CHAPTER III

AN ANALYSIS OF THE ENERGY LEVELS OF O^{18} AND O^{19}.

1. Introduction

In this chapter we consider somewhat more carefully, and also with a slightly different approach the energy levels of O^{18} and O^{19} and their relationship. It is obvious that if the same configuration space is assumed to describe the low-lying energy levels of O^{18} and O^{19}, the matrix elements of the Hamiltonian for different states of O^{19} can be directly expressed in terms of the matrix elements of O^{18}. Such an analysis - described in section 4 - is independent of any assumptions on the nature of the two-body interactions. We use such an analysis to obtain some sort of a check on the results obtained in the previous chapter. In section 3, a more direct approach is used, i.e. assuming the Hamiltonian matrix elements for O^{18} given by different types of interactions considered in the previous chapter. We evaluate the corresponding energy level spectrum of O^{19}, and compare with experiment to see if it is possible to discriminate between the several interactions which all give the same results for O^{18} level scheme. In section 2, we briefly discuss our results for O^{18} energies and wavefunctions comparing them with the results obtained by other authors who have previously made similar calculations.

2. Comparison with results of other authors

In view of the simplicity of structure, O^{18} and O^{19} have attracted considerable attention from several authors over many years. We mention the early work of Elliott and Flowers[13], and Redlich[34]. More recently Dawson, Talmi and Walecka[19] have used the Brueckner-Bethe-Goldstone[35] formalism to calculate the energy levels of O^{18}, using a reaction matrix calculated from the free nucleon-nucleon potential of Brueckner-Gammel-Thaler[36]. Some information on the wavefunctions of the various states involved is also derived from an analysis of the deuteron stripping reactions by Mcfarlane and French[37].

In the analysis of Elliott and Flowers, and Redlich, the configuration space included in addition to the $d_{5/2}$ and $s_{1/2}$ orbitals, also the $d_{3/2}$ orbital. As discussed in chapter II, we feel that in the spirit of the Brueckner theory, it appears unjustified to include the $d_{3/2}$ state, since it lies quite well above the $d_{5/2}$ state ($\sim$ 5 MeV). The calculations of Dawson, Talmi and Walecka also in the first approximation neglect the $d_{3/2}$ state, and their results as well as those of the earlier authors on the wavefunctions of the O^{18} - O^{19} states show quite clearly that even when the $d_{3/2}$ state is included in the configuration space, the components of the wavefunctions containing this state are rather small, and their effect on

the energy of the state would be even smaller. Therefore we can compare our results with those of the others neglecting the $d_{3/2}$ state, without introducing any important inconsistency.

Our calculated energy levels agree much better with experimental results than those of the other authors as shown in figure 1. This is inevitable since our potential is constructed in such a way as to give the best fit with the experimental results. A check of our calculations would be provided by the prediction of the $J = 3^+$ level $(s_{1/2}d_{5/2})$ in O^{18}. This we predict at 4.2 to 4.7 MeV (see figures 6 and 7 of chapter II). Dawson, Talmi and Walecka's calculation also predicts 3^+ level in this range. It is very desirable that this be checked experimentally. Location of this state is very important for determining the p-state forces, since it depends only on the p-state interaction. If the energy of this state is $E_3 < 4.8$ MeV, the matrix element of the interaction $\langle s_{1/2}d_{5/2}: 3|V_{12}|s_{1/2}d_{5/2}: 3\rangle$ will be < 0, whereas if $E_3 > 4.8$ MeV, the matrix element is > 0.† Thus in former case the p-state interaction would be attractive, whereas in the latter case it would be repulsive. Of course, in either case, it is certainly small in magnitude, but if, experimentally, it turns out to be repulsive, our analysis of the chapter II would have to be slightly revised.

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† This follows because $E_3 - E_o = \langle s_{1/2}d_{5/2}| V_{12}|s_{1/2}d_{5/2}\rangle_3 + (\Delta = 0.85 \text{ MeV}) + 3.92 \text{ MeV}$.

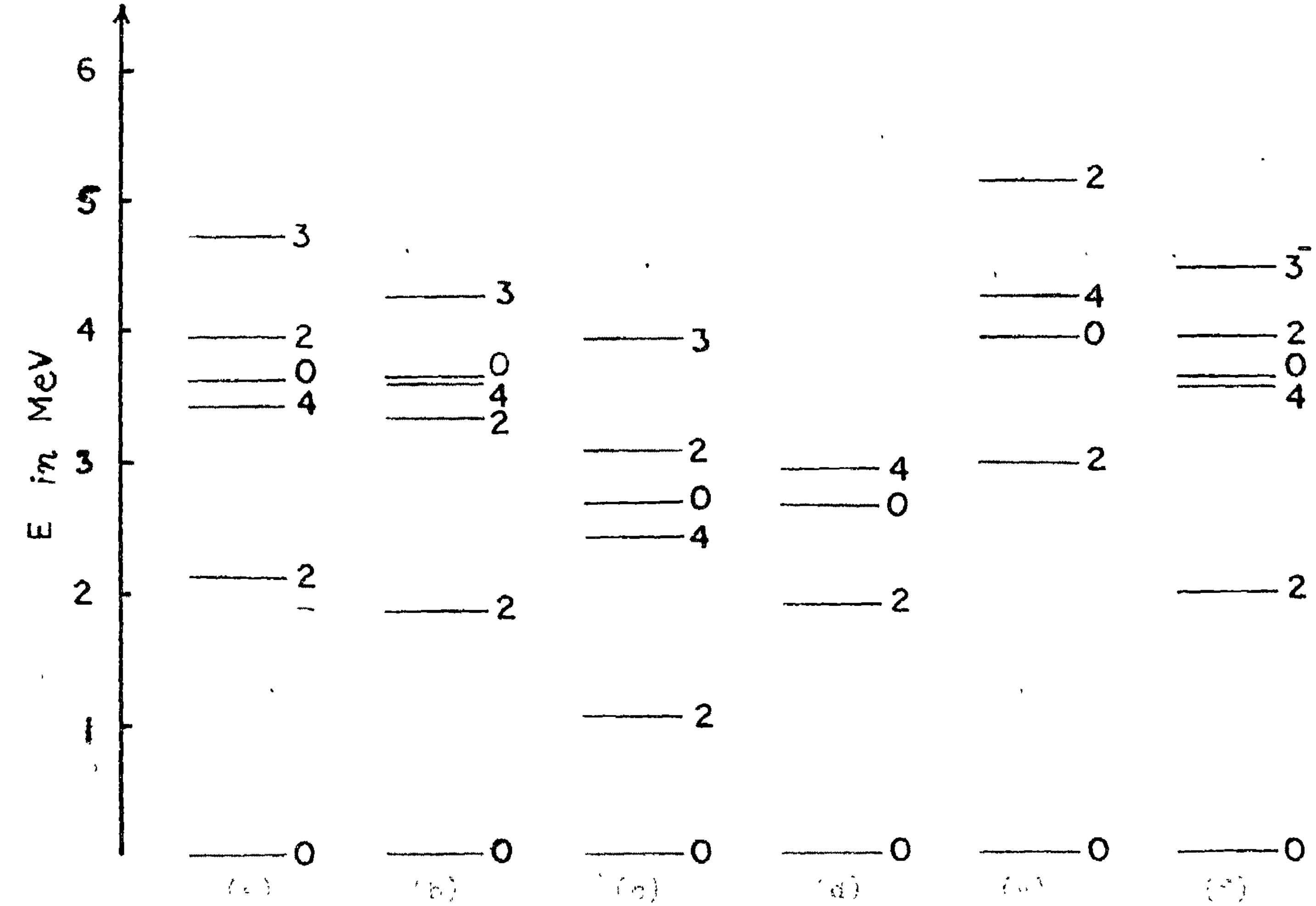

Fig. 1. Energy levels of [illegible]. (a) With parameters [illegible] (singlet [illegible] interactions only), λ [illegible] and [illegible] MeV (triplet [illegible] interactions). λ [illegible]. (b) With parameters [illegible] MeV [illegible] interactions). λ [illegible] MeV (triplet [illegible] interactions). λ [illegible]. (c) [illegible]

Since our interest is in devising a potential which would be a good model for the actual reaction matrix, it might be of interest to compare in detail the matrix elements calculated by our model and by the more involved reaction matrix calculation of Dawson, Talmi and Walecka[19]. Their calculations give for the Talmi integral $I_{os} = \langle os \mid V_{12} \mid os \rangle$ the best value of -8.3 MeV, which also explains fairly satisfactorily the observed spectrum. The values of I_{os} obtained by us using the parameters $V_o = -25$ MeV (singlet s-interactions only) and $\lambda = 1.0$ is -8.84 MeV and using parameters $V_o = -40$ MeV (singlet s- and d-interactions) and $\lambda = 0.70$ is -7.80 MeV. This compares very well with the value obtained by Dawson, Talmi and Walecka. The detailed comparison of the values of different $I_{n\ell}$ obtained by us with those of Dawson, Talmi and Walecka is given in table 1 (singlet even) and table 2 (triplet odd) for one set of parameters (a) only (i.e. $V_o = -25$ MeV, $\lambda = 1.0$). The results for other set of parameters (b) are rather similar. We also compare the relevant matrix elements for various states with those obtained by the above authors in table 3 for set of parameters (a) and in table 4 for set of parameters (b).

We further note that Dawson, Talmi and Walecka give also results for O^{18} spectrum calculated by using a simple non-singular central potential which has been used by Hulthén and Sugawara[38] to fit the low energy

Table 1

The values of $I_{n\ell} = \langle n\ell | V_{12} | n\ell \rangle$ for different n and ℓ for singlet potential.

$I_{n\ell}$	Present calculations with parameters V_o = -25 MeV λ = 1.0	Dawson, Talmi and Walecka
I_{os}	-8.840	-8.323
I_{1s}	-5.525	-5.988
I_{2s}	-4.352	-3.296
I_{op}	-4.420	-2.813
I_{od}	-2.210	-0.777
I_{1d}	-2.489	-1.328
I_{of}	-1.105	-0.267
I_{og}	-0.553	-0.106

Table 2

The values of $I_{n\ell} = \langle n\ell | V_{12} | n\ell \rangle$ for different n and ℓ for triplet odd potential.

$I_{n\ell}$ \ λ	0.5	0.8	1.0	1.2	Dawson, Talmi and Walecka.
I_{op}	-0.666	-0.864	-0.937	-0.990	-0.321
I_{1p}	-1.086	-0.962	-0.817	-0.758	-0.402
I_{od}	-0.134	-0.330	-0.469	-0.585	-0.118
I_{of}	-0.026	-0.129	-0.234	-0.345	-0.054

Table 3

The numerical values in MeV of the matrix elements of various states in O^{18} using our parameters (V_o = -25 MeV, λ = 1.0) for singlet even and those corresponding to figure 6 of chapter II for triplet odd, and those obtained by Dawson, Talmi and Walecka.

(a) $\langle d^2_{5/2} | V_{12} | d^2_{5/2} \rangle$

J	Present calculations singlet even	Dawson et al singlet even	Present calculations Triplet odd.	Dawson et al. Triplet odd.
0	-3.160	-3.209	λ =0.5, -0.350 λ =0.8, -0.352 λ =1.0, -0.351 λ =1.2, -0.350	+1.245
2	-0.870	-1.183	λ =0.5, -0.264 λ =0.8, -0.327 λ =1.0, -0.357 λ =1.2, -0.374	-0.419
4	-0.663	-0.671	λ =0.5, -0.034 λ =0.8, -0.043 λ =1.0, -0.047 λ =1.2, -0.050	-0.095

(b) $\langle d^2_{5/2} | V_{12} | s^2_{1/2} \rangle$

J	Present calculation singlet even	Dawson et al singlet even
0	-1.370	-0.952

Table 3 (Continued)

(c) $\langle s^2_{1/2} | V_{12} | s^2_{1/2} \rangle$

J	Present calculation singlet even	Dawson et al singlet even
0	-2.900	-3.019

(d) $\langle d^2_{5/2} | V_{12} | d_{5/2} s_{1/2} \rangle$

J	Present calculation singlet even.	Dawson et al singlet even	Present calculation Triplet odd.	Dawson et al Triplet odd
2	-0.895	-0.824	-	+0.044

(e) $\langle d_{5/2} s_{1/2} | V_{12} | d_{5/2} s_{1/2} \rangle$

J	Present calculation singlet even.	Dawson et al singlet even	Present calculation Triplet odd.	Dawson et al Triplet odd.
2	-1.595	-1.926	λ =0.5,-0.201 λ =0.8,-0.224 λ =1.0,-0.236 λ =1.2,-0.244	+0.237
3	-	-	λ =0.5,-0.249 λ =0.8,-0.280 λ =1.0,-0.296 λ =1.2,-0.308	-0.649

Table 4

The numerical values in MeV of the matrix elements of various states in O^{18} using parameters $V_o = -40$ MeV, $\lambda = 0.65$ for singlet even and those corresponding to figure 7 of chapter II for triplet odd.

(a) $\langle d^2_{5/2} | V_{12} | d^2_{5/2} \rangle$

J	Singlet even	Triplet odd	
0	-2.91	λ=0.5, -1.00 λ=1.0, -1.00	λ=0.8, -1.00 λ=1.2, -1.00
2	-0.95	λ=0.5, -0.76 λ=1.0, -1.02	λ=0.8, -0.93 λ=1.2, -1.07
4	-0.50	λ=0.5, -0.10 λ=1.0, -0.13	λ=0.8, -0.12 λ=1.2, -0.14

(b) $\langle d^2_{5/2} | V_{12} | s^2_{1/2} \rangle$

J	Singlet even
0	-1.01

(c) $\langle s^2_{1/2} | V_{12} | s^2_{1/2} \rangle$

J	Singlet even
0	-2.68

Table 4 (Continued)

(d) $\langle d^2_{5/2} | V_{12} | d_{5/2} s_{1/2} \rangle$

J	Singlet even
2	-0.68

(e) $\langle d_{5/2} s_{1/2} | V_{12} | d_{5/2} s_{1/2} \rangle$

J	Singlet even	Triplet odd
2	-1.48	λ =0.5, -0.57 λ =0.8, -0.64 λ =1.0, -0.68 λ =1.2, -0.70
3	-	λ =0.5, -0.71 λ =0.8, -0.80 λ =1.0, -0.84 λ =1.2, -0.88

nucleon-nucleon scattering data. For a Yukawa shape, their parameters are

$$\begin{aligned} V_{os} &= -47 \text{ MeV} & V_{ot} &= -52 \text{ MeV} \\ \ell_{os} &= 1.2 \text{ fm} & \ell_{ot} &= 1.4 \text{ fm} \end{aligned} \qquad (2.1)$$

It is somewhat remarkable that for our assumption of same interaction in all even states (i.e. $\ell = 0, 2$ here) we also obtain very similar parameters viz. $V_{os} = -40$ MeV, $\ell_{o\delta} = 1.2$ fm and although triplet parameters are not uniquely specified, the above values are not in disagreement (see chapter II, figure 7). The agreement is not very surprising, since for only two extra-core nucleons, and with such a simple non-singular potential the reaction matrix may not be drastically different from the potential.

Finally, we remark on the wavefunctions obtained by us. In table 5 we give the wavefunctions for the $J = 0$ ground state obtained by Elliott and Flowers, Redlich, Dawson, Talmi and Walecka, and by us. In table 6 are listed the wavefunctions for $J = 2$ state, neglecting however the states involving $d_{3/2}$ orbital. It may be noted that a considerable amount of configuration mixing is predicted by all theoretical calculations. On the other hand, Mcfarlane and French find from an analysis of stripping reaction data (O^{17} (d,p) O^{18}) a much smaller amount of mixing i.e. the amplitude of the two states are

$$\begin{aligned} A\,(d_{5/2}s_{1/2}) &\cong 0.36\text{-}0.43 \\ A\,(d_{5/2})^2 &\cong 0.89\text{-}0.93 \end{aligned} \qquad (2.2)$$

Table 5

The amplitudes of wavefunctions for J = 0, the ground state of O^{18}.

Configuration.	Redlich[34]	Elliott[13] & Flowers	Dawson[19] et al.	Present calculations Parameters (a)	(b)
$(d_{5/2})^2$	0.86	0.89	0.89	0.91	0.89
$(s_{1/2})^2$	0.31	0.24	0.39	0.42	0.45
$(d_{3/2})^2$	0.40	0.39	0.23	-	-

Table 6

The amplitudes of wavefunctions for J = 2 state of O^{18}.

Configuration.	Redlich[34]	Dawson[19] et al	Present calculations Parameters (a)	(b)
$(d_{5/2})^2$	0.71	0.79	0.74	0.76
$(d_{5/2}s_{1/2})$	0.64	0.54	0.68	0.65
$(d_{5/2}d_{3/2})$	-0.20	0.16	-	-
$(d_{3/2})^2$	0.14	0.10	-	-
$(d_{3/2}s_{1/2})$	0.20	-0.21	-	-

We do not discuss this discrepancy any further.

3. Energy levels of O^{19} (I)

In the previous chapter, we have discussed the nature of the effective nuclear interaction in T = 1 states of nuclei O^{18}, Ti^{50} etc. The parameters of the interaction in singlet even and triplet odd states of the two nucleons outside the closed shells were determined under two assumptions:

(a) the even state interactions are non-local and are effective in $\ell = 0$ (ℓ refers to relative orbital angular momentum), i.e. s-state only, and

(b) the even state interactions are the same in all states $\ell = 0, 2$ etc.

For O^{18}, the parameters of the even interactions (assumed to have Gaussian radial shape $V_o \exp(-r^2/r_o^2)$ were determined to be (a) $V_o = -25$ MeV, $\lambda = r_o/r_\ell = 1.0$ and (b) $V_o = -40$ MeV, $\lambda = 0.65$, and several sets of corresponding odd-state interactions which give a good fit to the O^{18} spectrum were also given. In this section we apply these different sets of parameters to calculate the energy level spectrum of O^{19}, to see if these additional data can help to distinguish between the different sets.

The configurations that we consider for the T = 3/2 states of O^{19} are

(a) $(d_{5/2})^3$, (b) $(d_{5/2})^2 s_{1/2}$ and (c) $d_{5/2}(s_{1/2})^2$

The wavefunction corresponding to the configuration (a) is just the one hole wavefunction. For the configurations (b) and (c) the antisymmetrised wavefunction for T = 3/2 state is given by[39]

$$\begin{aligned} &|(j)^2 J_0, j' : J\rangle \\ &= \frac{1}{\sqrt{N}} \left\{ |j^2 : J_0\rangle \times |j'\rangle \right. \\ &\left. + a^{-1} (-1)^{j+J} \sum_{J_1} (-1)^{1+J_1} U(jjJj'; J_0 J_1) |jj' : J_1\rangle \times |j\rangle \right\} \end{aligned} \tag{3.1}$$

where N is the normalisation coefficient given as

$$N = 3\left\{1 - 2(-1)^{j+J} U(jjJj'; J_0 J_0)\, \delta_{nn'}\, \delta_{ll'}\, \delta_{jj'}\right\} \tag{3.2}$$

and $$\begin{aligned} a = a_{jj'} &= 1/2 \quad \text{if} \quad j \text{ and } j' \text{ are equivalent} \\ &= 1/\sqrt{2} \text{ if} \quad j \text{ and } j' \text{ are nonequivalent} \end{aligned} \tag{3.3}$$

The matrix elements for the configuration (a) for various allowed states can be written with the help of coefficients of fractional parentage as

$$\begin{aligned} &\langle (d_{5/2})^3 J | V_{12} | (d_{5/2})^3 J \rangle \\ &= 3 \sum_{J_1} \langle (d_{5/2})^3 J | \} (d_{5/2})^2 J_1 \rangle^2 \langle (d_{5/2})^2 J_1 | V_{12} | (d_{5/2})^2 J_1 \rangle \end{aligned} \tag{3.4}$$

where $\langle (d_{5/2})^3 J \mid \} (d_{5/2})^2 J_1 \rangle$ are the coefficients of

fractional parentage[40]. For the configurations (b) and (c) the matrix elements can be written as

$$\begin{aligned}
&\langle (j)^2 J_0, j' : J \,|\, V_{12} \,|\, (j_1)^2 J_0', j_1' : J \rangle \\
&= \frac{3}{\sqrt{N_1 N_2}} \Big\{ \delta_{J_0 J_0'} \delta_{j' j_1'} \langle j^2 J_0 \,|\, V_{12} \,|\, j_1^2 J_0 \rangle \\
&- \delta_{j' j_1} (-1)^{j+J} a_1^{-1} U(j j_1 J j_1'; J_0' J_0) \langle j^2 J_0 \,|\, V_{12} \,|\, j_1 j_1' J_0 \rangle \\
&- \delta_{j_1' j} (-1)^{j+J} a^{-1} U(j j J j'; J_0 J_0') \langle j j' J_0' \,|\, V_{12} \,|\, j_1^2 J_0' \rangle \\
&+ \delta_{j j_1} (a a_1)^{-1} \sum_{J_1} U(j j J j'; J_0 J_1) U(j j J j_1'; J_0' J_1) \times \\
&\qquad \langle j j' J_1 \,|\, V_{12} \,|\, j j' J_1 \rangle \Big\}.
\end{aligned} \tag{3.5}$$

Thus we see that the matrix elements of states in O^{19} could be written in terms of the matrix elements of two-body states in O^{18} and one can employ the parameters of O^{18} to calculate the energy levels of O^{19}. The matrix elements for parameters (a) and (b) are given in tables 7 and 8. The matrix elements for triplet odd state interactions (triplet p- only) are also given in these tables for different ranges and strengths. The results of all calculations are given in figure 2. We can see that in case (a) the odd-state interactions appear to have only a small effect on the energies of the different levels and all the different sets of odd interactions of chapter II, give essentially the same results.

Table 7

The matrix elements of O^{19} for parameters (a) plus the triplet p-state interactions. Triplet p-interactions have different ranges and strengths. Single particle energy Δ is already included. The off-diagonal matrix elements do not have any contribution from odd-state interactions.

Configurations.	λ / V_1 / J	0.5 -37.23 MeV	0.8 -8.90 MeV	1.0 -5.30 MeV	1.2 -3.70 MeV
$\langle d^3_{5/2} \mid d^3_{5/2} \rangle$	3/2	-3.0282	-3.1694	-3.2381	-3.2772
	5/2	-4.3276	-4.3964	-4.4271	-4.4448
	9/2	-2.3694	-2.4306	-2.4597	-2.4774
$\langle d^2_{5/2} s_{1/2} \mid d^2_{5/2} s_{1/2} \rangle$	1/2	-3.6673	-3.7175	-3.7400	-3.7557
	3/2	-1.6290	-1.7172	-1.7668	-1.7969
	5/2	-1.0788	-1.1928	-1.2497	-1.2873
	7/2	-1.3929	-1.4313	-1.4503	-1.4634
	9/2	-0.4151	-0.4857	-0.5183	-0.5448
$\langle s^2_{1/2} d_{5/2} \mid s^2_{1/2} d_{5/2} \rangle$	5/2	-2.1798	-2.2275	-2.2511	-2.2682
$\langle d^3_{5/2} \mid d^2_{5/2} s_{1/2} \rangle$	3/2	1.5650	1.5650	1.5650	1.5650
$\langle d^3_{5/2} \mid d^2_{5/2} s_{1/2} \rangle$	9/2	-0.0600	-0.0600	-0.0600	-0.0600
$\langle d^3_{5/2} \mid d^2_{5/2} s_{1/2} \rangle$	5/2	-0.2975	-0.2975	-0.2975	-0.2975
$\langle d^3_{5/2} \mid s^2_{1/2} d_{5/2} \rangle$	5/2	-0.9125	-0.9125	-0.9125	-0.9125
$\langle d^2_{5/2} s_{1/2} \mid s^2_{1/2} d_{5/2} \rangle$	5/2	-0.5775	0.5775	0.5775	0.5775

Table 8

The matrix elements of O^{19} for parameters (b) plus the triplet p-state interactions. Triplet p-interactions have different ranges and strengths. Single particle energy Δ is already included. The off-diagonal matrix elements do not have any contribution from odd-state interactions.

Configurations.	J	0.5 -106.4 MeV	0.8 -25.30 MeV	1.0 -15.10 MeV	1.2 -10.60 MeV
$\langle d^3_{5/2} \vert d^3_{5/2} \rangle$	3/2	-4.1704	-4.5628	-4.7632	-4.8880
	5/2	-4.9204	-5.1083	-5.1994	-5.2595
	9/2	-2.5049	-2.6750	-2.7599	-2.8160
$\langle d^2_{5/2} s_{1/2} \vert d^2_{5/2} s_{1/2} \rangle$	1/2	-4.7122	-4.8440	-4.9130	-4.9705
	3/2	-2.6773	-2.9423	-3.0879	-3.1853
	5/2	-2.4084	-2.7221	-2.8892	-3.0105
	7/2	-1.6606	-1.7653	-1.8214	-1.8643
	9/2	-1.1564	-1.3493	-1.4459	-1.5315
$\langle s^2_{1/2} d_{5/2} \vert s^2_{1/2} d_{5/2} \rangle$	5/2	-2.5880	-2.7182	-2.7879	-2.8437
$\langle d^3_{5/2} \vert d^2_{5/2} s_{1/2} \rangle$	3/2	1.1880	1.1880	1.1880	1.1880
	9/2	-0.0440	-0.0440	-0.0440	-0.0440
	5/2	-0.2240	-0.2240	-0.2240	-0.2240
$\langle d^3_{5/2} \vert s^2_{1/2} d_{5/2} \rangle$	5/2	-0.6760	-0.6760	-0.6760	-0.6760
$\langle d^2_{5/2} s_{1/2} \vert s^2_{1/2} d_{5/2} \rangle$	5/2	0.4400	0.4400	0.4400	0.4400

On the other hand for case (b) the odd-state interactions change considerably the energies of the states and in particular the energy of the lowest J = 3/2 state is quite different for different odd-state forces which give equally good fit for O^{18}. One can easily see that the set (iii) gives the best fit for the lowest J = 3/2 state. We also give for comparison the results recently reported by Talmi and Unna[41].

On the experimental side the situation is not very happy. Indeed, the energies for a number of levels have been measured[42] but there is no information available on spin-parity assignments to these levels.

Looking at the results it seems that the spins and parities of the levels between 2 and 3.5 MeV would be very important for distinguishing between the various schemes. In particular, it would be possible to distinguish between the schemes (a) and (b) if the spin of the 3.16 MeV level is determined and the existence of the level at 3.05 MeV is confirmed.

The experiments at present report three states between 2 and 3.5 MeV excitation above the ground state. The choice (a) of interaction parameters predicts three states in this region, 9/2, 5/2 (close to each other) and 7/2 - in that order. The choice (b) predicts two additional states in this region viz. 5/2 and 3/2. Talmi and Unna's results show only two states in this region,

3/2 and 9/2. Our feeling is that an overall best fit is given by choice (a) of the parameters, with odd-state parameters remaining relatively indeterminate. Further experimental data would enable us to make more positive pronouncements.

4. Energy levels of O^{19} (II)

It is easy to see that if the low lying levels of O^{18} and O^{19} that we consider here are described only in terms of the $d_{5/2}$ and $s_{1/2}$ orbits, the nuclear spectroscopy involves only eight matrix elements of the effective interaction.

$$V_o = \left\langle d_{5/2}^2:0 \middle| V_{12} \middle| d_{5/2}^2:0 \right\rangle \quad V_o' = \left\langle (s_{1/2})^2:0 \middle| V_{12} \middle| (s_{1/2})^2:0 \right\rangle$$
$$V_2 = \left\langle d_{5/2}^2:2 \middle| V_{12} \middle| d_{5/2}^2:2 \right\rangle \quad V_2' = \left\langle s_{1/2}d_{5/2}:2 \middle| V_{12} \middle| s_{1/2}d_{5/2}:2 \right\rangle$$
$$V_4 = \left\langle d_{5/2}^2:4 \middle| V_{12} \middle| d_{5/2}^2:4 \right\rangle \quad V_3 = \left\langle s_{1/2}d_{5/2}:3 \middle| V_{12} \middle| s_{1/2}d_{5/2}:3 \right\rangle$$
$$V_{oo} = \left\langle (s_{1/2})^2:0 \middle| V_{12} \middle| (d_{5/2})^2:0 \right\rangle \qquad (4.1)$$
$$V_{22} = \left\langle d_{5/2}^2:2 \middle| V_{12} \middle| s_{1/2}d_{5/2}:2 \right\rangle.$$

In principle it should be possible to determine these matrix elements from the available information on the energies of lowest 5 states of O^{18} and the lowest 3 states of O^{19}. In practice, of course this would involve quite a bit of computation.† In this section we make this calculation

† In reference (41) Talmi and Unna assume identically $V_{oo} = V_{22} = 0$ for simplicity in computation.

to obtain the two-body matrix elements and compare them with the matrix elements calculated in chapter II with our various potential models.

The binding energy of the two extra-core neutrons in O^{18} ground state can be obtained from the binding energy data as[19]

$$\text{B.E. } (O^{18}) - \text{B.E. } (O^{16}) - 2\left[\text{B.E.}(O^{17}) - \text{B.E.}(O^{16})\right]$$

$$= -12.215 + 2\ (4.146) \text{ MeV} \qquad (4.2)$$

$$= -3.923 \text{ MeV}$$

and similarly the binding energy of the three neutrons in the ground state of O^{19} is given as

$$\text{B.E.}(O^{19}) - \text{B.E.}(O^{16}) - 3\left[\text{B.E } (O^{17}) - \text{B.E.}(O^{16})\right]$$

$$= -3.72 \text{ MeV.} \qquad (4.3)$$

Thus the absolute energies of states given experimentally in O^{18} and O^{19} are given in table 9.

Now obviously we obtain $V_4 = E_4 = -0.37$ MeV. Next, since the $J = 3^+$ state in O^{18} is not observed, there is no direct determination of V_3. Finally we note that E_o, E_o^* and E_2, E_2^* are the eigenvalues of the matrices

$$\begin{pmatrix} V_o & V_{oo} \\ & V_o' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} V_2 & V_{22} \\ & V_2' \end{pmatrix} \quad \text{respectively.}$$

Table 9

The absolute energies E_J of the states in O^{18} and O^{19}

O^{18}			O^{19}
J	E_J (MeV)	J	E_J (MeV)
0	$E_0 = -3.92$	5/2	$E_{5/2} = -3.72$
2	$E_2 = -1.94$	3/2	$E_{3/2} = -3.62$
4	$E_4 = -0.37$	1/2	$E_{1/2} = -2.20$
0^*	$E_0^* = -0.29$		
2^*	$E_2^* = 0.0$		

Therefore for each assumed value of V_{oo} and V_{22} one can calculate V_o, V_o' and V_2, V_2' by inversing the matrix. In table 10 we list the values of V_{oo}, V_{22} and corresponding values of V_o, V_o', V_2 and V_2'. Negative sign for these off-diagonal matrix elements is chosen to give phase of the wavefunctions in agreement with the results of other calculations[13,19]. We discuss the explicit wavefunction a little later. For $V_{oo} < -1.82$ MeV and $V_{22} < -0.97$ MeV, we would find in the wavefunction for the lowest $J = 0, 2$ states major components of $(s_{1/2})_o^2$ and $(s_{1/2}d_{5/2})_2$ states. Therefore we do not consider these values.

Table 10.

The values of V_o, V_o', V_2 and V_2' (in MeV) obtained by inverting the matrix for different values of V_{oo} and V_{22}.

V_{oo}	0	-0.5	-1.0	-1.5	-1.82
V_o	-3.92	-3.85	-3.62	-3.13	-2.11
V_o'	-2.03	-2.10	-2.33	-2.82	-3.85
V_{22}	0	-0.5	-0.75	-0.90	-0.97
V_2	-1.94	-1.80	-1.59	-1.33	-0.98
V_2'	-0.88	-1.02	-1.23	-1.49	-1.84

Thus we have obtained from the five known states of O^{18}, five matrix elements V_o, V_2, V_4, V_o' and V_2' in terms of three unknown matrices V_{oo}, V_{22} and V_3. Our next step is to construct Hamiltonian matrices for the states of O^{19} $J = 5/2$ (3 x 3), $J = 3/2$ (2 x 2) and $J = 1/2$ (1 x 1) for various sets of values of V_{oo}, V_{22} and V_3 and find the set which will best fit the energies of these states. The equations involved are already given in section 3. The results are summarised in figures 3, 4 and 5.

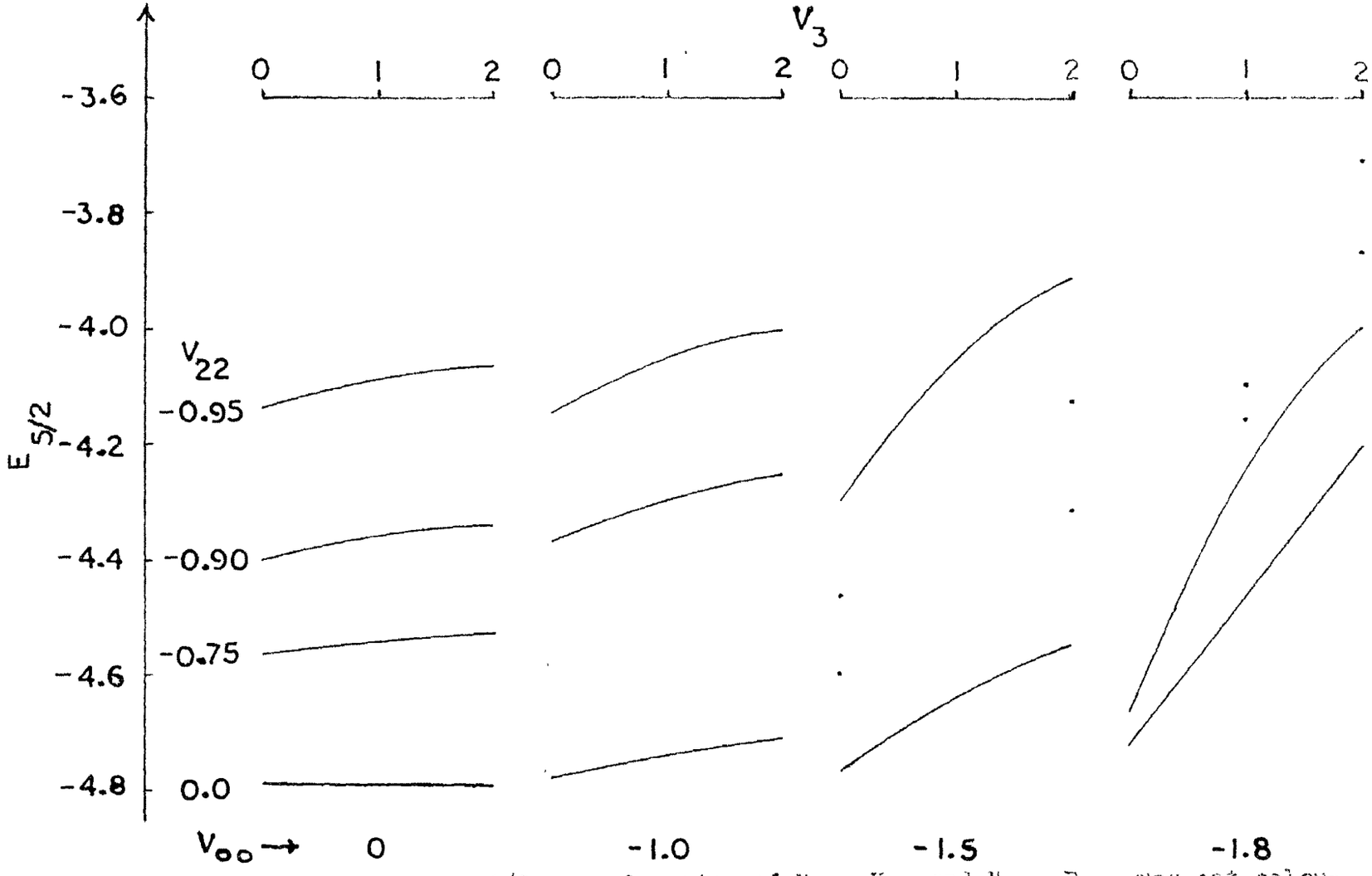

Fig. 7. The states $J = 5/2$ as a function of V_{00}, V_{22} and V_3. $E_{5/2}$ was not calculated for the cases (i) $V_{00} = -1.5$ and $V_{22} = -1.15$ and (ii) $V_{00} = -1.8$ and $V_{22} = -0.95$ and -1.15. For the cases $V_{00} = -1.5$ and -1.8 and $V_{22} = -0.95$ the eigenvectors were found to be $(d_{1/2})^2 d_{5/2}$ almost pure in character.

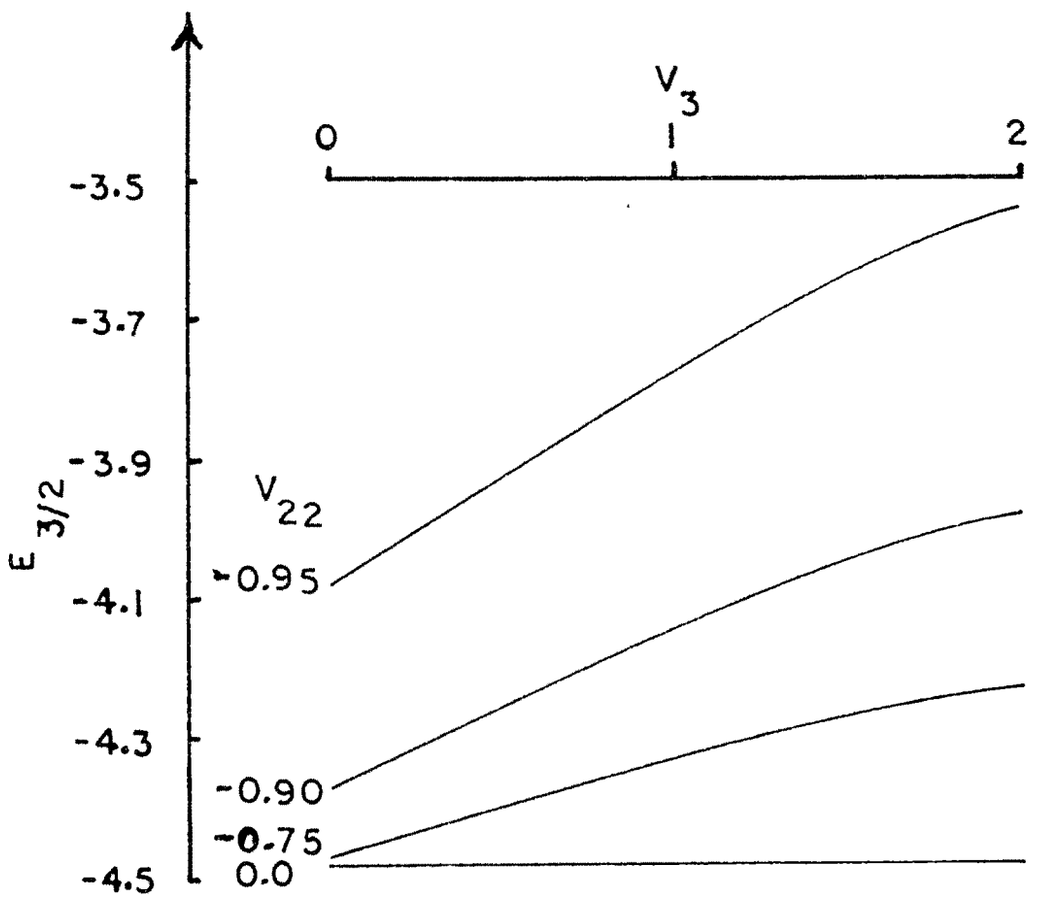

Fig.4. The state $J = 3/2$ as a function of $V_{00} : V_{22}$ and V_3 for all values of V_{01}.

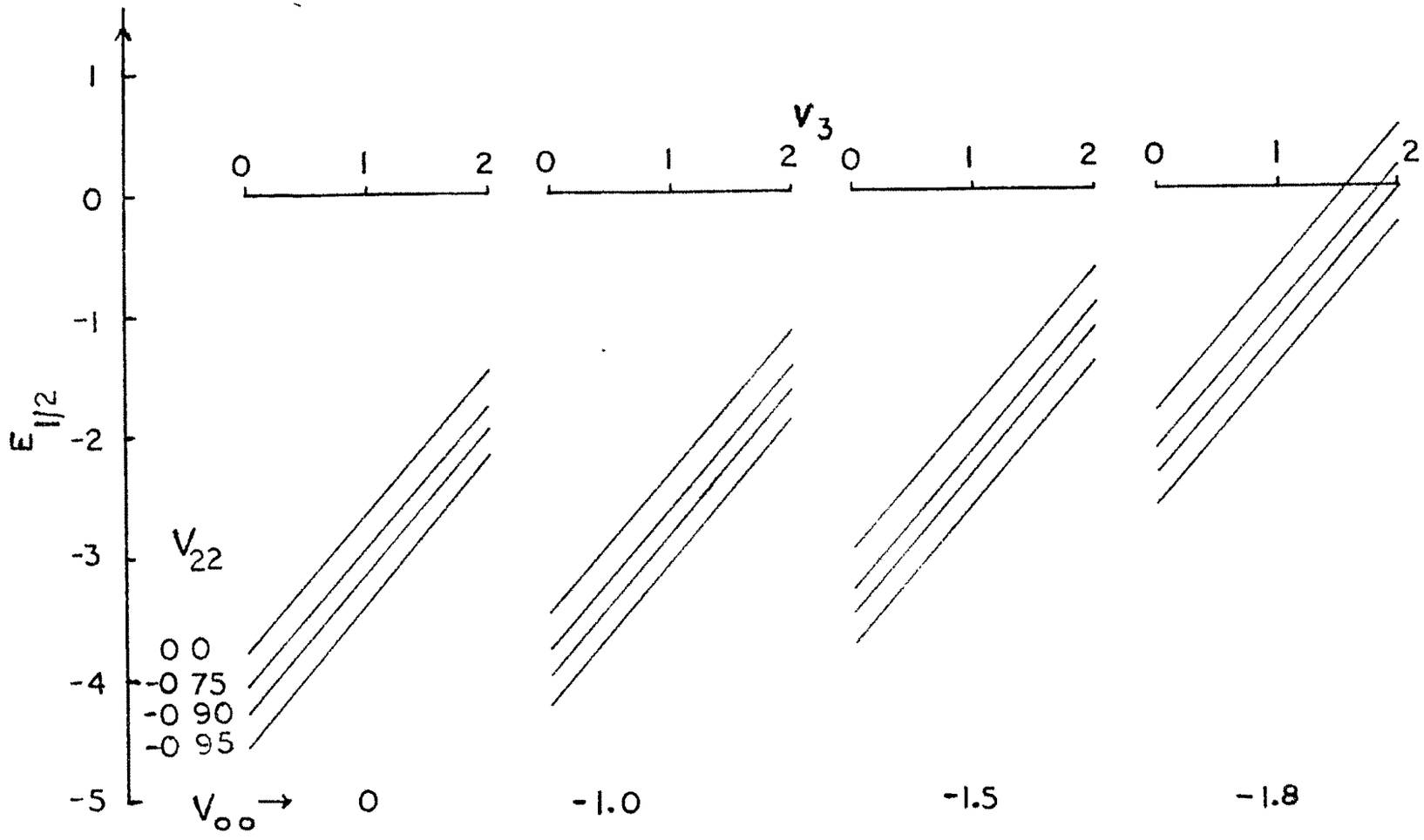

Fig. 1. The graphical representation of V_{00}, V_{22}, and V_3.

It seems to us that the best fit to O^{19} results is obtained for $V_{oo} = -1.5$ MeV, $V_{22} = -0.93$ MeV and $V_3 = +1.5$ MeV. This gives $E_{5/2} = -4.0$ MeV, $E_{3/2} = -3.85$ MeV, $E_{1/2} = -1.9$ MeV. We consider this agreement to be reasonably good since the calculation is heavily biased towards exact fitting of the five O^{18} states. A least square fit with equal bias to each of the 8 states would considerably improve the agreement.

To conclude we find that the following set of values provides a satisfactory fit to the known states of O^{18} and O^{19}.

$$\begin{aligned} V_o &= -3.0 \text{ MeV} & V_o &= -2.9 \text{ MeV} \\ V_2 &= -1.2 \text{ MeV} & V_2 &= -1.6 \text{ MeV} \\ V_4 &= -0.4 \text{ MeV} & V_3 &= +1.5 \text{ MeV} \\ V_{oo} &= -1.5 \text{ MeV} & V_{22} &= -0.93 \text{ MeV} \end{aligned} \qquad (4.5)$$

We remark that this predicts $J = 3^+$ state in O^{18} rather high i.e. at ~ 6 MeV. This we discuss below. Further $J = 9/2$ $(d_{5/2})^3$ state is also predicted by these parameters at about 2.1 MeV above the ground state 5/2.

Next we comment about the wavefunctions of the $J = 0, 2$ states of O^{18} given by these parameters. For $J = 0$ state the resulting wavefunction is reasonable, in the sense that it gives about 80 % $(d_{5/2})^2$ + 20% $(s_{1/2})^2$ mixing. This is in good agreement with other results.

On the other hand for $J = 2$ we find a rather large degree of mixing, about 55% $(d_{5/2})^2$ + 45% $(d_{5/2}s_{1/2})$. This appears to be rather large, especially in view of the results of Mcfarlane and French. Perhaps, if as mentioned above, a least square fitting is done, the values of the parameters may change a little, and may improve the result. But it is almost certain that a considerable mixing (greater than the values of Mcfarlane and French) will be predicted. We do not think this discrepancy is very serious.

The matrix elements obtained above agree well with the matrix elements obtained from our potential model and constitute another check on those results. (see tables 3 and 4 of section 1).

Two comments may be made. The matrix element V_o obtained from the potential model appears to be somewhat larger than that found in this section. (However, owing to large repulsive contribution of tensor and spin-orbit forces, Dawson, Talmi and Walecka find this matrix element much smaller i.e. -2.0 MeV). Secondly, the matrix element V_3 has changed sign. The value +1.5 MeV obtained here does appear to be somewhat large. In view of the lack of experimental knowledge of the position of the $J = 3^+$ state, we do not explore this point further. If future experiments show the state to have indeed a positive matrix element $V_3 > 0$, our potential model will have to be somewhat revised.

ooo 0 ooo