            | -9.06   |
| 60  | 82.0               | 0.631             | 0.629                  | 0.36   | 0.582                            | 7.75    |
| 61  | 105.0              | 0.808             | 0.717                  | 11.24  | 0.654                            | 19.06   |
| 62  | 127.0              | 0.977             | 0.820                  | 16.05  | 0.736                            | 24.63   |
| 63  | 145.0              | 1.115             | 0.952                  | 14.68  | 0.840                            | 24.70   |
| 64  | 79.2               | 0.240             | 0.205                  | 14.77  | 0.225                            | 6.10    |
| 65  | 105.6              | 0.320             | 0.301                  | 5.80   | 0.318                            | 0.75    |
| 66  | 125.4              | 0.380             | 0.364                  | 4.26   | 0.375                            | 1.29    |
| 67  | 155.1              | 0.470             | 0.463                  | 1.49   | 0.464                            | 1.21    |
| 68  | 178.2              | 0.540             | 0.572                  | -5.90  | 0.560                            | -3.64   |
| 69  | 198.0              | 0.600             | 0.677                  | -12.86 | 0.650                            | -8.32   |
| 70  | 121.8              | 0.600             | 0.676                  | -12.66 | 0.625                            | -4.21   |
| 71  | 148.1              | 0.730             | 0.860                  | -17.84 | 0.774                            | -6.02   |
| 72  | 76.0               | 0.205             | 0.184                  | 10.31  | 0.216                            | -4.94   |
| 73  | 114.0              | 0.308             | 0.241                  | 21.89  | 0.273                            | 11.38   |
| 74  | 133.0              | 0.359             | 0.310                  | 13.89  | 0.341                            | 5.10    |
| 75  | 148.2              | 0.401             | 0.345                  | 13.85  | 0.376                            | 6.24    |
| 76  | 171.0              | 0.462             | 0.394                  | 14.82  | 0.422                            | 8.69    |
| 77  | 114.0              | 0.308             | 0.246                  | 20.07  | 0.279                            | 9.58    |
| 78  | 125.4              | 0.339             | 0.291                  | 14.02  | 0.323                            | 4.60    |
| 79  | 144.4              | 0.390             | 0.355                  | 9.10   | 0.385                            | 1.40    |
| 80  | 163.4              | 0.442             | 0.424                  | 3.99   | 0.451                            | -2.02   |
| 81  | 182.4              | 0.493             | 0.513                  | -4.07  | 0.533                            | -8.19   |
| 82  | 190.0              | 0.514             | 0.552                  | -7.56  | 0.569                            | -10.87  |
| 83  | 66.0               | 0.347             | 0.384                  | -10.63 | 0.369                            | -6.33   |
| 84  | 93.0               | 0.489             | 0.466                  | 4.77   | 0.438                            | 10.49   |
| 85  | 100.0              | 0.526             | 0.533                  | -1.21  | 0.490                            | 6.87    |
| 86  | 108.0              | 0.568             | 0.567                  | 0.18   | 0.518                            | 8.82    |
| 87  | 132.0              | 0.695             | 0.613                  | 11.73  | 0.560                            | 19.39   |
| 88  | 130.0              | 0.684             | 0.706                  | -3.16  | 0.634                            | 7.30    |
| 89  | 130.0              | 0.684             | 0.646                  | 5.52   | 0.587                            | 14.24   |
| 90  | 27.0               | 0.142             | 0.153                  | -7.34  | 0.306                            | -115.16 |
| 91  | 36.0               | 0.189             | 0.190                  | -0.53  | 0.372                            | -96.41  |
| 92  | 169.0              | 0.367             | 0.305                  | 17.10  | 0.301                            | 18.03   |
| 93  | 206.0              | 0.448             | 0.351                  | 21.66  | 0.341                            | 23.79   |
| 94  | 227.0              | 0.493             | 0.387                  | 21.50  | 0.373                            | 24.50   |
| 95  | 258.0              | 0.561             | 0.418                  | 25.41  | 0.399                            | 28.89   |
| 96  | 268.0              | 0.583             | 0.446                  | 23.47  | 0.422                            | 27.58   |
| 97  | 284.0              | 0.617             | 0.493                  | 20.20  | 0.461                            | 25.35   |

Table - 7.3.3 ( contd. )

| No. | $a_c$<br>$m^2/m^3$ | $a_c/a_t$<br>exp. | $a_c/a_t$<br>This work | % err  | $a_c/a_t$<br>Puranik & Vogelpohl | % err   |
|-----|--------------------|-------------------|------------------------|--------|----------------------------------|---------|
| 98  | 72.6               | 0.220             | 0.196                  | 10.77  | 0.224                            | -1.81   |
| 99  | 107.3              | 0.325             | 0.294                  | 9.57   | 0.320                            | 1.51    |
| 100 | 136.0              | 0.412             | 0.374                  | 9.30   | 0.396                            | 3.91    |
| 101 | 208.0              | 0.630             | 0.471                  | 25.31  | 0.486                            | 22.95   |
| 102 | 24.0               | 0.073             | 0.082                  | -12.90 | 0.194                            | -166.75 |
| 103 | 43.0               | 0.130             | 0.123                  | 5.65   | 0.277                            | -112.78 |
| 104 | 49.5               | 0.150             | 0.156                  | -4.25  | 0.343                            | -128.67 |
| 105 | 65.4               | 0.198             | 0.197                  | 0.63   | 0.421                            | -112.25 |
| 106 | 33.0               | 0.100             | 0.100                  | 0.00   | 0.200                            | -100.28 |
| 107 | 46.5               | 0.141             | 0.149                  | -5.85  | 0.286                            | -103.14 |
| 108 | 59.6               | 0.181             | 0.190                  | -5.04  | 0.354                            | -96.07  |
| 109 | 81.0               | 0.245             | 0.239                  | 2.66   | 0.434                            | -76.92  |
| 110 | 107.5              | 0.326             | 0.304                  | 6.56   | 0.538                            | -65.17  |
| 111 | 92.7               | 0.201             | 0.209                  | -3.49  | 0.313                            | -55.10  |
| 112 | 107.4              | 0.234             | 0.230                  | 1.37   | 0.341                            | -46.14  |
| 113 | 121.9              | 0.265             | 0.249                  | 5.86   | 0.366                            | -38.22  |
| 114 | 131.8              | 0.286             | 0.266                  | 7.05   | 0.388                            | -35.44  |
| 115 | 60.7               | 0.194             | 0.202                  | -4.04  | 0.229                            | -17.86  |
| 116 | 92.0               | 0.294             | 0.295                  | -0.51  | 0.320                            | -8.96   |
| 117 | 112.7              | 0.360             | 0.376                  | -4.40  | 0.396                            | -10.09  |
| 118 | 102.1              | 0.338             | 0.380                  | -12.34 | 0.399                            | -17.99  |
| 119 | 140.4              | 0.465             | 0.466                  | -0.12  | 0.478                            | -2.72   |
| 120 | 153.4              | 0.508             | 0.530                  | -4.38  | 0.536                            | -5.49   |
| 121 | 35.5               | 0.110             | 0.106                  | 3.28   | 0.243                            | -121.45 |
| 122 | 43.6               | 0.135             | 0.129                  | 4.25   | 0.289                            | -114.34 |
| 123 | 33.6               | 0.115             | 0.104                  | 9.22   | 0.207                            | -79.83  |
| 124 | 44.3               | 0.152             | 0.153                  | -0.81  | 0.290                            | -91.11  |
| 125 | 57.8               | 0.198             | 0.195                  | 1.73   | 0.359                            | -81.21  |
| 126 | 28.2               | 0.097             | 0.105                  | -7.90  | 0.207                            | -113.66 |
| 127 | 50.6               | 0.174             | 0.195                  | -11.98 | 0.359                            | -106.41 |
| 128 | 59.3               | 0.204             | 0.239                  | -17.17 | 0.430                            | -110.95 |
| 129 | 81.5               | 0.280             | 0.272                  | 2.91   | 0.482                            | -72.20  |
| 130 | 28.0               | 0.090             | 0.103                  | -13.71 | 0.205                            | -126.73 |
| 131 | 56.0               | 0.181             | 0.191                  | -5.88  | 0.355                            | -96.51  |
| 132 | 77.5               | 0.250             | 0.235                  | 6.19   | 0.425                            | -70.07  |
| 133 | 93.0               | 0.300             | 0.267                  | 10.97  | 0.477                            | -59.00  |
| 134 | 23.6               | 0.082             | 0.105                  | -27.98 | 0.207                            | -153.13 |
| 135 | 53.0               | 0.184             | 0.195                  | -6.12  | 0.360                            | -95.38  |
| 136 | 67.0               | 0.233             | 0.239                  | -2.94  | 0.431                            | -85.12  |
| 137 | 51.4               | 0.176             | 0.180                  | -2.21  | 0.226                            | -28.58  |
| 138 | 77.1               | 0.264             | 0.263                  | 0.21   | 0.317                            | -20.14  |
| 139 | 97.8               | 0.335             | 0.335                  | 0.00   | 0.392                            | -17.14  |
| 140 | 44.5               | 0.153             | 0.180                  | -17.77 | 0.226                            | -48.10  |
| 141 | 137.9              | 0.474             | 0.411                  | 13.22  | 0.470                            | 0.78    |
| 142 | 116.4              | 0.400             | 0.468                  | -17.09 | 0.528                            | -31.88  |
| 143 | 114.0              | 0.368             | 0.329                  | 10.42  | 0.388                            | -5.58   |
| 144 | 137.6              | 0.444             | 0.404                  | 8.99   | 0.465                            | -4.77   |
| 145 | 140.7              | 0.454             | 0.460                  | -1.40  | 0.522                            | -14.99  |
| 146 | 95.0               | 0.330             | 0.336                  | -1.97  | 0.393                            | -19.22  |
| 147 | 105.7              | 0.367             | 0.412                  | -12.38 | 0.471                            | -28.34  |

Table - 7.3.3 ( contd. )

| No. | $a_c$<br>$m^2/m^3$ | $a_c/a_t$<br>exp. | $a_c/a_t$<br>This work | % err  | $a_c/a_t$<br>Puranik & Vogelpohl | % err  |
|-----|--------------------|-------------------|------------------------|--------|----------------------------------|--------|
| 148 | 109.7              | 0.345             | 0.327                  | 5.19   | 0.387                            | -12.05 |
| 149 | 135.0              | 0.425             | 0.401                  | 5.53   | 0.463                            | -9.06  |
| 150 | 150.0              | 0.472             | 0.457                  | 3.17   | 0.519                            | -10.12 |
| 151 | 43.7               | 0.151             | 0.143                  | 5.22   | 0.280                            | -86.22 |
| 152 | 52.8               | 0.182             | 0.181                  | 0.91   | 0.345                            | -89.48 |
| 153 | 61.4               | 0.212             | 0.211                  | 0.57   | 0.396                            | -86.79 |
| 154 | 71.2               | 0.246             | 0.228                  | 7.33   | 0.424                            | -72.54 |
| 155 | 199.0              | 0.510             | 0.557                  | -9.07  | 0.527                            | -3.25  |
| 156 | 175.5              | 0.450             | 0.485                  | -7.67  | 0.466                            | -3.57  |
| 157 | 144.0              | 0.369             | 0.393                  | -6.33  | 0.387                            | -4.79  |
| 158 | 123.0              | 0.315             | 0.322                  | -2.23  | 0.325                            | -3.06  |
| 159 | 215.8              | 0.553             | 0.551                  | 0.44   | 0.538                            | 2.80   |
| 160 | 201.0              | 0.515             | 0.509                  | 1.17   | 0.502                            | 2.64   |
| 161 | 171.0              | 0.438             | 0.428                  | 2.38   | 0.430                            | 1.89   |
| 162 | 151.0              | 0.387             | 0.378                  | 2.44   | 0.385                            | 0.52   |
| 163 | 132.8              | 0.341             | 0.322                  | 5.33   | 0.335                            | 1.68   |
| 164 | 161.0              | 0.345             | 0.406                  | -17.64 | 0.382                            | -10.67 |
| 165 | 176.0              | 0.377             | 0.428                  | -13.61 | 0.400                            | -6.21  |
| 166 | 188.0              | 0.403             | 0.449                  | -11.41 | 0.417                            | -3.60  |
| 167 | 204.0              | 0.437             | 0.485                  | -10.94 | 0.447                            | -2.24  |

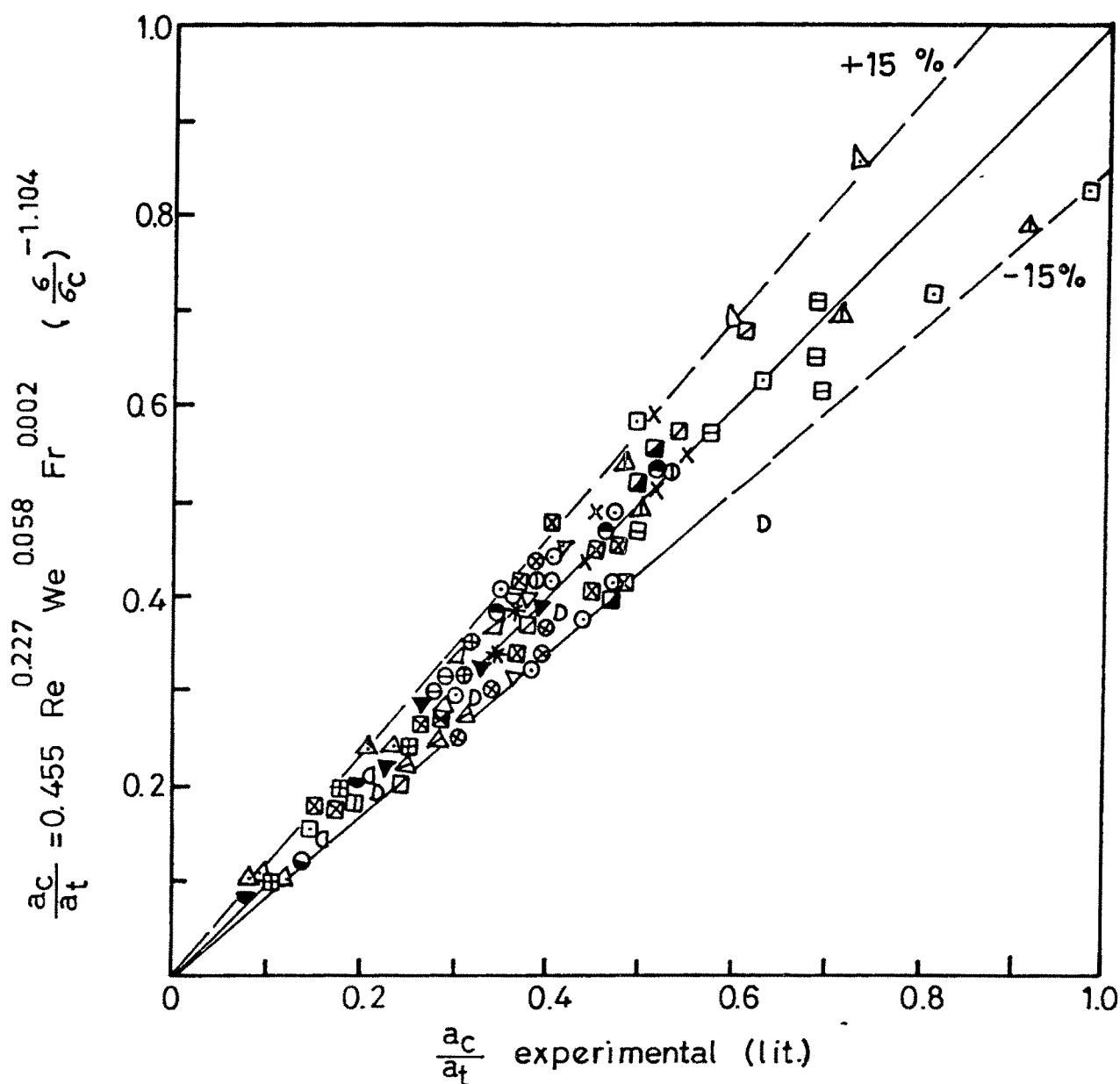


Fig.7.3 EFFECTIVE INTERFACIAL AREA DURING ABSORPTION  
WITH CHEMICAL REACTION.  
COMPARISON OF PREDICTED vs. EXPERIMENTAL  
(LIT.) DATA.

## LEGENDS FOR FIG. 7.3

| Data No. | Relevant Information                                                                                                         | Symbol  |
|----------|------------------------------------------------------------------------------------------------------------------------------|---------|
| 1        | Danckwerts (19), CO <sub>2</sub> -Na Buffer, C.R.R., 13mm                                                                    | ■       |
| 2-4      | Vassilatos (20), CO <sub>2</sub> -aq.NH <sub>3</sub> , C.R.R., 13mm.                                                         | ▽       |
| 5-9      | Richards (21), CO <sub>2</sub> -Na Buffer, C.R.R., 13mm.                                                                     | ▼       |
| 10-14    | Danckwerts (22), CO <sub>2</sub> -K Buffer, C.R.R., 38mm.                                                                    | △       |
| 15-17    | Genlawat (23), isobutylene-11.5 M H <sub>2</sub> SO <sub>4</sub> , C.R.R., 9.5mm.                                            | *       |
| 18-33    | Vidwans (24), CO <sub>2</sub> -NaOH, C.R.R., 9.5mm, 25mm.                                                                    | ○, ⊙    |
| 34-38    | Vidwans (24), CO <sub>2</sub> -MEA, C.R.R., 9.5mm.                                                                           | ⊖       |
| 39-47    | Jhaveri (25), O <sub>2</sub> -aq CuCl., C.R.R., 9.5mm.                                                                       | ⊗       |
| 48-50    | Jhaveri (25), O <sub>2</sub> -aq CuCl + 5 M HCl, C.R.R., 9.5mm.                                                              | ⊕       |
| 51-58    | Jhaveri (25), O <sub>2</sub> -Na dithionite, C.R.R., 9.5mm.                                                                  | △       |
| 59-63    | Danckwerts (27), Sulphite oxidation, C.R.R., 38mm.                                                                           | □       |
| 64-69    | Onda (28), Sulphite oxidation, C.R.R., 15mm.                                                                                 | ▣       |
| 70-71    | DeWaal (29), Sulphite oxidation, C.R.R., 25mm.                                                                               | ▤       |
| 72-82    | Puranik (30), CO <sub>2</sub> -NaOH, C.R.R., 13mm.                                                                           | ■       |
| 83-91    | Sahay (31), O <sub>2</sub> -Na dithionite, d <sub>p</sub> = 25mm,<br>C.R.R. and Polypropylene R.R.                           | ⊞, ⊞    |
| 92-97    | Andrieu (32), Sulphite oxidation, Pyrex R.R., 10 mm.                                                                         | ■       |
| 98-110   | Linek (33) Sulphite oxidation, d <sub>p</sub> = 15mm, C.R.R.,<br>Polypropylene R.R., Polyethylene R.R.                       | ⊙, ⊞    |
| 111-114  | Andrieu (34), Sulphite oxidation, Silicone Coated R.R. 10mm.                                                                 | △       |
| 115-150  | Linek (35), Sulphite oxidation, d <sub>p</sub> =15mm, Polypropylene R.R.<br>C.R.R., Polyethylene R.R. and Hydrophilised R.R. | ●, △, ⊞ |
| 151-154  | Alper (36), CO <sub>2</sub> -K Buffer, Polyethylene R.R., 16mm.                                                              | ○       |
| 155-163  | Rizzuti (37), CO <sub>2</sub> -K Buffer, Glass R.R., 10mm.                                                                   | x       |
| 163-167  | Augugliaro (39), CO <sub>2</sub> -K Buffer, Steel.R.R., 10mm.                                                                | ⊙       |

equation (7.10) can correlate satisfactorily all the data obtained from various sources covering a wide range of variables as mentioned earlier.

No generalised correlation is available in the literature for predicting the values of  $a_c$ , except that of Puranik and Vogelpohl (106) [Equation (2.10)]. The values of  $(a_c/a_t)$  obtained by using this correlation are also reported in Table (7.3.3). The detailed statistical analysis of correlation by Puranik and Vogelpohl can be described as follows:

(i) The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were -21.3, 27.5 and 8.8 respectively.

(ii) From the 167 data points 73 % of the predicted values of  $a_c$  were within  $\pm 20$  % of the experimental data values, 68 % were within  $\pm 15$  % and only 50 % were within  $\pm 10$  %.

The comparison of the statistical analysis for both the correlations reveals that the value of %  $E_{abs}$  (also %  $E_{avg}$ ) for the correlation of Puranik and Vogelpohl is substantially higher than that for correlation developed in this investigation being 27.5 and 8.25 respectively.

During the period when the correlation for predicting the values of  $(a_c/a_t)$  was proposed by Puranik and Vogelpohl, the data bank of  $(a_c/a_t)$  for plastic packings was not available in the literature, that is why the correlation of Puranik and Vogelpohl totally fails to predict the values of  $(a_c/a_t)$  for plastic packings within reasonable limits of accuracy. Practically for all the data

of  $(a_c/a_t)$  for plastic packings, the predicted values are over 50 % higher than the experimentally obtained values with the values of maximum error as high as -166 %; Whereas the maximum error in the correlation [Equation (7.10)] developed in this investigation is only -28 %.

Since the generalised correlation proposed by Puranik and Vogelpohl happens to be a correlation to be utilised also for predicting the value of  $(a_w/a_t)$ , the data fit only for the values of  $a_c$  might have been inferior. Further, the index of the dominant parameter  $(\sigma / \sigma_c)$  probably would not have been appropriately optimised in that correlation. Hence, the correlation proposed in this investigations [Equation (7.10)] is expected to be superior in comparison to the existing generalised correlation of Puranik and Vogelpohl.

#### 7.4.0 RESULTS AND DISCUSSION FOR STATIC AREA ( $a_{st}$ ) :

As has been already discussed in Chapter (6) : Mathematical modelling of effective interfacial areas and mass transfer coefficients; equation (6.4) can be used to estimate the values of static area. The use of equation (6.4) requires the knowledge of  $a_w$  and  $a_p$  under otherwise identical conditions. However, it is observed that the experimental values of  $a_w$  or  $a_p$  reported in the literature are not available under otherwise identical conditions. Hence, the following approaches can be used for estimating the values of  $a_{st}$  :-

APPROACH (I) : Using the data bank of  $a_p$  and obtaining the values of

$a_w$  under otherwise identical conditions by use of generalised correlation developed in this investigation i.e.,

$$a_{st} = a_w \text{ by correlation} - a_p \text{ from data bank} \quad (7.11)$$

APPROACH (II) : Using the data bank of  $a_w$  and obtaining the values of  $a_p$  under otherwise identical conditions by use of generalised correlation i.e.,

$$a_{st} = a_w \text{ from data bank} - a_p \text{ by correlation} \quad (7.12)$$

#### 7.4.1 Results for $a_{st}$ by approach (I) : Use of equation (7.11) :-

The generalised correlation for  $a_w/a_t$  [Equation (7.3)] developed in this investigation could be used conveniently to obtain the value of  $a_w$  under otherwise identical conditions mentioned for the data bank of experimental values of  $a_p$ . The results obtained for the observed values of  $a_{st}$  by approach (I) are reported in Table (7.4.1) which consists of 100 data points covering systems for physical absorption such as ammonia - water,  $CO_2$  - water,  $CO_2$  - methanol,  $CO_2$  - cane sugar solutions, methanol vapour - water etc. and includes 18 variations. These observed values of  $a_{st}$  could be used appropriately for analysing the effect of different variables on the values of  $a_{st}$ .

#### 7.4.2 Critical analysis of data and mathematical modelling for $a_{st}$ :-

As has been discussed already, the values of  $a_{st}$  get affected by the liquid flow rate (L), the packing size ( $d_p$ ) and its wettability and the physical properties of the liquid/absorption media like density, viscosity and surface tension etc.

Table - ( 7.4.1 )

Observed values of static area during absorption ( $a_{st}$ )  
based on  $(a_p/a_t)_{exp}$

| No | T    | $a_t$     | $\rho_L$ | $\mu_L$   | $\sigma$ | L        | $a_p$ | $a_w$     | $a_{st}$ |
|----|------|-----------|----------|-----------|----------|----------|-------|-----------|----------|
|    | °C   | $m^2/m^3$ | $kg/m^3$ | $mNs/m^2$ | $mN/m$   | $kg/m^2$ | exp.  | pred.     | obs.     |
|    |      |           |          |           |          |          |       | $m^2/m^3$ |          |
| 1  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.678    | 28.1  | 45.9      | 17.8     |
| 2  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 1.350    | 39.6  | 57.8      | 18.2     |
| 3  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 2.034    | 48.2  | 66.4      | 18.1     |
| 4  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 5.400    | 68.4  | 92.1      | 23.7     |
| 5  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 6.103    | 67.8  | 95.9      | 28.1     |
| 6  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 6.800    | 78.0  | 99.5      | 21.5     |
| 7  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.111    | 12.2  | 25.0      | 12.8     |
| 8  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.204    | 11.6  | 30.7      | 19.1     |
| 9  | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.312    | 14.5  | 35.4      | 20.9     |
| 10 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.440    | 21.0  | 39.7      | 18.7     |
| 11 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.622    | 19.4  | 44.6      | 25.2     |
| 12 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 1.392    | 30.8  | 58.4      | 27.6     |
| 13 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 4.716    | 52.2  | 88.0      | 35.8     |
| 14 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 7.947    | 72.0  | 104.8     | 32.8     |
| 15 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.146    | 16.0  | 27.4      | 11.4     |
| 16 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.358    | 19.7  | 37.0      | 17.3     |
| 17 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 0.598    | 26.6  | 44.0      | 17.4     |
| 18 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 1.240    | 33.5  | 56.2      | 22.7     |
| 19 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 2.043    | 45.4  | 66.5      | 21.0     |
| 20 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 3.116    | 57.2  | 76.6      | 19.3     |
| 21 | 20.0 | 195.0     | 998.2    | 1.008     | 72.8     | 6.265    | 70.9  | 96.8      | 25.9     |
| 22 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.156    | 9.9   | 45.5      | 35.6     |
| 23 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.302    | 15.0  | 56.8      | 41.8     |
| 24 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.585    | 22.3  | 70.9      | 48.5     |
| 25 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.725    | 32.5  | 76.2      | 43.6     |
| 26 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.927    | 30.1  | 82.7      | 52.6     |
| 27 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 1.514    | 30.1  | 97.5      | 67.4     |
| 28 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 3.188    | 54.4  | 125.2     | 70.8     |
| 29 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 5.131    | 63.5  | 146.9     | 83.4     |
| 30 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 5.846    | 58.8  | 153.4     | 94.7     |
| 31 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 7.947    | 75.5  | 170.1     | 94.6     |
| 32 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.203    | 13.9  | 49.7      | 35.8     |
| 33 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.529    | 27.7  | 68.5      | 40.8     |
| 34 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 0.845    | 32.3  | 80.2      | 47.9     |
| 35 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 2.043    | 43.9  | 107.8     | 63.9     |
| 36 | 20.0 | 348.0     | 998.2    | 1.008     | 72.8     | 4.235    | 64.4  | 137.7     | 73.3     |
| 37 | 15.0 | 348.0     | 796.0    | 0.669     | 23.0     | 0.290    | 17.5  | 117.1     | 99.7     |
| 38 | 15.0 | 348.0     | 796.0    | 0.669     | 23.0     | 0.409    | 37.7  | 131.5     | 93.9     |
| 39 | 15.0 | 348.0     | 796.0    | 0.669     | 23.0     | 0.539    | 40.6  | 144.3     | 103.6    |
| 40 | 15.0 | 330.0     | 999.0    | 1.144     | 73.5     | 0.280    | 21.8  | 52.6      | 30.8     |
| 41 | 15.0 | 330.0     | 999.0    | 1.144     | 73.5     | 0.550    | 29.6  | 66.0      | 36.4     |
| 42 | 15.0 | 330.0     | 999.0    | 1.144     | 73.5     | 2.220    | 58.7  | 105.4     | 46.7     |
| 43 | 15.0 | 330.0     | 999.0    | 1.144     | 73.5     | 2.780    | 64.0  | 113.6     | 49.7     |
| 44 | 15.0 | 330.0     | 999.0    | 1.144     | 73.5     | 5.500    | 87.4  | 142.9     | 55.5     |
| 45 | 25.0 | 330.0     | 787.7    | 0.523     | 21.9     | 0.280    | 54.6  | 114.4     | 59.8     |

Table - 7.4.1 ( contd.)

| No | T                  | $a_t$                   | $\rho_L$               | $\mu_L$                 | $\sigma$             | L                      | $a_p$ | $a_w$                   | $a_{st}$ |
|----|--------------------|-------------------------|------------------------|-------------------------|----------------------|------------------------|-------|-------------------------|----------|
|    | $^{\circ}\text{C}$ | $\text{m}^2/\text{m}^3$ | $\text{kg}/\text{m}^3$ | $\text{mNs}/\text{m}^2$ | $\text{mN}/\text{m}$ | $\text{kg}/\text{m}^2$ | exp.  | pred.                   | obs.     |
|    |                    |                         |                        |                         |                      |                        |       | $\text{m}^2/\text{m}^3$ |          |
| 46 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 0.550                  | 65.5  | 143.4                   | 77.9     |
| 47 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 0.830                  | 71.8  | 164.6                   | 92.9     |
| 48 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 1.120                  | 93.6  | 182.1                   | 88.5     |
| 49 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 1.670                  | 109.2 | 208.2                   | 99.0     |
| 50 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 2.220                  | 124.8 | 229.0                   | 104.2    |
| 51 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 2.780                  | 131.0 | 247.0                   | 115.9    |
| 52 | 25.0               | 330.0                   | 787.7                  | 0.523                   | 21.9                 | 5.500                  | 188.8 | 310.5                   | 121.7    |
| 53 | 25.0               | 207.0                   | 787.7                  | 0.523                   | 21.9                 | 0.550                  | 64.2  | 97.1                    | 33.0     |
| 54 | 25.0               | 207.0                   | 787.7                  | 0.523                   | 21.9                 | 0.830                  | 82.8  | 111.5                   | 28.7     |
| 55 | 25.0               | 207.0                   | 787.7                  | 0.523                   | 21.9                 | 1.120                  | 89.0  | 123.3                   | 34.3     |
| 56 | 25.0               | 207.0                   | 787.7                  | 0.523                   | 21.9                 | 1.670                  | 105.6 | 141.0                   | 35.4     |
| 57 | 25.0               | 207.0                   | 787.7                  | 0.523                   | 21.9                 | 2.220                  | 124.2 | 155.1                   | 30.9     |
| 58 | 25.0               | 330.0                   | 894.0                  | 1.427                   | 30.0                 | 2.780                  | 109.2 | 199.4                   | 90.2     |
| 59 | 25.0               | 330.0                   | 953.0                  | 1.600                   | 40.0                 | 2.780                  | 90.5  | 165.6                   | 75.2     |
| 60 | 25.0               | 330.0                   | 981.0                  | 1.185                   | 55.0                 | 2.780                  | 78.0  | 135.9                   | 57.9     |
| 61 | 15.0               | 330.0                   | 1100.0                 | 3.500                   | 70.0                 | 1.390                  | 49.9  | 91.1                    | 41.2     |
| 62 | 15.0               | 330.0                   | 1100.0                 | 3.500                   | 70.0                 | 2.780                  | 65.5  | 115.0                   | 49.5     |
| 63 | 15.0               | 330.0                   | 1100.0                 | 3.500                   | 70.0                 | 5.560                  | 90.5  | 145.1                   | 54.6     |
| 64 | 15.0               | 330.0                   | 1100.0                 | 3.500                   | 70.0                 | 4.200                  | 74.9  | 132.1                   | 57.2     |
| 65 | 15.0               | 330.0                   | 1400.0                 | 3.730                   | 67.0                 | 1.390                  | 49.9  | 89.8                    | 39.9     |
| 66 | 15.0               | 330.0                   | 1400.0                 | 3.730                   | 67.0                 | 2.780                  | 65.5  | 113.4                   | 47.8     |
| 67 | 15.0               | 330.0                   | 1400.0                 | 3.730                   | 67.0                 | 4.200                  | 74.9  | 130.2                   | 55.3     |
| 68 | 15.0               | 330.0                   | 1400.0                 | 3.730                   | 67.0                 | 5.560                  | 90.5  | 143.0                   | 52.5     |
| 69 | 13.2               | 348.0                   | 999.0                  | 1.200                   | 73.8                 | 0.681                  | 31.1  | 74.0                    | 42.9     |
| 70 | 12.7               | 348.0                   | 999.0                  | 1.216                   | 73.8                 | 0.682                  | 27.6  | 74.0                    | 46.4     |
| 71 | 13.8               | 348.0                   | 999.0                  | 1.180                   | 73.7                 | 0.692                  | 26.6  | 74.4                    | 47.8     |
| 72 | 13.6               | 348.0                   | 999.0                  | 1.186                   | 73.7                 | 0.670                  | 28.2  | 73.6                    | 45.4     |
| 73 | 14.2               | 348.0                   | 999.0                  | 1.168                   | 73.6                 | 1.363                  | 39.1  | 93.4                    | 54.3     |
| 74 | 14.2               | 348.0                   | 999.0                  | 1.168                   | 73.6                 | 1.372                  | 40.5  | 93.7                    | 53.2     |
| 75 | 12.3               | 348.0                   | 999.0                  | 1.216                   | 73.9                 | 1.310                  | 38.8  | 92.0                    | 53.2     |
| 76 | 12.6               | 348.0                   | 999.0                  | 1.226                   | 73.9                 | 1.238                  | 34.9  | 90.3                    | 55.3     |
| 77 | 12.0               | 348.0                   | 999.0                  | 1.239                   | 73.9                 | 1.311                  | 32.1  | 92.0                    | 59.9     |
| 78 | 16.3               | 348.0                   | 999.0                  | 1.106                   | 73.3                 | 0.290                  | 19.6  | 55.8                    | 36.2     |
| 79 | 17.0               | 348.0                   | 999.0                  | 1.087                   | 73.2                 | 0.681                  | 27.9  | 74.3                    | 46.4     |
| 80 | 13.3               | 348.0                   | 999.0                  | 1.196                   | 73.7                 | 0.703                  | 27.0  | 74.7                    | 47.7     |
| 81 | 13.7               | 348.0                   | 999.0                  | 1.184                   | 73.7                 | 0.700                  | 28.3  | 74.7                    | 46.4     |
| 82 | 13.3               | 348.0                   | 999.0                  | 1.196                   | 73.7                 | 0.665                  | 24.3  | 73.4                    | 49.1     |
| 83 | 14.9               | 348.0                   | 999.0                  | 1.147                   | 73.5                 | 1.397                  | 45.4  | 94.3                    | 48.9     |
| 84 | 14.7               | 348.0                   | 999.0                  | 1.153                   | 73.5                 | 1.390                  | 36.4  | 94.1                    | 57.7     |
| 85 | 13.5               | 348.0                   | 999.0                  | 1.190                   | 73.7                 | 1.313                  | 37.2  | 92.2                    | 55.0     |
| 86 | 13.4               | 348.0                   | 999.0                  | 1.193                   | 73.7                 | 1.360                  | 31.5  | 93.3                    | 61.8     |
| 87 | 13.6               | 348.0                   | 999.0                  | 1.187                   | 73.8                 | 1.370                  | 34.1  | 93.5                    | 59.4     |
| 88 | 13.5               | 195.0                   | 999.2                  | 1.190                   | 73.7                 | 0.681                  | 23.1  | 45.6                    | 22.5     |
| 89 | 15.0               | 195.0                   | 999.2                  | 1.144                   | 73.4                 | 0.708                  | 27.2  | 46.3                    | 19.1     |
| 90 | 15.0               | 195.0                   | 999.2                  | 1.144                   | 73.5                 | 0.673                  | 23.9  | 45.5                    | 21.6     |
| 91 | 14.8               | 195.0                   | 999.2                  | 1.150                   | 73.6                 | 0.662                  | 26.5  | 45.2                    | 18.7     |
| 92 | 17.0               | 195.0                   | 999.2                  | 1.087                   | 73.1                 | 0.703                  | 27.2  | 46.3                    | 19.1     |
| 93 | 17.0               | 195.0                   | 999.2                  | 1.087                   | 73.2                 | 0.703                  | 30.7  | 46.3                    | 15.5     |
| 94 | 17.2               | 195.0                   | 999.2                  | 1.082                   | 73.2                 | 0.700                  | 30.9  | 46.2                    | 15.3     |
| 95 | 17.2               | 195.0                   | 999.2                  | 1.082                   | 73.2                 | 0.684                  | 27.2  | 45.9                    | 18.7     |

Table - 7.4.1 ( contd.)

| No  | T                  | $a_t$                   | $\rho_L$               | $\mu_L$                 | $\sigma$             | L                              | $a_p$ | $a_v$                   | $a_{st}$ |
|-----|--------------------|-------------------------|------------------------|-------------------------|----------------------|--------------------------------|-------|-------------------------|----------|
|     | $^{\circ}\text{C}$ | $\text{m}^2/\text{m}^3$ | $\text{kg}/\text{m}^3$ | $\text{mNs}/\text{m}^2$ | $\text{mN}/\text{m}$ | $\text{kg}/\text{m}^2\text{s}$ | exp.  | pred.                   | obs.     |
|     |                    |                         |                        |                         |                      |                                | ----- | $\text{m}^2/\text{m}^3$ | -----    |
| 96  | 17.4               | 195.0                   | 999.2                  | 1.077                   | 73.1                 | 0.705                          | 27.1  | 46.4                    | 19.3     |
| 97  | 13.7               | 195.0                   | 999.2                  | 1.184                   | 73.7                 | 0.713                          | 30.5  | 46.3                    | 15.8     |
| 98  | 13.2               | 195.0                   | 999.2                  | 1.200                   | 73.8                 | 0.745                          | 24.6  | 47.0                    | 22.4     |
| 99  | 14.2               | 195.0                   | 999.2                  | 1.168                   | 73.6                 | 0.703                          | 26.1  | 46.1                    | 20.0     |
| 100 | 14.8               | 195.0                   | 999.2                  | 1.120                   | 73.3                 | 1.496                          | 41.6  | 59.6                    | 18.0     |

Relevant details regarding Table - ( 7.4.1 )

| Data No. | System and Packing characteristics.                                                                                                                                                                                                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1-6      | Absorption of Ammonia in water. Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$<br>$d_p = 0.025 \text{ m}$ , Ref.(13).                                                                                                                                     |
| 7-36     | Absorption of $\text{CO}_2$ in water. Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$<br>No 7-21, $d_p = 0.025 \text{ m}$ , No 22-36, $d_p = 0.015 \text{ m}$ , Ref.(14).<br>(Packed depths : No.7-14 & 22-31 = 0.4m, 15-21 & 32-36 = 0.2m)                |
| 37-39    | Absorption of $\text{CO}_2$ in methanol. Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$<br>$d_p = 0.015 \text{ m}$ . Ref.(14).                                                                                                                            |
| 40-44    | Absorption of $\text{CO}_2$ in water. Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$<br>$d_p = 0.015 \text{ m}$ , Ref.(15).                                                                                                                               |
| 45-57    | Absorption of $\text{CO}_2$ in methanol. Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$<br>No. 45-52, $d_p = 0.015 \text{ m}$ , No. 53-57, $d_p = 0.025 \text{ m}$ , Ref.(15).                                                                            |
| 58-60    | Absorption of $\text{CO}_2$ in aqueous methanol solutions.<br>Methanol concentrations :- 60%, 30%, 10% ,for no.58,59,60<br>respectively.Ceramic R.R., $\sigma_c = 61 \text{ mN/m}$ . $d_p = 0.015 \text{ m}$ , Ref.(15).                              |
| 61-68    | Absorption of $\text{CO}_2$ in cane sugar solutions. No.61-64, 5%<br>No. 65-68, 10%, ceramic R.R., $\sigma_c = 61 \text{ mN/m}$ , $d_p = 0.015 \text{ m}$<br>Ref.(15).                                                                                |
| 69-100   | Absorption of methanol vapours in water. Ceramic R.R.<br>$\sigma_c = 61 \text{ mN/m}$ , No. 69-87, $d_p = 0.015 \text{ m}$ , 88-100, $d_p = 0.025 \text{ m}$<br>Ref.(16,17). (Packed depth : No.69-77 & 88-93 = 0.2m, No.<br>78-87 & 94-100 = 0.1m ). |

The critical analysis of the observed values of  $a_{st}$  obtained by using approach (I) which are reported in Table (7.4.1) reveals the following :

(I) Effect of liquid flow rate (L) :-

Under otherwise identical conditions with an increase in L the values of  $a_{st}$  increase. However at high values of L, the effect of L on the values of  $a_{st}$  appears to be marginal. Thus for example for a set of observation numbers 22 to 31, under otherwise identical conditions with an increase in the value of L from 0.156 to 5.131  $\text{kg/m}^2\text{-s}$ , the value of  $a_{st}$  increases from 35.6  $\text{m}^2/\text{m}^3$  to 83.4  $\text{m}^2/\text{m}^3$ . With a further increase in L from 5.131  $\text{kg/m}^2\text{-s}$  to 7.947  $\text{kg/m}^2\text{-s}$ , the value of  $a_{st}$  increases marginally from 83.4  $\text{m}^2/\text{m}^3$  to 94.6  $\text{m}^2/\text{m}^3$ . Also, for all the other systems/variations under consideration similar conclusion can be drawn.

(II) Effect of packing size ( $d_p$ ) :-

The packing size has profound effect on the values of  $a_{st}$ . With an increase in the size of packing the values of  $a_{st}$  decrease considerably under otherwise identical conditions. Thus, for example for observation numbers (5) and (30) under otherwise identical conditions of  $\sigma_c = 61 \text{ mN/m}$ ,  $\sigma = 72.8 \text{ mN/m}$ ,  $\mu_L = 1.008 \text{ mNs/m}^2$ ,  $\rho_L = 998.2 \text{ kg/m}^3$  and  $L = 5.8$  to  $6.1 \text{ kg/m}^2\text{-s}$ ; with an increase in the packing size of ceramic Raschig rings from 0.015m to 0.025m the value of  $a_{st}$  decreases considerably from 94.7  $\text{m}^2/\text{m}^3$  to 28.1  $\text{m}^2/\text{m}^3$ . Under otherwise identical conditions similar trend is observed for all the other systems/variations under consideration.

(III) Effect of physical properties  $\mu_L$ ,  $\rho_L$  and  $\sigma$  inclusive of  $\sigma_c$  :

Under otherwise comparable conditions with an increase in the viscosity, the value of  $a_{st}$  also decreases. Thus for example for observation numbers 74 and 61 under otherwise comparable conditions of  $a_t = 330$  to  $348 \text{ m}^2/\text{m}^3$ ,  $\rho_L = 999$  to  $1100 \text{ kg/m}^3$ ,  $\sigma = 70$  to  $73.6 \text{ mN/m}$  and  $L = 1.372$  to  $1.390 \text{ kg/m}^2\text{-s}$ , with an increase in  $\mu_L$  from  $1.168$  to  $3.50 \text{ mNs/m}^2$ , the value of  $a_{st}$  decreases from  $53.2 \text{ m}^2/\text{m}^3$  to  $41.2 \text{ m}^2/\text{m}^3$ . With an increase in  $\rho_L$  also, the value of  $a_{st}$  do decrease but marginally, as could be observed from sets of observations 61 to 64 and 65 to 68. Under otherwise identical conditions, it is also observed that with an increase in the value of  $\sigma$ , the value of  $a_{st}$  decreases as indicated by observations 51, 58, 59 and 60.

(IV) Effect of packed bed height :-

Packed bed height appears to have no effect on the values of  $a_{st}$ . Table (7.4.1) also gives the values of  $a_{st}$  from Cases (I) and (II).

CASE (I) : Set of observations - 68 to 93 having packed bed height 0.2 m.

CASE (II) : Set of observations - 94 to 100 having packed bed height 0.1 m.

Under otherwise comparable operating conditions, the values of  $a_{st}$  obtained for Case I & II are practically comparable indicating that the packed height appears to have no effect on the values of  $a_{st}$ .

The probable effects of different parameters on the values of

$a_{st}$  have been already analysed in detail giving justifications in chapter (6). The observed values of  $a_{st}$  obtained under different sets of conditions do confirm all these conclusions. Thus, the static area model proposed in this investigation elucidates the different mechanisms of mass transfer during gas absorption in a packed column.

In order to correlate the data for  $a_{st}$  in terms of model parameters - system parameters and hydrodynamic parameters, one can propose the following equation :-

$$a_{st} = f(L, a_t^{-1}, \rho_L^{-1}, \mu_L^{-1}, \sigma^{-1}, \sigma_c) \quad (7.13)$$

With the help of dimensional analysis, these model parameters can be grouped in terms of the following dimensionless numbers/groups :-

$Re = L/a_t \mu_L$ ,  $Fr = L^2 a_t / \rho_L^2 g$ ,  $We = L^2/a_t \rho_L \sigma$  and ratio  $\sigma / \sigma_c$ . Therefore, equation (7.13) for predicting the values of  $a_{st}$  takes the following form :-

$$a_s/a_t = C (Re)^\alpha (Fr/We)^\beta (\sigma / \sigma_c)^\delta \quad (7.14)$$

Hence, the different dimensionless groups were calculated using the data for  $a_{st}$  and the relevant corresponding physical properties of the systems reported in Table - (7.4.1). These dimensionless numbers/parameters required for the processing of the data for  $a_{st}$  are reported in Table (7.4.2).

#### 7.4.3 Statistical analysis for different correlations :

Basis : Approach (I) Using equation (7.11) :

Processing of the data reported in Table (7.4.2) was done by two methodologies namely (1) DSC - Powell algorithm (2)

Table - ( 7.4.2 )

Results inclusive of processing parameters for  $a_{st}$  based on  $(a_p/a_t)_{exp}$   
 Based on Equation (7.11) :  $a_{st}/a_t = (a_w/a_t)_{pred} - (a_p/a_t)_{exp}$

| No. | L<br>kg/m <sup>2</sup> s | $a_w/a_t$<br>pred. | $a_p/a_t$<br>exp. | Re     | Fr/We | $\sigma/\sigma_c$ | $a_{st}/a_t$<br>obs. | $a_{st}/a_t$<br>pred. | % err  |
|-----|--------------------------|--------------------|-------------------|--------|-------|-------------------|----------------------|-----------------------|--------|
| 1   | 0.678                    | 0.235              | 0.144             | 3.450  | 0.283 | 1.193             | 0.091                | 0.093                 | -1.96  |
| 2   | 1.350                    | 0.297              | 0.203             | 6.868  | 0.283 | 1.193             | 0.094                | 0.105                 | -11.99 |
| 3   | 2.034                    | 0.340              | 0.247             | 10.350 | 0.283 | 1.193             | 0.093                | 0.112                 | -20.86 |
| 4   | 5.400                    | 0.472              | 0.351             | 27.473 | 0.283 | 1.193             | 0.121                | 0.133                 | -9.58  |
| 5   | 6.103                    | 0.492              | 0.348             | 31.049 | 0.283 | 1.193             | 0.144                | 0.136                 | 5.76   |
| 6   | 6.800                    | 0.510              | 0.400             | 34.595 | 0.283 | 1.193             | 0.110                | 0.138                 | -25.70 |
| 7   | 0.111                    | 0.128              | 0.062             | 0.562  | 0.283 | 1.193             | 0.066                | 0.068                 | -3.42  |
| 8   | 0.204                    | 0.157              | 0.060             | 1.039  | 0.283 | 1.193             | 0.098                | 0.076                 | 22.64  |
| 9   | 0.312                    | 0.181              | 0.074             | 1.585  | 0.283 | 1.193             | 0.107                | 0.081                 | 23.97  |
| 10  | 0.440                    | 0.204              | 0.108             | 2.239  | 0.283 | 1.193             | 0.096                | 0.086                 | 10.10  |
| 11  | 0.622                    | 0.229              | 0.099             | 3.163  | 0.283 | 1.193             | 0.129                | 0.092                 | 29.08  |
| 12  | 1.392                    | 0.300              | 0.158             | 7.081  | 0.283 | 1.193             | 0.142                | 0.105                 | 25.67  |
| 13  | 4.716                    | 0.451              | 0.268             | 23.992 | 0.283 | 1.193             | 0.184                | 0.130                 | 29.20  |
| 14  | 7.947                    | 0.538              | 0.369             | 40.432 | 0.283 | 1.193             | 0.168                | 0.142                 | 15.41  |
| 15  | 0.146                    | 0.141              | 0.082             | 0.741  | 0.283 | 1.193             | 0.058                | 0.071                 | -22.34 |
| 16  | 0.358                    | 0.190              | 0.101             | 1.820  | 0.283 | 1.193             | 0.089                | 0.083                 | 6.13   |
| 17  | 0.598                    | 0.226              | 0.137             | 3.043  | 0.283 | 1.193             | 0.089                | 0.091                 | -2.09  |
| 18  | 1.240                    | 0.288              | 0.172             | 6.311  | 0.283 | 1.193             | 0.116                | 0.103                 | 11.31  |
| 19  | 2.043                    | 0.341              | 0.233             | 10.393 | 0.283 | 1.193             | 0.108                | 0.113                 | -4.35  |
| 20  | 3.116                    | 0.393              | 0.293             | 15.851 | 0.283 | 1.193             | 0.099                | 0.121                 | -22.02 |
| 21  | 6.265                    | 0.496              | 0.364             | 31.871 | 0.283 | 1.193             | 0.133                | 0.137                 | -2.97  |
| 22  | 0.156                    | 0.131              | 0.028             | 0.445  | 0.901 | 1.193             | 0.102                | 0.117                 | -13.83 |
| 23  | 0.302                    | 0.163              | 0.043             | 0.861  | 0.901 | 1.193             | 0.120                | 0.131                 | -8.67  |
| 24  | 0.585                    | 0.204              | 0.064             | 1.667  | 0.901 | 1.193             | 0.139                | 0.146                 | -4.97  |
| 25  | 0.725                    | 0.219              | 0.093             | 2.066  | 0.901 | 1.193             | 0.125                | 0.152                 | -21.17 |
| 26  | 0.927                    | 0.238              | 0.087             | 2.641  | 0.901 | 1.193             | 0.151                | 0.159                 | -4.92  |
| 27  | 1.514                    | 0.280              | 0.087             | 4.317  | 0.901 | 1.193             | 0.194                | 0.173                 | 10.90  |
| 28  | 3.188                    | 0.360              | 0.156             | 9.089  | 0.901 | 1.193             | 0.203                | 0.196                 | 3.52   |
| 29  | 5.131                    | 0.422              | 0.182             | 14.628 | 0.901 | 1.193             | 0.240                | 0.213                 | 11.11  |
| 30  | 5.846                    | 0.441              | 0.169             | 16.667 | 0.901 | 1.193             | 0.272                | 0.218                 | 19.91  |
| 31  | 7.947                    | 0.489              | 0.217             | 22.655 | 0.901 | 1.193             | 0.272                | 0.230                 | 15.46  |
| 32  | 0.203                    | 0.143              | 0.040             | 0.578  | 0.901 | 1.193             | 0.103                | 0.122                 | -18.65 |
| 33  | 0.529                    | 0.197              | 0.080             | 1.508  | 0.901 | 1.193             | 0.117                | 0.144                 | -22.61 |
| 34  | 0.845                    | 0.230              | 0.093             | 2.409  | 0.901 | 1.193             | 0.138                | 0.156                 | -13.33 |
| 35  | 2.043                    | 0.310              | 0.126             | 5.823  | 0.901 | 1.193             | 0.184                | 0.182                 | 1.07   |
| 36  | 4.235                    | 0.396              | 0.185             | 12.074 | 0.901 | 1.193             | 0.211                | 0.206                 | 2.12   |
| 37  | 0.290                    | 0.337              | 0.050             | 1.244  | 0.357 | 0.377             | 0.286                | 0.202                 | 29.47  |
| 38  | 0.409                    | 0.378              | 0.108             | 1.757  | 0.357 | 0.377             | 0.270                | 0.214                 | 20.51  |
| 39  | 0.539                    | 0.415              | 0.117             | 2.316  | 0.357 | 0.377             | 0.298                | 0.225                 | 24.49  |
| 40  | 0.280                    | 0.159              | 0.066             | 0.742  | 0.818 | 1.205             | 0.093                | 0.120                 | -29.08 |
| 41  | 0.550                    | 0.200              | 0.090             | 1.457  | 0.818 | 1.205             | 0.110                | 0.135                 | -22.80 |
| 42  | 2.220                    | 0.319              | 0.178             | 5.880  | 0.818 | 1.205             | 0.142                | 0.172                 | -21.55 |
| 43  | 2.780                    | 0.344              | 0.194             | 7.364  | 0.818 | 1.205             | 0.151                | 0.179                 | -18.86 |
| 44  | 5.500                    | 0.433              | 0.265             | 14.569 | 0.818 | 1.205             | 0.168                | 0.201                 | -19.69 |
| 45  | 0.280                    | 0.347              | 0.165             | 1.622  | 0.309 | 0.359             | 0.181                | 0.204                 | -12.54 |
| 46  | 0.550                    | 0.435              | 0.199             | 3.187  | 0.309 | 0.359             | 0.236                | 0.229                 | 2.99   |
| 47  | 0.830                    | 0.499              | 0.217             | 4.809  | 0.309 | 0.359             | 0.281                | 0.246                 | 12.66  |
| 48  | 1.120                    | 0.552              | 0.284             | 6.489  | 0.309 | 0.359             | 0.268                | 0.259                 | 3.41   |

Table - 7.4.2 ( contd.)

| No. | L<br>kg/m <sup>2</sup> | a <sub>v</sub> /a <sub>t</sub><br>pred. | a <sub>p</sub> /a <sub>t</sub><br>exp. | Re     | Fr/We | o/o <sub>c</sub> | a <sub>st</sub> /a <sub>t</sub><br>obs. | a <sub>st</sub> /a <sub>t</sub><br>pred. | % err  |
|-----|------------------------|-----------------------------------------|----------------------------------------|--------|-------|------------------|-----------------------------------------|------------------------------------------|--------|
| 49  | 1.670                  | 0.631                                   | 0.331                                  | 9.676  | 0.309 | 0.359            | 0.300                                   | 0.277                                    | 7.50   |
| 50  | 2.220                  | 0.694                                   | 0.378                                  | 12.863 | 0.309 | 0.359            | 0.316                                   | 0.291                                    | 7.74   |
| 51  | 2.780                  | 0.748                                   | 0.397                                  | 16.108 | 0.309 | 0.359            | 0.351                                   | 0.303                                    | 13.78  |
| 52  | 5.500                  | 0.941                                   | 0.572                                  | 31.867 | 0.309 | 0.359            | 0.369                                   | 0.341                                    | 7.61   |
| 53  | 0.550                  | 0.469                                   | 0.310                                  | 5.080  | 0.122 | 0.359            | 0.159                                   | 0.156                                    | 2.24   |
| 54  | 0.830                  | 0.539                                   | 0.400                                  | 7.667  | 0.122 | 0.359            | 0.139                                   | 0.167                                    | -20.51 |
| 55  | 1.120                  | 0.596                                   | 0.430                                  | 10.345 | 0.122 | 0.359            | 0.166                                   | 0.176                                    | -6.26  |
| 56  | 1.670                  | 0.681                                   | 0.510                                  | 15.426 | 0.122 | 0.359            | 0.171                                   | 0.189                                    | -10.34 |
| 57  | 2.220                  | 0.749                                   | 0.600                                  | 20.506 | 0.122 | 0.359            | 0.149                                   | 0.198                                    | -32.68 |
| 58  | 2.780                  | 0.604                                   | 0.331                                  | 5.903  | 0.373 | 0.492            | 0.273                                   | 0.223                                    | 18.53  |
| 59  | 2.780                  | 0.502                                   | 0.274                                  | 5.265  | 0.466 | 0.656            | 0.228                                   | 0.198                                    | 12.95  |
| 60  | 2.780                  | 0.412                                   | 0.236                                  | 7.109  | 0.623 | 0.902            | 0.176                                   | 0.192                                    | -9.15  |
| 61  | 1.390                  | 0.276                                   | 0.151                                  | 1.203  | 0.707 | 1.148            | 0.125                                   | 0.126                                    | -0.98  |
| 62  | 2.780                  | 0.348                                   | 0.199                                  | 2.407  | 0.707 | 1.148            | 0.150                                   | 0.142                                    | 5.17   |
| 63  | 5.560                  | 0.440                                   | 0.274                                  | 4.814  | 0.707 | 1.148            | 0.165                                   | 0.160                                    | 3.17   |
| 64  | 4.200                  | 0.400                                   | 0.227                                  | 3.636  | 0.707 | 1.148            | 0.173                                   | 0.153                                    | 11.90  |
| 65  | 1.390                  | 0.272                                   | 0.151                                  | 1.129  | 0.532 | 1.098            | 0.121                                   | 0.112                                    | 7.70   |
| 66  | 2.780                  | 0.344                                   | 0.199                                  | 2.259  | 0.532 | 1.098            | 0.145                                   | 0.126                                    | 13.18  |
| 67  | 4.200                  | 0.394                                   | 0.227                                  | 3.412  | 0.532 | 1.098            | 0.168                                   | 0.135                                    | 19.36  |
| 68  | 5.560                  | 0.433                                   | 0.274                                  | 4.517  | 0.532 | 1.098            | 0.159                                   | 0.142                                    | 10.92  |
| 69  | 0.681                  | 0.213                                   | 0.089                                  | 1.630  | 0.912 | 1.209            | 0.123                                   | 0.145                                    | -17.96 |
| 70  | 0.682                  | 0.213                                   | 0.079                                  | 1.612  | 0.913 | 1.210            | 0.133                                   | 0.145                                    | -8.85  |
| 71  | 0.692                  | 0.214                                   | 0.076                                  | 1.684  | 0.911 | 1.208            | 0.137                                   | 0.146                                    | -6.38  |
| 72  | 0.670                  | 0.211                                   | 0.081                                  | 1.623  | 0.912 | 1.208            | 0.131                                   | 0.145                                    | -11.30 |
| 73  | 1.363                  | 0.269                                   | 0.112                                  | 3.353  | 0.911 | 1.207            | 0.156                                   | 0.165                                    | -5.46  |
| 74  | 1.372                  | 0.269                                   | 0.116                                  | 3.377  | 0.911 | 1.207            | 0.153                                   | 0.165                                    | -7.90  |
| 75  | 1.310                  | 0.264                                   | 0.111                                  | 3.096  | 0.914 | 1.211            | 0.153                                   | 0.162                                    | -6.10  |
| 76  | 1.238                  | 0.259                                   | 0.100                                  | 2.902  | 0.914 | 1.211            | 0.159                                   | 0.161                                    | -0.92  |
| 77  | 1.311                  | 0.264                                   | 0.092                                  | 3.041  | 0.915 | 1.212            | 0.172                                   | 0.162                                    | 5.95   |
| 78  | 0.290                  | 0.160                                   | 0.056                                  | 0.754  | 0.907 | 1.202            | 0.104                                   | 0.127                                    | -22.63 |
| 79  | 0.681                  | 0.214                                   | 0.080                                  | 1.800  | 0.905 | 1.200            | 0.133                                   | 0.148                                    | -11.07 |
| 80  | 0.703                  | 0.215                                   | 0.078                                  | 1.688  | 0.912 | 1.209            | 0.137                                   | 0.146                                    | -6.68  |
| 81  | 0.700                  | 0.215                                   | 0.081                                  | 1.698  | 0.912 | 1.208            | 0.133                                   | 0.146                                    | -9.92  |
| 82  | 0.665                  | 0.211                                   | 0.070                                  | 1.597  | 0.912 | 1.209            | 0.141                                   | 0.145                                    | -2.72  |
| 83  | 1.397                  | 0.271                                   | 0.131                                  | 3.500  | 0.909 | 1.205            | 0.140                                   | 0.166                                    | -18.16 |
| 84  | 1.390                  | 0.270                                   | 0.105                                  | 3.464  | 0.910 | 1.206            | 0.166                                   | 0.166                                    | 0.00   |
| 85  | 1.313                  | 0.265                                   | 0.107                                  | 3.170  | 0.912 | 1.208            | 0.158                                   | 0.163                                    | -3.20  |
| 86  | 1.360                  | 0.268                                   | 0.090                                  | 3.277  | 0.912 | 1.208            | 0.178                                   | 0.164                                    | 7.72   |
| 87  | 1.370                  | 0.269                                   | 0.098                                  | 3.316  | 0.912 | 1.209            | 0.171                                   | 0.164                                    | 3.80   |
| 88  | 0.681                  | 0.234                                   | 0.118                                  | 2.934  | 0.286 | 1.208            | 0.115                                   | 0.090                                    | 21.89  |
| 89  | 0.708                  | 0.237                                   | 0.140                                  | 3.173  | 0.285 | 1.203            | 0.098                                   | 0.091                                    | 6.59   |
| 90  | 0.673                  | 0.233                                   | 0.122                                  | 3.015  | 0.285 | 1.205            | 0.111                                   | 0.091                                    | 18.25  |
| 91  | 0.662                  | 0.232                                   | 0.136                                  | 2.951  | 0.286 | 1.207            | 0.096                                   | 0.090                                    | 5.84   |
| 92  | 0.703                  | 0.238                                   | 0.139                                  | 3.314  | 0.284 | 1.198            | 0.098                                   | 0.092                                    | 5.94   |
| 93  | 0.703                  | 0.237                                   | 0.158                                  | 3.314  | 0.284 | 1.200            | 0.080                                   | 0.092                                    | -15.76 |
| 94  | 0.700                  | 0.237                                   | 0.158                                  | 3.317  | 0.284 | 1.199            | 0.079                                   | 0.092                                    | -17.28 |
| 95  | 0.684                  | 0.235                                   | 0.139                                  | 3.240  | 0.284 | 1.199            | 0.096                                   | 0.092                                    | 4.30   |
| 96  | 0.705                  | 0.238                                   | 0.139                                  | 3.358  | 0.284 | 1.199            | 0.099                                   | 0.092                                    | 6.68   |
| 97  | 0.713                  | 0.238                                   | 0.156                                  | 3.090  | 0.286 | 1.208            | 0.081                                   | 0.091                                    | -12.18 |
| 98  | 0.745                  | 0.241                                   | 0.126                                  | 3.182  | 0.286 | 1.209            | 0.115                                   | 0.091                                    | 20.33  |
| 99  | 0.703                  | 0.236                                   | 0.134                                  | 3.084  | 0.286 | 1.207            | 0.102                                   | 0.091                                    | 11.22  |
| 100 | 1.496                  | 0.306                                   | 0.213                                  | 6.849  | 0.285 | 1.202            | 0.092                                   | 0.104                                    | -13.10 |

Optimization by modified simplex algorithm of Nelder and Mead. The results obtained using these two methodologies are reported in Table (7.4.3) and are discussed herewith.

CASE (I) : Initially the plausible values of indices  $\alpha$   $\beta$   $\delta$  and proportionality constant  $C$  were obtained by performing graphical analysis. The different plots of  $a_{st}$  Vs  $Re$  or  $(Fr/We)$  or  $(\sigma / \sigma_c)$  etc were obtained (not shown herewith) and the following approximate values of indices were estimated :-

$$\alpha = 0.172, \quad \beta = 0.33, \quad \delta = -0.666 \text{ and } C = 0.1659.$$

The statistical analysis of data fit based on the graphical estimate [step (1)] was as under :-

$$\% E_{avg} = -17.7, \quad \% E_{abs} = 20.2 \text{ and } \% S_{dev} = 2.67.$$

CASE (II) : Using the graphically obtained values of indices as the satisfactory guesses, the DSC - Powell algorithm was utilised to obtain the optimum values of indices. The regression of data as per step (2) yielded the following correlation :-

$$a_s/a_t = 0.1659 (Re)^{0.0758} (Fr/We)^{0.3372} (\sigma / \sigma_c)^{-0.6728} \quad (7.15)$$

By changing the direction of search sequences, it was observed that a correlation with superior statistical analysis could be obtained.

The direction of search sequence in the step (2) was as under :-

$$Re \text{ ---> } Fr/We \text{ ---> } (\sigma / \sigma_c) \text{ ---> } C$$

and that in step (5) was

$$(\sigma / \sigma_c) \text{ ---> } Fr/We \text{ ---> } Re \text{ ---> } C.$$

Table - ( 7.4.3 )

Statistical analysis for different correlations of  $a_{st}$   
based on  $(a_p/a_t)_{exp}$

$$\text{Equation 7.11 : } a_{st}/a_t = (a_w/a_t)_{pred} - (a_p/a_t)_{exp}$$

$$\text{Correlation Structure : } a_{st}/a_t = C \cdot Re^\alpha \cdot (Fr/We)^\beta \cdot (\sigma/\sigma_c)^\delta.$$

| No. | C | $\alpha$ | $\beta$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|

Case 1 :- Graphical estimation of data indicated the plausible values of indices  $\alpha$ ,  $\beta$ ,  $\delta$  and C were 0.172, 0.333, -0.666, and 0.1659 respectively. The Statistics of fit is indicated below.

|   |        |        |       |        |       |      |        |
|---|--------|--------|-------|--------|-------|------|--------|
| 1 | 0.1659 | 0.1722 | 0.333 | -0.666 | -17.7 | 20.2 | 2.674. |
|---|--------|--------|-------|--------|-------|------|--------|

Case 2 :- Optimisation by the D.S.C. - Powell algorithm.

Initialisation were based on guesses of indices,  $\alpha$ ,  $\beta$ ,  $\delta$ , and C obtained as 0.172, 0.333, -0.666, 0.1659 respectively from graphical estimation of the data. Direction of search sequence  $\alpha$ ,  $\beta$ ,  $\delta$ , C. respectively.

|   |        |        |        |         |       |       |       |
|---|--------|--------|--------|---------|-------|-------|-------|
| 2 | 0.1659 | 0.0758 | 0.3372 | -0.6728 | -3.26 | 15.18 | 3.05. |
|---|--------|--------|--------|---------|-------|-------|-------|

Direction of search sequence changed to  $\beta$ ,  $\alpha$ ,  $\delta$ , C. Initialisation as in step no 2.

|   |        |      |        |        |       |       |       |
|---|--------|------|--------|--------|-------|-------|-------|
| 3 | 0.1659 | 0.17 | 0.5047 | -0.665 | -2.95 | 13.45 | 2.64. |
|---|--------|------|--------|--------|-------|-------|-------|

Direction of search sequence changed to C,  $\delta$ ,  $\beta$ ,  $\alpha$ ,. Initialisation as in step no 2.

|   |        |      |        |        |       |       |      |
|---|--------|------|--------|--------|-------|-------|------|
| 4 | 0.1423 | 0.17 | 0.3327 | -0.737 | -1.12 | 13.27 | 2.79 |
|---|--------|------|--------|--------|-------|-------|------|

Direction of search sequence changed to  $\delta$ ,  $\beta$ ,  $\alpha$ , C. Initialisation as in step no. 2.

|    |        |        |      |        |       |       |      |
|----|--------|--------|------|--------|-------|-------|------|
| 5* | 0.1605 | 0.1726 | 0.50 | -0.725 | -0.40 | 12.34 | 2.42 |
|----|--------|--------|------|--------|-------|-------|------|

Value of the index  $\delta$  fixed as -0.442 similar to the correlations for wetted surface area and effective interfacial area during physical absorption. Direction of search sequence changed to  $\alpha$ ,  $\beta$ , C. Initialisation as in step no. 2.

|   |        |        |        |        |      |       |       |
|---|--------|--------|--------|--------|------|-------|-------|
| 6 | 0.1397 | 0.0220 | 0.3796 | -0.442 | 1.98 | 14.37 | 3.51. |
|---|--------|--------|--------|--------|------|-------|-------|

Table - 7.4.3 ( contd.)

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| No. | C | $\alpha$ | $\beta$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|

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Value of index  $\alpha$  fixed as 0.099 similar to the correlation for effective interfacial area during physical absorption . Direction of search sequence changed to  $\beta$ ,  $\delta$  C. Initialisation as in step no. 2.

|   |        |       |       |        |       |       |       |
|---|--------|-------|-------|--------|-------|-------|-------|
| 7 | 0.1699 | 0.099 | 0.385 | -0.653 | -4.99 | 15.09 | 2.87. |
|---|--------|-------|-------|--------|-------|-------|-------|

Value of index  $\alpha$  fixed as 0.0014 similar to the correlation for wetted surface area. Direction of search sequence changed to  $\beta$ ,  $\delta$ , C. Initialisation as in step no. 2.

|   |        |        |        |        |       |       |       |
|---|--------|--------|--------|--------|-------|-------|-------|
| 8 | 0.1868 | 0.0014 | 0.3937 | -0.849 | 1.998 | 17.09 | 3.56. |
|---|--------|--------|--------|--------|-------|-------|-------|

Case 3 : - Optimisation by the Modified Simplex algorithm of Nelder and Mead

All parameters are iterated, base values as obtained from step no 2. Each indice value perturbed 50 % at a time.

|   |        |       |       |        |      |       |       |
|---|--------|-------|-------|--------|------|-------|-------|
| 9 | 0.1575 | 0.164 | 0.482 | -0.822 | 0.89 | 11.97 | 2.34. |
|---|--------|-------|-------|--------|------|-------|-------|

$\beta$  value fixed as 0.50. Other parameters iterated as in step no. 9.

|    |       |       |      |        |      |       |       |
|----|-------|-------|------|--------|------|-------|-------|
| 10 | 0.150 | 0.208 | 0.50 | -0.767 | 0.69 | 11.77 | 2.34. |
|----|-------|-------|------|--------|------|-------|-------|

Value of index  $\delta$  fixed as -0.442 similar to the wetted surface area correlations. Other parameters iterated as in step no. 9.

|    |      |       |       |        |      |       |       |
|----|------|-------|-------|--------|------|-------|-------|
| 11 | 0.36 | 0.220 | 0.407 | -0.442 | 0.10 | 13.93 | 3.50. |
|----|------|-------|-------|--------|------|-------|-------|

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Note : The values of indices in the finalised generalised correlation are as per regression step marked by ( \* )

When the regression was done as per step (5), the mathematical modelling of the data yielded the following correlations :-

$$(a_s/a_t) = 0.1605 (Re)^{0.1726} (Fr/We)^{0.5} (\sigma/\sigma_c)^{-0.725} \quad (7.16)$$

The statistical analysis for both the correlations [Equations (7.15) and (7.16)] was as under :-.

The values of %  $E_{abs}$ , %  $E_{avg}$  and  $S_{dev}$  for the correlation (7.15) were 15.18, -3.26 and 3.05 respectively, while those for the correlation (7.16) were 12.34, -0.40, and 2.42 respectively. Thus by changing the direction of search sequence one can decrease the values of %  $E_{abs}$ , %  $E_{avg}$  and %  $S_{dev}$  to some extent. Therefore it can be concluded that one has to be careful in selecting a suitable methodology while performing mathematical modelling of data.

Further, by prefixing the indices of some groups as done in steps (6) to (8) yielded different correlations. The statistical analysis of these correlations was relatively inferior as compared to step (5). Therefore, the indices of all the numbers in equation (7.16) inclusive of the value of proportionality constant are such that the correlation under consideration predicts the values of  $a_{st}$  with the least amount of error.

CASE (III) : Using the optimization technique : Modified simplex algorithm of Nelder and Mead and by prefixing the indices of different numbers did not alter the statistical analysis appreciably for regression steps (9) to (11) wherein the values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  remained in the ranges of (0.10 to 0.89), (11.72 to 13.93) and (2.3 to 3.5) respectively.

When the index of  $(\sigma / \sigma_c)$  group was fixed to the value of -0.442 [similar to the index of  $(\sigma / \sigma_c)$  group obtained for the correlations of  $(a_w/a_t)$  and  $(a_p/a_t)$ ] and the regression of data was performed as per step (11). It resulted in the undermentioned correlation :-

$$(a_{st}/a_t) = 0.36 (Re)^{0.22} (Fr/We)^{0.407} (\sigma / \sigma_c)^{-0.442} \quad (7.17)$$

The values of %  $E_{avg}$ , %  $E_{abs}$  and %  $S_{dev}$  were 0.1, 13.93, and 3.5 respectively. The data fit of  $(a_{st}/a_t)$  by equation (7.17) is expected to be to some extent inferior than the data fit of  $(a_{st}/a_t)$  by equation (7.16). However, this correlation [Equation (7.17)] could also be considered as a satisfactory correlation for predicting the values of  $(a_{st}/a_t)$ . It is interesting to observe that the index of  $(\sigma / \sigma_c)$  in equations (7.3), (7.7) and (7.17) is the same and the value is -0.442.

#### 7.4.4 Results for $a_{st}$ by approach (II) : Use of equation (7.12) :-

The generalised correlation developed in this investigation for  $(a_p/a_t)$  [Equation (7.7)] could also be utilised conveniently to obtain the values of  $a_p$  under otherwise identical conditions mentioned for the data bank of experimental values of  $a_w$ . The data bank of  $a_w$  and the relevant conditions mentioned therein in Table (5.1) were utilised to obtain the predicted values of  $(a_p)$  under otherwise identical conditions. The values of  $a_{st}$  were estimated accordingly by using equation (7.12). The observed value of  $a_{st}$  obtained by using the approach (II) are reported in Table (7.4.4) which consists of 100 data points covering 21 systems/variations.

The critical analysis of these results indicate the following:-

- (i) With an increase in  $L$ , the values of  $a_{st}$  increase.
- (ii) With an increase in  $\rho_L$ ,  $\mu_L$  and  $\sigma / \sigma_c$  the values of  $a_{st}$  decrease.
- (iii) With an increase in the packing size, the values of  $a_{st}$  decrease considerably.

The parametric dependencies of different variables and their effect on the values of  $a_{st}$  observed for approach (II) and that observed for approach (I) are thus identical.

In order to correlate the observed values of  $a_{st}$  by approach (II), the equation (7.14) was utilised. The different dimensionless groups required for the processing of the data for  $a_{st}$  by equation (7.14) are reported in Table (7.4.4).

#### 7.4.5 Statistical analysis of different correlations :

Basis - Approach (II) Using equation (7.12) :

Processing of the data reported in Table - (7.4.4) was also done by the same methodologies which were utilised while processing the data of  $a_{st}$  obtained by using approach (I) : Use of equation (7.11). The results obtained using these methodologies are reported in Table (7.4.5) and are discussed herewith.

It was observed that by using the DSC-Powell algorithm, and by performing regressions as per steps (2) to (4), by changing the direction of search sequence, altered the values of  $\% E_{abs}$ ,  $\% E_{avg}$  and  $\% S_{dev}$  considerably.

Table - ( 7.4.4 )

Results inclusive of processing parameters for  $a_{st}$  based on  $(a_w/a_t)_{exp}$ .  
 Based on Equation (7.12) :  $a_{st}/a_t = (a_w/a_t)_{exp} - (a_p/a_t)_{pred}$

| No. | L<br>kg/m <sup>2</sup> | $a_w/a_t$<br>exp. | $a_p/a_t$<br>pred. | Re      | Fr/We | $\sigma/\sigma_c$ | $a_{st}/a_t$<br>obs. | $a_{st}/a_t$<br>pred. | % err   |
|-----|------------------------|-------------------|--------------------|---------|-------|-------------------|----------------------|-----------------------|---------|
| 1   | 6.781                  | 0.460             | 0.369              | 40.477  | 0.219 | 1.510             | 0.091                | 0.130                 | -42.92  |
| 2   | 12.206                 | 0.470             | 0.509              | 72.859  | 0.219 | 1.510             | -0.039               | 0.136                 | 446.38  |
| 3   | 13.562                 | 0.642             | 0.539              | 80.954  | 0.219 | 1.510             | 0.103                | 0.137                 | -33.08  |
| 4   | 1.356                  | 0.279             | 0.120              | 3.763   | 1.015 | 1.510             | 0.159                | 0.136                 | 14.67   |
| 5   | 3.390                  | 0.367             | 0.198              | 9.407   | 1.015 | 1.510             | 0.169                | 0.145                 | 14.13   |
| 6   | 6.781                  | 0.451             | 0.289              | 18.816  | 1.015 | 1.510             | 0.162                | 0.153                 | 5.66    |
| 7   | 1.356                  | 0.320             | 0.174              | 12.106  | 0.098 | 1.510             | 0.146                | 0.106                 | 27.45   |
| 8   | 3.390                  | 0.400             | 0.287              | 30.265  | 0.098 | 1.510             | 0.113                | 0.113                 | -0.48   |
| 9   | 6.781                  | 0.500             | 0.420              | 60.539  | 0.098 | 1.510             | 0.080                | 0.119                 | -48.49  |
| 10  | 13.562                 | 0.560             | 0.613              | 121.078 | 0.098 | 1.510             | -0.053               | 0.125                 | 335.55  |
| 11  | 4.230                  | 0.445             | 0.271              | 12.788  | 1.009 | 1.000             | 0.174                | 0.182                 | -4.41   |
| 12  | 5.830                  | 0.509             | 0.323              | 17.625  | 1.009 | 1.000             | 0.186                | 0.186                 | 0.00    |
| 13  | 10.850                 | 0.685             | 0.453              | 32.801  | 1.009 | 1.000             | 0.232                | 0.195                 | 15.87   |
| 14  | 13.080                 | 0.720             | 0.502              | 39.543  | 1.009 | 1.000             | 0.218                | 0.198                 | 9.31    |
| 15  | 5.400                  | 0.500             | 0.295              | 14.362  | 1.584 | 0.972             | 0.205                | 0.198                 | 2.99    |
| 16  | 8.100                  | 0.600             | 0.369              | 21.543  | 1.584 | 0.972             | 0.231                | 0.204                 | 11.58   |
| 17  | 0.850                  | 0.240             | 0.121              | 1.641   | 0.839 | 0.833             | 0.119                | 0.167                 | -40.69  |
| 18  | 3.400                  | 0.396             | 0.259              | 6.564   | 0.839 | 0.833             | 0.137                | 0.185                 | -35.04  |
| 19  | 5.100                  | 0.455             | 0.324              | 9.846   | 0.839 | 0.833             | 0.131                | 0.190                 | -44.83  |
| 20  | 6.800                  | 0.500             | 0.379              | 13.127  | 0.839 | 0.833             | 0.121                | 0.194                 | -60.32  |
| 21  | 0.500                  | 0.279             | 0.111              | 1.795   | 0.243 | 0.833             | 0.168                | 0.141                 | 16.45   |
| 22  | 1.190                  | 0.342             | 0.178              | 4.271   | 0.243 | 0.833             | 0.164                | 0.150                 | 8.67    |
| 23  | 2.270                  | 0.441             | 0.253              | 8.148   | 0.243 | 0.833             | 0.188                | 0.157                 | 16.28   |
| 24  | 7.200                  | 0.602             | 0.476              | 25.844  | 0.243 | 0.833             | 0.126                | 0.171                 | -36.10  |
| 25  | 0.140                  | 0.210             | 0.072              | 0.475   | 0.379 | 0.472             | 0.138                | 0.180                 | -30.03  |
| 26  | 0.280                  | 0.280             | 0.105              | 0.949   | 0.379 | 0.472             | 0.175                | 0.189                 | -7.97   |
| 27  | 0.550                  | 0.350             | 0.151              | 1.864   | 0.379 | 0.472             | 0.199                | 0.199                 | 0.00    |
| 28  | 1.110                  | 0.410             | 0.222              | 3.762   | 0.379 | 0.472             | 0.188                | 0.209                 | -11.62  |
| 29  | 1.380                  | 0.450             | 0.250              | 4.678   | 0.379 | 0.472             | 0.200                | 0.213                 | -6.66   |
| 30  | 2.780                  | 0.600             | 0.367              | 9.423   | 0.379 | 0.472             | 0.233                | 0.224                 | 3.67    |
| 31  | 0.278                  | 0.187             | 0.081              | 1.637   | 0.240 | 0.903             | 0.106                | 0.134                 | -26.25  |
| 32  | 0.550                  | 0.217             | 0.117              | 3.238   | 0.240 | 0.903             | 0.100                | 0.141                 | -41.51  |
| 33  | 1.110                  | 0.290             | 0.172              | 6.535   | 0.240 | 0.903             | 0.118                | 0.148                 | -26.17  |
| 34  | 2.780                  | 0.375             | 0.285              | 16.366  | 0.240 | 0.903             | 0.090                | 0.159                 | -76.13  |
| 35  | 5.500                  | 0.490             | 0.414              | 32.380  | 0.240 | 0.903             | 0.076                | 0.167                 | -118.92 |
| 36  | 0.140                  | 0.180             | 0.059              | 0.824   | 0.222 | 0.833             | 0.121                | 0.131                 | -7.95   |
| 37  | 0.278                  | 0.200             | 0.085              | 1.637   | 0.222 | 0.833             | 0.115                | 0.138                 | -20.13  |
| 38  | 0.550                  | 0.250             | 0.124              | 3.238   | 0.222 | 0.833             | 0.126                | 0.145                 | -14.86  |
| 39  | 1.110                  | 0.300             | 0.182              | 6.535   | 0.222 | 0.833             | 0.118                | 0.153                 | -29.12  |
| 40  | 1.380                  | 0.350             | 0.205              | 8.124   | 0.222 | 0.833             | 0.145                | 0.155                 | -6.79   |
| 41  | 2.780                  | 0.420             | 0.300              | 16.366  | 0.222 | 0.833             | 0.120                | 0.163                 | -36.52  |
| 42  | 0.140                  | 0.200             | 0.066              | 0.824   | 0.185 | 0.694             | 0.134                | 0.140                 | -4.37   |
| 43  | 0.278                  | 0.250             | 0.096              | 1.637   | 0.185 | 0.694             | 0.154                | 0.147                 | 4.44    |
| 44  | 0.550                  | 0.310             | 0.140              | 3.238   | 0.185 | 0.694             | 0.170                | 0.154                 | 9.25    |
| 45  | 1.110                  | 0.380             | 0.205              | 6.535   | 0.185 | 0.694             | 0.175                | 0.163                 | 6.93    |
| 46  | 1.380                  | 0.400             | 0.231              | 8.124   | 0.185 | 0.694             | 0.169                | 0.165                 | 2.10    |
| 47  | 2.780                  | 0.520             | 0.339              | 16.366  | 0.185 | 0.694             | 0.181                | 0.174                 | 3.81    |
| 48  | 0.140                  | 0.230             | 0.079              | 0.824   | 0.140 | 0.528             | 0.151                | 0.154                 | -2.11   |
| 49  | 0.278                  | 0.300             | 0.115              | 1.637   | 0.140 | 0.528             | 0.185                | 0.162                 | 12.33   |

Table - 7.4.4 ( contd.)

| No. | L<br>kg/m <sup>2</sup> | $a_w/a_t$<br>exp. | $a_p/a_t$<br>pred. | Re     | Fr/We | $\sigma/\sigma_c$ | $a_{st}/a_t$<br>obs. | $a_{st}/a_t$<br>pred. | % err  |
|-----|------------------------|-------------------|--------------------|--------|-------|-------------------|----------------------|-----------------------|--------|
| 50  | 0.550                  | 0.380             | 0.168              | 3.238  | 0.140 | 0.528             | 0.212                | 0.170                 | 19.88  |
| 51  | 1.110                  | 0.480             | 0.246              | 6.535  | 0.140 | 0.528             | 0.234                | 0.179                 | 23.39  |
| 52  | 1.380                  | 0.520             | 0.277              | 8.124  | 0.140 | 0.528             | 0.243                | 0.182                 | 24.99  |
| 53  | 2.780                  | 0.620             | 0.407              | 16.366 | 0.140 | 0.528             | 0.213                | 0.192                 | 10.11  |
| 54  | 0.140                  | 0.150             | 0.053              | 0.439  | 0.211 | 0.861             | 0.097                | 0.122                 | -25.94 |
| 55  | 0.280                  | 0.180             | 0.077              | 0.877  | 0.211 | 0.861             | 0.103                | 0.129                 | -25.27 |
| 56  | 0.550                  | 0.268             | 0.112              | 1.723  | 0.211 | 0.861             | 0.156                | 0.135                 | 13.44  |
| 57  | 1.400                  | 0.300             | 0.186              | 4.386  | 0.211 | 0.861             | 0.114                | 0.145                 | -27.45 |
| 58  | 2.780                  | 0.400             | 0.271              | 8.709  | 0.211 | 0.861             | 0.129                | 0.152                 | -18.27 |
| 59  | 4.150                  | 0.448             | 0.338              | 13.001 | 0.211 | 0.861             | 0.110                | 0.157                 | -42.19 |
| 60  | 0.140                  | 0.160             | 0.046              | 0.194  | 0.213 | 0.917             | 0.114                | 0.112                 | 1.71   |
| 61  | 0.280                  | 0.185             | 0.068              | 0.388  | 0.213 | 0.917             | 0.117                | 0.118                 | -0.17  |
| 62  | 0.560                  | 0.275             | 0.099              | 0.776  | 0.213 | 0.917             | 0.176                | 0.124                 | 29.78  |
| 63  | 1.400                  | 0.310             | 0.163              | 1.939  | 0.213 | 0.917             | 0.147                | 0.132                 | 9.98   |
| 64  | 1.670                  | 0.345             | 0.179              | 2.313  | 0.213 | 0.917             | 0.166                | 0.134                 | 19.00  |
| 65  | 2.100                  | 0.380             | 0.203              | 2.909  | 0.213 | 0.917             | 0.177                | 0.136                 | 22.77  |
| 66  | 2.780                  | 0.410             | 0.237              | 3.850  | 0.213 | 0.917             | 0.173                | 0.139                 | 19.47  |
| 67  | 4.150                  | 0.450             | 0.295              | 5.748  | 0.213 | 0.917             | 0.155                | 0.143                 | 7.37   |
| 68  | 0.140                  | 0.200             | 0.055              | 0.279  | 0.093 | 0.833             | 0.145                | 0.107                 | 26.19  |
| 69  | 0.280                  | 0.250             | 0.081              | 0.558  | 0.093 | 0.833             | 0.169                | 0.112                 | 33.59  |
| 70  | 1.400                  | 0.400             | 0.195              | 2.791  | 0.093 | 0.833             | 0.205                | 0.127                 | 38.40  |
| 71  | 2.780                  | 0.410             | 0.283              | 5.542  | 0.093 | 0.833             | 0.127                | 0.133                 | -4.91  |
| 72  | 0.560                  | 0.195             | 0.074              | 0.862  | 2.344 | 1.098             | 0.121                | 0.161                 | -32.69 |
| 73  | 1.160                  | 0.265             | 0.110              | 1.786  | 2.344 | 1.098             | 0.155                | 0.170                 | -9.40  |
| 74  | 1.680                  | 0.300             | 0.135              | 2.587  | 2.344 | 1.098             | 0.165                | 0.174                 | -5.41  |
| 75  | 2.240                  | 0.324             | 0.158              | 3.450  | 2.344 | 1.098             | 0.166                | 0.178                 | -6.98  |
| 76  | 2.780                  | 0.343             | 0.177              | 4.281  | 2.344 | 1.098             | 0.166                | 0.181                 | -9.19  |
| 77  | 0.560                  | 0.234             | 0.096              | 0.948  | 1.610 | 0.754             | 0.138                | 0.185                 | -33.53 |
| 78  | 1.160                  | 0.310             | 0.142              | 1.963  | 1.610 | 0.754             | 0.168                | 0.195                 | -16.32 |
| 79  | 1.680                  | 0.350             | 0.174              | 2.843  | 1.610 | 0.754             | 0.176                | 0.200                 | -14.08 |
| 80  | 2.780                  | 0.453             | 0.230              | 4.705  | 1.610 | 0.754             | 0.223                | 0.208                 | 6.90   |
| 81  | 0.560                  | 0.290             | 0.091              | 0.472  | 1.620 | 0.738             | 0.199                | 0.178                 | 10.69  |
| 82  | 1.160                  | 0.345             | 0.136              | 0.977  | 1.620 | 0.738             | 0.209                | 0.187                 | 10.46  |
| 83  | 1.680                  | 0.395             | 0.166              | 1.415  | 1.620 | 0.738             | 0.229                | 0.192                 | 15.84  |
| 84  | 2.240                  | 0.428             | 0.195              | 1.886  | 1.620 | 0.738             | 0.233                | 0.197                 | 15.76  |
| 85  | 2.780                  | 0.452             | 0.219              | 2.341  | 1.620 | 0.738             | 0.233                | 0.200                 | 14.27  |
| 86  | 0.560                  | 0.180             | 0.062              | 0.383  | 2.448 | 1.230             | 0.118                | 0.144                 | -22.40 |
| 87  | 1.160                  | 0.240             | 0.093              | 0.793  | 2.448 | 1.230             | 0.147                | 0.152                 | -3.22  |
| 88  | 1.680                  | 0.278             | 0.114              | 1.149  | 2.448 | 1.230             | 0.164                | 0.156                 | 5.01   |
| 89  | 2.240                  | 0.320             | 0.133              | 1.532  | 2.448 | 1.230             | 0.187                | 0.160                 | 14.73  |
| 90  | 2.780                  | 0.318             | 0.150              | 1.901  | 2.448 | 1.230             | 0.168                | 0.162                 | 3.77   |
| 91  | 0.560                  | 0.165             | 0.053              | 0.089  | 2.277 | 1.230             | 0.112                | 0.128                 | -14.38 |
| 92  | 1.160                  | 0.195             | 0.079              | 0.184  | 2.277 | 1.230             | 0.116                | 0.135                 | -16.42 |
| 93  | 1.680                  | 0.245             | 0.097              | 0.266  | 2.277 | 1.230             | 0.148                | 0.139                 | 6.40   |
| 94  | 2.240                  | 0.270             | 0.113              | 0.355  | 2.277 | 1.230             | 0.157                | 0.142                 | 9.60   |
| 95  | 2.780                  | 0.285             | 0.127              | 0.440  | 2.277 | 1.230             | 0.158                | 0.144                 | 8.63   |
| 96  | 0.560                  | 0.170             | 0.058              | 0.255  | 2.328 | 1.246             | 0.112                | 0.138                 | -23.69 |
| 97  | 1.160                  | 0.230             | 0.087              | 0.529  | 2.328 | 1.246             | 0.143                | 0.146                 | -1.82  |
| 98  | 1.680                  | 0.260             | 0.107              | 0.766  | 2.328 | 1.246             | 0.153                | 0.150                 | 2.51   |
| 99  | 2.240                  | 0.280             | 0.125              | 1.021  | 2.328 | 1.246             | 0.155                | 0.153                 | 1.61   |
| 100 | 2.780                  | 0.300             | 0.140              | 1.267  | 2.328 | 1.246             | 0.160                | 0.155                 | 2.77   |

Table ( 7.4.5 )

Statistical analysis for different correlations of  $a_{st}$   
based on  $(a_w / a_t)_{exp}$

$$\text{Equation 7.12 : } a_{st} / a_t = (a_w / a_t)_{exp} - (a_p / a_t)_{pred}$$

$$\text{Correlation Structure : } a_{st} / a_t = C \cdot Re^\alpha \cdot (Fr/We)^\beta \cdot (\sigma / \sigma_c)^\delta$$

| No. | C | $\alpha$ | $\beta$ | $\delta$ | % E <sub>avg</sub> | % E <sub>abs</sub> | % S <sub>dev</sub> |
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|
|-----|---|----------|---------|----------|--------------------|--------------------|--------------------|

Case 1 :- Optimisation by the D.S.C. - Powell algorithm.

Initialisation were based on guesses of indices,  $\alpha$ ,  $\beta$ ,  $\delta$ , and C obtained as 0.172, 0.333, -0.666, 0.1659 respectively from graphical estimation of the data. Direction of search sequence  $\alpha$ ,  $\beta$ ,  $\delta$ , C. respectively.

|   |      |      |        |         |       |      |       |
|---|------|------|--------|---------|-------|------|-------|
| 1 | 0.17 | 0.02 | 0.3395 | -0.6678 | -2.60 | 25.9 | 4.82. |
|---|------|------|--------|---------|-------|------|-------|

Direction of search sequence changed to  $\beta$ ,  $\alpha$ ,  $\delta$ , C. Initialisation as in step no 1

|   |        |        |       |         |      |       |       |
|---|--------|--------|-------|---------|------|-------|-------|
| 2 | 0.1507 | 0.0734 | 0.143 | -0.4937 | -5.5 | 17.82 | 3.16. |
|---|--------|--------|-------|---------|------|-------|-------|

Direction of search sequence changed to C,  $\delta$ ,  $\beta$ ,  $\alpha$ ,. Initialisation as in step no 1

|   |        |      |       |         |       |       |       |
|---|--------|------|-------|---------|-------|-------|-------|
| 3 | 0.1522 | 0.17 | 0.497 | -0.7158 | -0.90 | 33.87 | 6.34. |
|---|--------|------|-------|---------|-------|-------|-------|

Direction of search sequence changed to  $\delta$ ,  $\beta$ ,  $\alpha$ , C. Initialisation as in step no. 1

|   |        |        |      |         |       |       |       |
|---|--------|--------|------|---------|-------|-------|-------|
| 4 | 0.1284 | 0.2338 | 0.28 | -0.0043 | -6.20 | 32.45 | 5.77. |
|---|--------|--------|------|---------|-------|-------|-------|

Case 2 : - Optimisation by the Modified Simplex algorithm of Nelder and Mead

All parameters are iterated, base values as obtained from step no 1. Each index value perturbed 50 % at a time.

|   |        |      |        |        |       |      |       |
|---|--------|------|--------|--------|-------|------|-------|
| 5 | 0.1519 | 0.07 | 0.1458 | -0.564 | -6.27 | 18.4 | 3.15. |
|---|--------|------|--------|--------|-------|------|-------|

Note : Data points 2 and 10 of Table ( 7.4.4 ) wherein the values of  $(a_p)_{pred} > (a_w)_{exp}$  were excluded from statistical analysis hence the values of various errors listed here are for 98 data points.

The direction of search sequence in step (2) was as under :-

$$(Fr/We) \rightarrow (Re) \rightarrow (\sigma / \sigma_c) \rightarrow C$$

When the regression was done as per step (2), the mathematical modelling of the data yielded the following correlation having statistical analysis as %  $E_{abs} = 17.82$ , %  $E_{avg} = -5.5$  and %  $S_{dev} = 3.16$  :-

$$(a_s/a_t) = 0.1507 (Re)^{0.07} (Fr/We)^{0.143} (\sigma / \sigma_c)^{-0.49} \quad (7.18)$$

The direction of search sequence in step (4) was as under :-

$$(\sigma / \sigma_c) \rightarrow (Fr/We) \rightarrow Re \rightarrow C.$$

When regression was done as per step (4), the mathematical modelling of the data yielded the following correlation having statistical analysis as %  $E_{abs} = 32.45$ , %  $E_{avg} = -6.2$  and %  $S_{dev} = 5.57$  :-

$$(a_{st}/a_t) = 0.1284 (Re)^{0.2338} (Fr/We)^{0.28} (\sigma / \sigma_c)^{-0.0043} \quad (7.19)$$

The data fit of  $(a_{st}/a_t)$  by equation (7.19) is expected to be highly unsatisfactory as could be observed from the statistical analysis.

Thus, sometimes misleading results can be obtained. Inconsistent results are also likely to be obtained. Also the correlations with highly inferior statistical analysis are likely to be developed. Hence one must be very careful while using different optimization techniques for obtaining generalised correlations by mathematical modelling.

Using optimization technique the modified simplex algorithm of Nelder and Mead, by performing regression of the data as per step (5), yielded the following correlation having statistical analysis as  $\% E_{avg} = 18.4$ ,  $\% E_{avg} = -6.27$ , and  $\% S_{dev} = 3.15$  :-

$$(a_{st}/a_t) = 0.1519 (Re)^{0.07} (Fr/We)^{0.1458} (\sigma / \sigma_c)^{-0.564} \quad (7.20)$$

The statistical analysis of the correlations [equations (7.18) and (7.20)] is comparable. Hence both the correlations can be used satisfactorily for predicting the values of  $(a_{st})$  by approach (II).

#### 7.4.6 Comparison of Correlations for predicting the values of $a_{st}$ by approaches (I) and (II) :

The values of  $(a_{st}/a_t)$  can be predicted satisfactorily by the correlation [Equation (7.16)] using approach (I). The detailed statistical analysis of the correlation [Equation (7.16)] can be described as follows :-

(i) The values of  $\% E_{abs}$ ,  $\% E_{avg}$  and  $\% S_{dev}$  were 12.34, 0.4 and 2.42 respectively.

(ii) From the 100 data points, 75 % of the predicted values of  $a_{st}$  were within  $\pm 25$  % of the observed data values, 63 % within  $\pm 15$  % and 40 % within  $\pm 10$  %.

Thus, while obtaining a generalised correlation for predicting the values of  $a_{st}$  by mathematical modelling, the following two approaches have been used :

(i) Approach (I) based on equation (7.11).

(ii) Approach (II) based on equation (7.12).

The methodology used in approach (I) has yielded the correlation : Equation (7.16) Further the methodology used in approach (II) has yielded the correlation : Equation (7.18). The comparison of the statistical analysis of the correlations indicates that the equation (7.16) for predicting the values of  $a_{st}$  ( $\% E_{abs} = 12.34$ ) appears to be more reliable and appropriate than the equation (7.18) for predicting the values of  $a_{st}$  ( $\% E_{av} = 17.82$ ). Therefore, the basis used for predicting the values of  $a_{st}$  in approach (I) also appears to be more reliable and appropriate than that used in approach (II).

#### 7.4.7 Comparison between observed values and predicted values of $a_{st}$ :

The observed values of  $(a_{st}/a_t)$  obtained by using data bank of  $a_p$  by approach (I) and the predicted value of  $(a_{st}/a_t)$  obtained by correlation [Equation (7.16)] are tabulated in Table (7.4.6) and are plotted in Figure (7.4).

Thus, figure (7.4) shows a parity plot of equation (7.16) wherein the values of  $(a_{st}/a_t)$  predicted have been plotted versus  $(a_{st}/a_t)$  observed values. The satisfactory correlation fit in Figure (7.4) clearly reflects that equation (7.16) can correlate satisfactorily all the data on  $a_{st}$ .

The generalised correlation proposed by Puranik and Vogelpohl (106) [Equation (2.11)] happens to be the only correlation available

Table ( 7.4.6 )

Comparison between observed and predicted values of  $a_{st}$   
 based on  $(a_p/a_t)_{\text{expt.}}$

| No. | $a_w$<br>-----<br>pred. | $a_p$<br>$\frac{m^2}{m^3}$<br>exp. | $a_{st}$<br>obs. | $a_{st}/a_t$<br>obs. | $a_{st}/a_t$<br>pred.<br>This work | % err. | $a_{st}/a_t$<br>pred.<br>Puranik & Vogelpohl | % err   |
|-----|-------------------------|------------------------------------|------------------|----------------------|------------------------------------|--------|----------------------------------------------|---------|
| 1   | 45.9                    | 28.1                               | 17.8             | 0.091                | 0.093                              | -1.96  | 0.114                                        | -25.02  |
| 2   | 57.8                    | 39.6                               | 18.2             | 0.094                | 0.105                              | -11.99 | 0.114                                        | -21.93  |
| 3   | 66.4                    | 48.2                               | 18.1             | 0.093                | 0.112                              | -20.86 | 0.114                                        | -22.61  |
| 4   | 92.1                    | 68.4                               | 23.7             | 0.121                | 0.133                              | -9.58  | 0.114                                        | 6.07    |
| 5   | 95.9                    | 67.8                               | 28.1             | 0.144                | 0.136                              | 5.76   | 0.114                                        | 20.91   |
| 6   | 99.5                    | 78.0                               | 21.5             | 0.110                | 0.138                              | -25.70 | 0.114                                        | -3.54   |
| 7   | 25.0                    | 12.2                               | 12.8             | 0.066                | 0.068                              | -3.42  | 0.114                                        | -73.44  |
| 8   | 30.7                    | 11.6                               | 19.1             | 0.098                | 0.076                              | 22.64  | 0.114                                        | -16.69  |
| 9   | 35.4                    | 14.5                               | 20.9             | 0.107                | 0.081                              | 23.97  | 0.114                                        | -6.62   |
| 10  | 39.7                    | 21.0                               | 18.7             | 0.096                | 0.086                              | 10.10  | 0.114                                        | -18.78  |
| 11  | 44.6                    | 19.4                               | 25.2             | 0.129                | 0.092                              | 29.08  | 0.114                                        | 11.72   |
| 12  | 58.4                    | 30.8                               | 27.6             | 0.142                | 0.105                              | 25.67  | 0.114                                        | 19.50   |
| 13  | 88.0                    | 52.2                               | 35.8             | 0.184                | 0.130                              | 29.20  | 0.114                                        | 37.88   |
| 14  | 104.4                   | 72.0                               | 32.3             | 0.167                | 0.142                              | 15.03  | 0.113                                        | 32.14   |
| 15  | 27.3                    | 16.0                               | 11.3             | 0.058                | 0.071                              | -22.46 | 0.113                                        | -95.03  |
| 16  | 36.9                    | 19.7                               | 17.1             | 0.088                | 0.083                              | 6.15   | 0.113                                        | -28.00  |
| 17  | 44.0                    | 26.6                               | 17.4             | 0.089                | 0.091                              | -2.09  | 0.114                                        | -27.92  |
| 18  | 56.2                    | 33.5                               | 22.7             | 0.116                | 0.103                              | 11.31  | 0.114                                        | 2.01    |
| 19  | 66.5                    | 45.4                               | 21.0             | 0.108                | 0.113                              | -4.35  | 0.114                                        | -5.78   |
| 20  | 76.6                    | 57.2                               | 19.3             | 0.099                | 0.121                              | -22.02 | 0.114                                        | -15.00  |
| 21  | 96.8                    | 70.9                               | 25.9             | 0.133                | 0.137                              | -2.97  | 0.114                                        | 13.98   |
| 22  | 45.5                    | 9.9                                | 35.6             | 0.102                | 0.117                              | -13.83 | 0.219                                        | -114.31 |
| 23  | 56.8                    | 15.0                               | 41.8             | 0.120                | 0.131                              | -8.67  | 0.219                                        | -82.56  |
| 24  | 70.9                    | 22.3                               | 48.5             | 0.139                | 0.146                              | -4.97  | 0.219                                        | -57.36  |
| 25  | 76.2                    | 32.5                               | 43.6             | 0.125                | 0.152                              | -21.17 | 0.219                                        | -75.02  |
| 26  | 82.7                    | 30.1                               | 52.6             | 0.151                | 0.159                              | -4.92  | 0.219                                        | -45.26  |
| 27  | 97.5                    | 30.1                               | 67.4             | 0.194                | 0.173                              | 10.90  | 0.219                                        | -13.33  |
| 28  | 125.2                   | 54.4                               | 70.8             | 0.203                | 0.196                              | 3.52   | 0.219                                        | -7.92   |
| 29  | 146.9                   | 63.5                               | 83.4             | 0.240                | 0.213                              | 11.11  | 0.219                                        | 8.41    |
| 30  | 153.4                   | 58.8                               | 94.7             | 0.272                | 0.218                              | 19.91  | 0.219                                        | 19.31   |
| 31  | 170.1                   | 75.5                               | 94.6             | 0.272                | 0.230                              | 15.46  | 0.219                                        | 19.23   |
| 32  | 49.7                    | 13.9                               | 35.8             | 0.103                | 0.122                              | -18.65 | 0.219                                        | -113.54 |
| 33  | 68.5                    | 27.7                               | 40.8             | 0.117                | 0.144                              | -22.61 | 0.219                                        | -86.99  |
| 34  | 80.2                    | 32.3                               | 47.9             | 0.138                | 0.156                              | -13.33 | 0.219                                        | -59.42  |
| 35  | 107.8                   | 43.9                               | 63.9             | 0.184                | 0.182                              | 1.07   | 0.219                                        | -19.50  |
| 36  | 137.7                   | 64.4                               | 73.3             | 0.211                | 0.206                              | 2.12   | 0.219                                        | -4.25   |
| 37  | 117.1                   | 17.5                               | 99.7             | 0.286                | 0.202                              | 29.47  | 0.135                                        | 52.76   |
| 38  | 131.5                   | 37.7                               | 93.9             | 0.270                | 0.214                              | 20.51  | 0.135                                        | 49.84   |
| 39  | 144.3                   | 40.6                               | 103.6            | 0.298                | 0.225                              | 24.49  | 0.135                                        | 54.57   |
| 40  | 52.6                    | 21.8                               | 30.8             | 0.093                | 0.120                              | -29.08 | 0.211                                        | -125.84 |
| 41  | 66.0                    | 29.6                               | 36.4             | 0.110                | 0.135                              | -22.80 | 0.211                                        | -91.22  |
| 42  | 105.4                   | 58.7                               | 46.7             | 0.142                | 0.172                              | -21.55 | 0.211                                        | -48.77  |
| 43  | 113.6                   | 64.0                               | 49.7             | 0.151                | 0.179                              | -18.86 | 0.211                                        | -39.94  |
| 44  | 142.9                   | 87.4                               | 55.5             | 0.168                | 0.201                              | -19.69 | 0.211                                        | -25.26  |
| 45  | 114.4                   | 54.6                               | 59.8             | 0.181                | 0.204                              | -12.54 | 0.122                                        | 32.57   |
| 46  | 143.4                   | 65.5                               | 77.9             | 0.236                | 0.229                              | 2.99   | 0.122                                        | 48.27   |
| 47  | 164.6                   | 71.8                               | 92.9             | 0.281                | 0.246                              | 12.66  | 0.122                                        | 56.62   |
| 48  | 182.1                   | 93.6                               | 88.5             | 0.268                | 0.259                              | 3.41   | 0.122                                        | 54.44   |

Table - 7.4.6 ( contd.)

| No. | $a_v$<br>-----<br>pred. | $a_{2p3}$<br>$m/m$<br>exp. | $a_{st}$<br>-----<br>obs. | $a_{st}/a_t$<br>obs. | $a_{st}/a_t$<br>pred.<br>This work | % err  | $a_{st}/a_t$<br>pred.<br>Puranik & Vogelpohl | % err   |
|-----|-------------------------|----------------------------|---------------------------|----------------------|------------------------------------|--------|----------------------------------------------|---------|
| 49  | 208.2                   | 109.2                      | 99.0                      | 0.300                | 0.277                              | 7.50   | 0.122                                        | 59.28   |
| 50  | 229.0                   | 124.8                      | 104.2                     | 0.316                | 0.291                              | 7.74   | 0.122                                        | 61.33   |
| 51  | 247.0                   | 131.0                      | 115.9                     | 0.351                | 0.303                              | 13.78  | 0.122                                        | 65.24   |
| 52  | 310.5                   | 188.8                      | 121.7                     | 0.369                | 0.341                              | 7.61   | 0.122                                        | 66.89   |
| 53  | 97.1                    | 64.2                       | 33.0                      | 0.159                | 0.156                              | 2.24   | 0.037                                        | 76.62   |
| 54  | 111.5                   | 82.8                       | 28.7                      | 0.139                | 0.167                              | -20.51 | 0.037                                        | 73.16   |
| 55  | 123.3                   | 89.0                       | 34.3                      | 0.166                | 0.176                              | -6.26  | 0.037                                        | 77.52   |
| 56  | 141.0                   | 105.6                      | 35.4                      | 0.171                | 0.189                              | -10.34 | 0.037                                        | 78.22   |
| 57  | 155.1                   | 124.2                      | 30.9                      | 0.149                | 0.198                              | -32.68 | 0.037                                        | 75.06   |
| 58  | 199.4                   | 109.2                      | 90.2                      | 0.273                | 0.223                              | 18.53  | 0.139                                        | 49.07   |
| 59  | 165.6                   | 90.5                       | 75.2                      | 0.228                | 0.198                              | 12.95  | 0.160                                        | 29.93   |
| 60  | 135.9                   | 78.0                       | 57.9                      | 0.176                | 0.192                              | -9.15  | 0.186                                        | -5.94   |
| 61  | 91.1                    | 49.9                       | 41.2                      | 0.125                | 0.126                              | -0.98  | 0.197                                        | -58.11  |
| 62  | 115.0                   | 65.5                       | 49.5                      | 0.150                | 0.142                              | 5.17   | 0.197                                        | -31.74  |
| 63  | 145.1                   | 90.5                       | 54.6                      | 0.165                | 0.160                              | 3.17   | 0.197                                        | -19.35  |
| 64  | 132.1                   | 74.9                       | 57.2                      | 0.173                | 0.153                              | 11.90  | 0.197                                        | -13.98  |
| 65  | 89.8                    | 49.9                       | 39.9                      | 0.121                | 0.112                              | 7.70   | 0.172                                        | -41.79  |
| 66  | 113.4                   | 65.5                       | 47.8                      | 0.145                | 0.126                              | 13.18  | 0.172                                        | -18.33  |
| 67  | 130.2                   | 74.9                       | 55.3                      | 0.168                | 0.135                              | 19.36  | 0.172                                        | -2.36   |
| 68  | 143.0                   | 90.5                       | 52.5                      | 0.159                | 0.142                              | 10.92  | 0.172                                        | -7.73   |
| 69  | 74.0                    | 31.1                       | 42.9                      | 0.123                | 0.145                              | -17.96 | 0.221                                        | -79.07  |
| 70  | 74.0                    | 27.6                       | 46.4                      | 0.133                | 0.145                              | -8.85  | 0.221                                        | -65.67  |
| 71  | 74.4                    | 26.6                       | 47.8                      | 0.137                | 0.146                              | -6.38  | 0.221                                        | -60.46  |
| 72  | 73.6                    | 28.2                       | 45.4                      | 0.131                | 0.145                              | -11.30 | 0.221                                        | -69.00  |
| 73  | 93.4                    | 39.1                       | 54.3                      | 0.156                | 0.165                              | -5.46  | 0.220                                        | -41.18  |
| 74  | 93.7                    | 40.5                       | 53.2                      | 0.153                | 0.165                              | -7.90  | 0.220                                        | -44.28  |
| 75  | 92.0                    | 38.8                       | 53.2                      | 0.153                | 0.162                              | -6.10  | 0.221                                        | -44.37  |
| 76  | 90.3                    | 34.9                       | 55.3                      | 0.159                | 0.161                              | -0.92  | 0.221                                        | -38.83  |
| 77  | 92.0                    | 32.1                       | 59.9                      | 0.172                | 0.162                              | 5.95   | 0.221                                        | -28.40  |
| 78  | 55.8                    | 19.6                       | 36.2                      | 0.104                | 0.127                              | -22.63 | 0.220                                        | -111.83 |
| 79  | 74.3                    | 27.9                       | 46.4                      | 0.133                | 0.148                              | -11.07 | 0.220                                        | -64.95  |
| 80  | 74.7                    | 27.0                       | 47.7                      | 0.137                | 0.146                              | -6.68  | 0.221                                        | -60.95  |
| 81  | 74.7                    | 28.3                       | 46.4                      | 0.133                | 0.146                              | -9.92  | 0.221                                        | -65.60  |
| 82  | 73.4                    | 24.3                       | 49.1                      | 0.141                | 0.145                              | -2.72  | 0.221                                        | -56.47  |
| 83  | 94.3                    | 45.4                       | 48.9                      | 0.140                | 0.166                              | -18.16 | 0.220                                        | -56.88  |
| 84  | 94.1                    | 36.4                       | 57.7                      | 0.166                | 0.166                              | 0.00   | 0.220                                        | -32.96  |
| 85  | 92.2                    | 37.2                       | 55.0                      | 0.158                | 0.163                              | -3.20  | 0.221                                        | -39.63  |
| 86  | 93.3                    | 31.5                       | 61.8                      | 0.178                | 0.164                              | 7.72   | 0.221                                        | -24.14  |
| 87  | 93.5                    | 34.1                       | 59.4                      | 0.171                | 0.164                              | 3.80   | 0.221                                        | -29.19  |
| 88  | 45.6                    | 23.1                       | 22.5                      | 0.115                | 0.090                              | 21.89  | 0.115                                        | 0.00    |
| 89  | 46.3                    | 27.2                       | 19.1                      | 0.098                | 0.091                              | 6.59   | 0.115                                        | -17.24  |
| 90  | 45.5                    | 23.9                       | 21.6                      | 0.111                | 0.091                              | 18.25  | 0.115                                        | -3.65   |
| 91  | 45.2                    | 26.5                       | 18.7                      | 0.096                | 0.090                              | 5.84   | 0.115                                        | -20.07  |
| 92  | 46.3                    | 27.2                       | 19.1                      | 0.098                | 0.092                              | 5.94   | 0.114                                        | -16.68  |
| 93  | 46.3                    | 30.7                       | 15.5                      | 0.080                | 0.092                              | -15.76 | 0.115                                        | -43.78  |
| 94  | 46.2                    | 30.9                       | 15.3                      | 0.079                | 0.092                              | -17.28 | 0.114                                        | -45.57  |
| 95  | 45.9                    | 27.2                       | 18.7                      | 0.096                | 0.092                              | 4.30   | 0.114                                        | -19.26  |
| 96  | 46.4                    | 27.1                       | 19.3                      | 0.099                | 0.092                              | 6.68   | 0.114                                        | -15.54  |
| 97  | 46.3                    | 30.5                       | 15.8                      | 0.081                | 0.091                              | -12.18 | 0.115                                        | -41.98  |
| 98  | 47.0                    | 24.6                       | 22.4                      | 0.115                | 0.091                              | 20.33  | 0.115                                        | 0.00    |
| 99  | 46.1                    | 26.1                       | 20.0                      | 0.102                | 0.091                              | 11.22  | 0.115                                        | -12.30  |
| 100 | 59.6                    | 41.6                       | 18.0                      | 0.092                | 0.104                              | -13.10 | 0.115                                        | -24.12  |

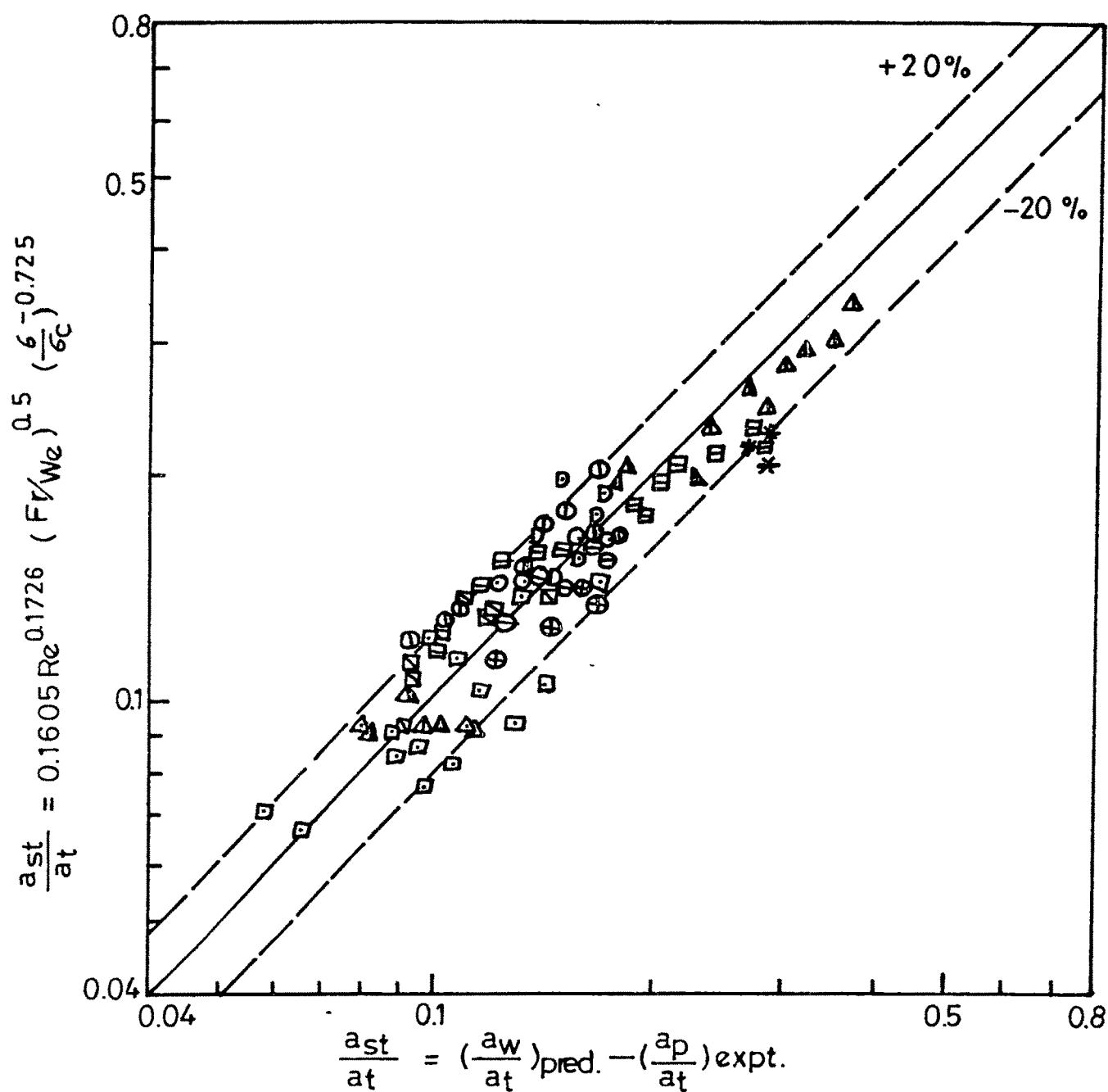


Fig. 7.4. STATIC AREA IN PACKED COLUMN.  
COMPARISON OF PREDICTED vs. CALCULATED VALUES.

## LEGENDS FOR FIG. 7.4

| Data No. | Relevant Information                                                                                             | Symbol |
|----------|------------------------------------------------------------------------------------------------------------------|--------|
| 1-6      | Fellinger (13), $\text{NH}_3$ -Water, C.R.R., 25mm.                                                              | ◻      |
| 7-36     | Yoshida (14), $\text{CO}_2$ -Water, C.R.R., 25mm, 15mm.                                                          | ◻ ◻    |
| 37-39    | Yoshida (14), $\text{CO}_2$ -Methanol, C.R.R., 15mm.                                                             | *      |
| 40-44    | Hikita (15), $\text{CO}_2$ -Water, C.R.R., 15mm.                                                                 | ⊙      |
| 45-57    | Hikita (15), $\text{CO}_2$ -Methanol, C.R.R., 15mm, 25mm.                                                        | △ D    |
| 58-60    | Hikita (15), $\text{CO}_2$ -aq. Methanol, C.R.R., $d_p=15\text{mm}$ ,<br>Methanol concentrations, 60%, 30%, 10%. | △ ▲ ▽  |
| 61-68    | Hikita (15), $\text{CO}_2$ -cane sugar solution, C.R.R.,<br>$d_p=15\text{mm}$ , sugar concentrations, 5%, 10%.   | ⊖ ⊖    |
| 69-100   | Yoshida (16,17), Methanol vapour-Water,<br>C.R.R., 15mm, 25mm.                                                   | ○ △    |

in literature for predicting the values of  $a_{st}$ . The values of  $(a_{st}/a_t)$  predicted by the correlation of Puranik and Vogelpohl are also reported in Table (7.4.6). The detailed statistical analysis of this correlation can be described as follows :-

(i) The values of %  $E_{abs}$ , %  $E_{avg}$  and %  $S_{dev}$  were 40.98, -15.93 and 8.23 respectively.

(ii) Out of 100 data points of the observed values of  $(a_{st}/a_t)$  only 14 % of the predicted values were within  $\pm 10$  % of the observed value, 21 % were within  $\pm 15$  % and 39 % were within  $\pm 25$  %.

The comparison of statistical analysis for both these generalised correlations reveals that the values of %  $E_{abs}$  for the correlation of Puranik and Vogelpohl and that developed in this investigation [Equation (7.16)] are 40.98 and 12.34 respectively. The corresponding values of %  $E_{avg}$  are -15.93 and -0.4 respectively and the corresponding values of %  $S_{dev}$  are 8.23 and 2.42 respectively. Hence, it can be concluded <sup>that</sup> the generalised correlation developed in this investigation is expected to be superior in comparison to the existing generalised correlation of Puranik and Vogelpohl.

The lack of data fit in the generalised correlation of Puranik and Vogelpohl may be attributed to the following reasons :-

(i) As has been already discussed in Chapter (6) section 6.1.5 the quantification of static area as  $a_{st} = (a_c - a_p)$  under certain conditions appears to be inappropriate. Hence, the static area model based on this approach is expected to generate under certain

conditions, the wrong observed values of  $a_{st}$ . Necessarily the data fit obtained by this correlation is expected to be inferior.

(ii) For the mathematical modelling of data for  $a_{st}$ , only a single model parameter - namely  $(We/Fr)$  was used by these investigators. As has been also discussed in Chapter (6), the values of  $a_{st}$  do get affected by altering the values of  $L$ ,  $\mu_L$ , and  $(\sigma / \sigma_c)$ . Hence the additional model parameters like  $Re$  and  $(\sigma / \sigma_c)$  should have been included by these investigators in their generalised correlation.

#### 7.5.0 PREDICTION OF INTERFACIAL AREAS :

Thus, in the present chapter (7) a software package developed in this investigation the salient features of which have been mentioned in Chapter (3), has been successfully used to perform mathematical modelling of data on interfacial areas.

The following generalised correlation [Equation (7.3)] could be used satisfactorily to predict the value of wetted surface area :-

$$a_w/a_t = 1.431 (Re)^{0.0014} (We)^{0.165} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442}$$

The following generalised correlation [Equation (7.7)] can be used to predict the values of interfacial area during physical absorption :-

$$a_p/a_t = 1.08 (Re)^{0.099} (We)^{0.22} (Fr)^{0.002} (\sigma / \sigma_c)^{-0.442}$$

Further, the following generalised correlation can also be used satisfactorily to predict the values of  $(a_w/a_t)$  and  $(a_p/a_t)$  by using

appropriate values of index for 'We' :-

$$a_w/a_t \text{ or } a_p/a_t = 1.431 (\text{Re})^{0.0014} (\text{We})^\beta (\text{Fr})^{0.002} (\sigma / \sigma_c)^{-0.442}$$

where  $\beta = 0.165$  for wetted surface area and 0.237 for physical absorption.

The following generalised correlation [Equation (7.10)] could be utilised to predict the values of effective interfacial area during chemical absorption :-

$$a_c/a_t = 0.455 (\text{Re})^{0.227} (\text{We})^{0.058} (\text{Fr})^{0.002} (\sigma / \sigma_c)^{-1.104}$$

The values of  $(a_{st}/a_t)$  can be predicted satisfactorily by the following generalised correlation [Equation (7.16)] :-

$$a_{st}/a_t = 0.1605 (\text{Re})^{0.1726} (\text{Fr}/\text{We})^{0.5} (\sigma / \sigma_c)^{-0.725}$$

The static area model developed in this investigation can be used successfully to predict the values of  $(a_p/a_t)$  by using the following correlation :-

$$a_p/a_t = (a_w/a_t)_{\text{pred}} - (a_{st}/a_t)_{\text{pred}} \quad (7.21)$$

The detailed statistical analysis of the correlation is as under :-

$$\% E_{\text{avg}} = 4.76, \% E_{\text{abs}} = 18.23 \text{ and } \% S_{\text{dev}} = 5.04.$$

Using equations (7.3) and (7.16) one can predict the values of  $(a_w/a_t)$  and  $(a_{st}/a_t)$  under otherwise identical conditions and then subtracting  $(a_{st}/a_t)$  from  $(a_w/a_t)$ , the values of  $(a_p/a_t)$  can be predicted. This generalised correlation [Equation (7.21)] developed

in this investigation is not merely an empirical correlation but it elucidates the mechanism of mass transfer during physical absorption.

The static area model developed in this investigation can also be used successfully to predict the values of  $(a_c/a_t)$  by using the following correlation :-

$$a_c/a_t = (a_p/a_t)_{\text{pred}} + (a_{st}/a_t)_{\text{pred}} \quad (7.22)$$

The detailed statistical analysis of the correlation is as under :

$$\% E_{\text{avg}} = 2.29, \% E_{\text{abs}} = 20.28 \text{ and } \% S_{\text{dev}} = 0.95$$

Using equation (7.7) and (7.16) one can predict the values of  $(a_p/a_t)$  and  $(a_{st}/a_t)$  under otherwise identical conditions and then adding  $(a_p/a_t)$  to  $(a_{st}/a_t)$ , the values of  $(a_c/a_t)$  can be predicted. This additional generalised correlation [Equation (7.22)] developed in this investigation is not merely an empirical correlation but it also elucidates the mechanism of mass transfer during chemical absorption.

Thus, the static area model developed in this investigation (186,187) could be utilised conveniently for predicting the values of effective interfacial areas during physical absorption as well as during chemical absorption.