

# CHAPTER

8

---

EMHD FLUID FLOW WITH SLIP EFFECTS

Content of this chapter is published in:

**Heat Transfer (Wiley) 2022 Volume 51(2) 1449-1467 (Scopus)**

# Chapter 8

## EMHD fluid flow with slip effects

Electromagnetohydrodynamic (EMHD) is the area that concerns the study of dynamics of electrically conducting fluids under the influence of magnetic and electric fields. EMHD has raised quite an interest over the years due to its versatile application in geophysics, engineering, biomedical engineering, magnetic drug targeting, and many others. The non-adherence of the fluid to a solid boundary, known as velocity slip, occurs under certain circumstances. Fluids displaying slip are essential for technologies such as internal cavities and in the artificial cardiac valve polishing.

### 8.1 Introduction of the Problem

The general goal of electro-MHD, as described in RanjitShit2019 and Ghosh2020, is to improve fluid flows by using electric fields, magnetic fields, or suitable combinations of these fields. Micropumps are frequently used to implement this phenomenon. Several researchers have conducted promising research on magnetohydrodynamic micro pumps; see, for example, [13, 51]. The results of experiments suggest that combined EMHD effects might be used to improve liquid flow in microchannels. EMHD mixed convection flow of fluid over a stretching sheet is discussed by [138].

In circumstances involving wire nettings and perforated plates, lubricated hydrophobic surfaces, porous and abrasive surfaces, and coated surfaces, slip condition must unavoidably be taken into account. The tangential fluid velocity at the surface and the velocity gradient normal to the surface are related by the velocity slip model. Daniel et al. [138] scrutinized the effect of velocity slip on EMHD dual stratified flow of nanofluid. Soomro et al. [32] examined the impact of velocity slip on MHD Williamson fluid along the vertical surface. Eringen [9] introduced the simple micropolar fluids theory. According to his theory, simple micropolar fluids is a fluent medium whose properties and behavior are affected by the local motion of the fluid fundamentals. Keeping this in mind, Eringen [10] simplified micropolar

fluids theory and obtained a subclass of these fluids called micro-polar fluids. This theory is one of the best established theories of fluids with microstructure. This theory is useful in explaining the features of certain fluids such as liquid crystals, suspensions and animal blood.

Kataria and Patel [41, 42] explored MHD Casson fluid flow considering radiation, thermal diffusion etc. Anantha Kumar et al. [52] investigated electromagnetic effects on non-Newtonian fluid. Kataria et al. [44] studied a uniform magnetic field in which motion is produced by the buoyancy force of incompressible micropolar fluid. Eringen's proposed mathematical model for examining the flow of conductive fluids was taken into consideration by Lund et al. [63]. The mathematical modelling for Brownian movement and thermophoresis effects on MHD flow with nonlinear thermal radiation was developed by Mittal and Patel [16]. The impact of the electrically driven micropolar fluid through a permeable stretched sheet was studied by Pradhan et al. [125]. Rashad et al. [17] evaluated the influence of chemical reaction impact on mixed convection boundary layer micropolar fluid flow over a constantly moving vertical surface. The differential process was used by Rashidi and Pour [86] to seek a completely analytical solution including radiation. Sheikholeslami et al. [89] examined the unsteady electrically conducting MHD fluid flow. Li et al. [143] investigated the migration of nanofluid through a permeable pipe taking Lorentz forces into account.

## 8.2 Novelty of the Problem

In this research, the aim of this investigation is to analyze EMHD flow of Micropolar fluid over a stretching sheet. The energy equation includes nonlinear thermal radiation and joule heating effects. Slip condition and convective boundary conditions are considered.

## 8.3 Mathematical Formulation of the Problem

Two dimensional incompressible Micropolar fluid flow through a horizontal sheet is considered. Figure 8.1 shows velocity of the stretching sheet is taken as  $u = ax + U_{Slip}$ . A magnetic field of strength  $\mathcal{B}$  is normally applied with the conjecture of lower Reynolds number, so that the induced magnetic field may be ignored. Viscous dissipation and Joule heating and nonlinear thermal radiation effects are accounted. The surface of the stretching plate is in contact with another hot plate of temperature  $T_w$  and concentration  $C_w$  with  $h_{ft}$  and  $h_{fc}$  are the heat transfer coefficient and mass

transfer coefficient. The volume of particle concentration and the temperature of the Micropolar fluid far away from the plate is supposed to be  $C_\infty$  and  $T_\infty$  respectively.

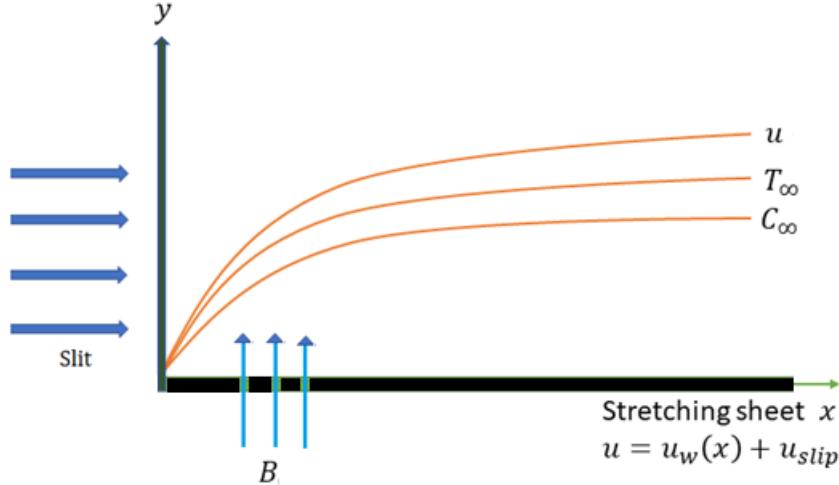


Figure 8.1: Physical Sketch of the Problem

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (8.3.1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho} \frac{\partial G}{\partial y} + \frac{\sigma}{\rho} (E_0 B_0 - B_0^2 u), \quad (8.3.2)$$

$$u \frac{\partial G}{\partial x} + v \frac{\partial G}{\partial y} = \frac{\gamma^*}{\rho j} \frac{\partial^2 G}{\partial y^2} - \frac{k}{\rho j} \left( 2G + \frac{\partial u}{\partial y} \right), \quad (8.3.3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_P} \left( \frac{\partial^2 T}{\partial y^2} \right) + \frac{(u B_0 - E_0)^2 \sigma}{\rho C_P} - \frac{1}{\rho C_P} \frac{\partial q_r}{\partial y} + \left( \frac{\mu + k}{\rho} \right) \left( \frac{\partial u}{\partial y} \right)^2, \quad (8.3.4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_M \frac{\partial^2 C}{\partial y^2}, \quad (8.3.5)$$

The variable formula can also be used here to identify these differential equations because it has a energy integral that is useful for the structure of the solution.

$$\begin{aligned} u &= U_w(x) + U_{slip} = ax + \alpha^* \left[ (\mu + k) \frac{\partial u}{\partial y} + kG \right], \quad v = v_w, \quad G = -n \frac{\partial u}{\partial y}, \\ -\kappa \frac{\partial T}{\partial y} &= h_{ft} (T_f - T), \quad -D_M \frac{\partial C}{\partial y} = h_{fc} (C_f - C) \text{ at } y = 0, \\ u(\infty) &= G(\infty) = 0, \quad T(\infty) = T_\infty, \quad C(\infty) = C_\infty. \end{aligned} \quad (8.3.6)$$

Heat flux is given by Rossland [126]:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (8.3.7)$$

where  $\sigma^*$  and  $k^*$  are Stephen Boltzmann's constant and coefficient of mean absorption respectively.

Instance  $n = 0$ ,  $G = 0$  on the surface is indicated. It shows a focused particle flow that cannot turn the micro-components nearest to the wall. This situation is often referred to as the high microelement concentration. Because the antisymmetric component of the stress tensor vanishes, the probability  $n = 0.5$  implies a low concentration of microelements. The turbulent boundary layer flows are modelled using the case  $n = 1$ . Consider the following similarity transformation.

$$\left. \begin{aligned} u &= axf'(\eta), v = -\sqrt{a\nu}f(\eta), \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \eta &= \sqrt{\frac{a}{\nu}}y, G = ax\sqrt{\frac{a}{\nu}}h(\eta). \end{aligned} \right\} \quad (8.3.8)$$

However, Equations (8.3.2) - (8.3.6) are translated in the following dimensional equations. The completion of the continuity equation (8.3.1) is self-evident.

$$(1 + K)f''' + ff'' - f'^2 + Kh' - M^2f' + M^2E = 0, \quad (8.3.9)$$

$$\left(1 + \frac{K}{2}\right)h'' + fh' - f'h - K(2h + f'') = 0, \quad (8.3.10)$$

$$\begin{aligned} &\left(1 + \frac{4}{3}Rd\{1 + (\theta_w - 1)\theta\}^3\right)\theta'' + Prf\theta' + 4Rd\{1 + (\theta_w - 1)\theta\}^2(\theta_w - 1)\theta'^2 \\ &+ M^2EcPr(f'^2 + E^2 - 2Ef') + (1 + K)PrEc\theta'^2 = 0 \end{aligned} \quad (8.3.11)$$

$$\phi'' + Sc f\phi' = 0, \quad (8.3.12)$$

Revised boundary conditions now

$$\begin{aligned} f(0) &= f_w, \quad f'(0) = 1 + \gamma(1 + K(1 - n))f''(0), \quad h(0) = -nf''(0), \\ \theta'(0) &= -\lambda_1(1 - \theta(0)), \quad \phi'(0) = -\lambda_2(1 - \phi(0)), \quad \text{at } y = 0, \\ f'(y) &\rightarrow 0, \quad h(y) \rightarrow 0, \quad \theta(y) \rightarrow 0, \quad \phi(y) \rightarrow 0, \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (8.3.13)$$

where  $Pr = \frac{\mu C_P}{\kappa}$ ,  $K = \frac{k}{\mu}$ ,  $M^2 = \frac{\sigma B_0^2}{a\rho}$ ,  $f_w = -(av)^{-\frac{1}{2}}v_w$ ,  $Re_x = \frac{ax^2}{\nu}$ ,  $\theta_w = \frac{T_w}{T_\infty}$ ,

$Sc = \frac{\nu}{D_M}$ ,  $\gamma = \alpha^* \mu \sqrt{\frac{a}{\nu}}$ ,  $E = \frac{E_0}{U_w B_0}$ ,  $Ec = \frac{U_w^2}{C_P(T_w - T_\infty)}$ ,  $\lambda_1 = \frac{h_{ft}}{\kappa} \sqrt{\frac{\nu}{a}}$ ,  $\lambda_2 = \frac{h_{fc}}{D_M} \sqrt{\frac{\nu}{a}}$  Coefficient of skin friction, wall couple stress, Nusselt number and Sherwood number in dimensional form are

$$C_{fx} = \frac{2\tau_w}{\rho U_w^2(x)}, \quad Mw_x = \frac{m_w}{\rho a^2 x^3}, \quad Sh_x = \frac{xq_m}{D_M(C_w - C_\infty)}, \quad Nu_x = \frac{xq_w}{\kappa(T_w - T_\infty)} \quad (8.3.14)$$

where,

$$\tau_w = \left[ (\mu + k) \frac{\partial u}{\partial y} + kG \right]_{y=0}, \quad m_w = \gamma^* \frac{\partial G}{\partial y} \Big|_{y=0}, \quad q_m = -D_M \frac{\partial C}{\partial y} \Big|_{y=0}, \quad q_w = -\kappa \frac{\partial T}{\partial y} \Big|_{y=0}, \quad (8.3.15)$$

These quantities of physical interest take on a dimensionless equation

$$\begin{aligned} Nu_x Re_x^{-\frac{1}{2}} &= - \left( 1 + \frac{4}{3} Rd ((\theta_w - 1) \theta(0) + 1)^3 \right) \theta'(0), \\ Mw_x Re_x &= \left( 1 + \frac{K}{2} \right) h'(0), \quad \frac{1}{2} C_{fx} Re_x^{\frac{1}{2}} = (1 + (1 - n) K) f''(0), \\ Sh_x Re_x^{-\frac{1}{2}} &= -\phi'(0), \quad (8.3.16) \end{aligned}$$

## 8.4 Solution by Homotopy Analysis Method

Liao [120] developed Homotopy analysis method for locating resolution of the governing equations. The initial guess and auxiliary linear operator can be chosen freely in the Homotopy analysis approach.

Initial guesses are

$$\begin{aligned} f_0(\eta) &= f_w + \frac{1}{1 + \gamma(1 + K(1 - n))} (1 - e^{-\eta}), \quad \theta_0(\eta) = \frac{\lambda_1}{1 + \lambda_1} e^{-\eta}, \\ h_0(\eta) &= \frac{n}{1 + \gamma(1 + K(1 - n))} e^{-\eta}, \quad \phi_0(\eta) = \frac{\lambda_2}{1 + \lambda_2} e^{-\eta}, \quad (8.4.1) \end{aligned}$$

with auxiliary linear operators

$$\mathcal{L}_f = \frac{\partial^3 f}{\partial \eta^3} - \frac{\partial f}{\partial \eta}, \quad \mathcal{L}_h = \frac{\partial^2 h}{\partial \eta^2} - h, \quad \mathcal{L}_\theta = \frac{\partial^2 \theta}{\partial \eta^2} - \theta, \quad \mathcal{L}_\phi = \frac{\partial^2 \phi}{\partial \eta^2} - \phi, \quad (8.4.2)$$

satisfying

$$\begin{aligned} \mathcal{L}_f(k_1 + k_2 e^\eta + k_3 e^{-\eta}) &= 0, \quad \mathcal{L}_h(k_4 e^\eta + k_5 e^{-\eta}) = 0, \\ \mathcal{L}_\theta(k_6 e^\eta + k_7 e^{-\eta}) &= 0, \quad \mathcal{L}_\phi(k_8 e^\eta + k_9 e^{-\eta}) = 0, \quad (8.4.3) \end{aligned}$$

Here  $k_i$ , ( $i = 1, 2, \dots, 9$ ) are the arbitrary constants .

### 8.4.1 Zero-th order deformation

The following is the problem of zeroth order deformation:

$$\left. \begin{aligned} (1-q) \mathcal{L}_f [F(\eta; q) - f_0(\eta)] &= q \hbar_f \mathcal{N}_f [F(\eta; q)], \\ (1-q) \mathcal{L}_h [H(\eta; q) - h_0(\eta)] &= q \hbar_h \mathcal{N}_h [H(\eta; q)], \\ (1-q) \mathcal{L}_\theta [\Theta(\eta; q) - \theta_0(\eta)] &= q \hbar_\theta \mathcal{N}_\theta [\Theta(\eta; q)], \\ (1-q) \mathcal{L}_\phi [\Phi(\eta; q) - \phi_0(\eta)] &= q \hbar_\phi \mathcal{N}_\phi [\Phi(\eta; q)], \end{aligned} \right\} \quad (8.4.4)$$

The following is a list of nonlinear operators:

$$\mathcal{N}_f [F(\eta; q)] = (1+K) \frac{\partial^3 F}{\partial \eta^3} + F \frac{\partial^2 F}{\partial \eta^2} - \left\{ \frac{\partial F}{\partial \eta} \right\}^2 + K \frac{\partial H}{\partial \eta} - M^2 \frac{\partial F}{\partial \eta} + M^2 E, \quad (8.4.5)$$

$$\mathcal{N}_h [H(\eta; q)] = \left( 1 + \frac{K}{2} \right) \frac{\partial^2 H}{\partial \eta^2} + F \frac{\partial H}{\partial \eta} - H \frac{\partial F}{\partial \eta} - K \left( 2H + \frac{\partial^2 F}{\partial \eta^2} \right), \quad (8.4.6)$$

$$\begin{aligned} \mathcal{N}_\theta [\Theta(\eta; q)] &= \left( 1 + \frac{4}{3} R d \{ 1 + (\theta_w - 1) \Theta \}^3 \right) \frac{\partial^2 \Theta}{\partial \eta^2} + (1+K) P r E c \left( \frac{\partial^2 F}{\partial \eta^2} \right)^2 \\ &\quad + P r F \frac{\partial \Theta}{\partial \eta} + 4 R d \{ 1 + (\theta_w - 1) \Theta \}^2 (\theta_w - 1) \left( \frac{\partial \Theta}{\partial \eta} \right)^2 \\ &\quad + M^2 E c P r \left( \left\{ \frac{\partial F}{\partial \eta} \right\}^2 + E^2 - 2E \frac{\partial F}{\partial \eta} \right), \end{aligned} \quad (8.4.7)$$

$$\mathcal{N}_\phi [\Phi(\eta; q)] = \frac{\partial^2 \Phi}{\partial \eta^2} + S c F \frac{\partial \Phi}{\partial \eta}, \quad (8.4.8)$$

Boundary conditions subject to

$$\begin{aligned} F(0; q) &= f_w, \quad F'(0; q) = 1 + \gamma(1+K(1-n))F''(0; q); \quad F'(+\infty; q) = 0, \\ H(0; q) &= -nF''(0; q), \quad H(+\infty; q) = 0, \quad \Theta'(0; q) = -\lambda_1(1 - \Theta(0; q)), \\ \Theta(+\infty; q) &= 0, \quad \Phi'(0; q) = -\lambda_2(1 - \Phi(0; q)), \quad \Phi(+\infty; q) = 0, \end{aligned} \quad (8.4.9)$$

where  $F(\eta; q)$ ,  $H(\eta; q)$ ,  $\Theta(\eta; q)$  and  $\Phi(\eta; q)$  are unknown in terms of functions  $\eta$  and  $q$ .  $\hbar_f$ ,  $\hbar_h$ ,  $\hbar_\theta$  and  $\hbar_\phi$  are zero non-auxiliary parameters and  $N_f$ ,  $N_h$ ,  $N_\theta$  and  $N_\phi$  the

non-linear operators. Furthermore, where  $q \in (0, 1)$  is a parameter embedding.

$$\left. \begin{array}{ll} F(\eta; 0) = f_0(\eta), & F(\eta; 1) = f(\eta), \\ H(\eta; 0) = h_0(\eta), & H(\eta; 1) = h(\eta), \\ \Theta(\eta; 0) = \theta_0(\eta), & \Theta(\eta; 1) = \theta(\eta), \\ \Phi(\eta; 0) = \phi_0(\eta), & \Phi(\eta; 1) = \phi(\eta), \end{array} \right\} \quad (8.4.10)$$

If  $q$  is 0 to 1, the variation will be  $F(\eta; q)$ ,  $H(\eta; q)$ ,  $\Theta(\eta; q)$  and  $\Phi(\eta; q)$  differ from  $f_0(\eta)$ ,  $h_0(\eta)$ ,  $\theta_0(\eta)$  and  $\phi_0(\eta)$  to  $f(\eta)$ ,  $h(\eta)$ ,  $\theta(\eta)$ , and  $\phi(\eta)$ . So one can obtain

$$\left. \begin{array}{l} F(\eta; q) = f_0(\eta) + \sum_{i=1}^{\infty} f_i(\eta) q^i, \\ H(\eta; q) = h_0(\eta) + \sum_{i=1}^{\infty} h_i(\eta) q^i, \\ \Theta(\eta; q) = \theta_0(\eta) + \sum_{i=1}^{\infty} \theta_i(\eta) q^i, \\ \Phi(\eta; q) = \phi_0(\eta) + \sum_{i=1}^{\infty} \phi_i(\eta) q^i, \end{array} \right\} \quad (8.4.11)$$

where

$$\left. \begin{array}{l} f_i(\eta) = \frac{1}{i!} \frac{\partial^i f(\eta; q)}{\partial \eta^i} \Big|_{q=0}, \\ h_i(\eta) = \frac{1}{i!} \frac{\partial^i h(\eta; q)}{\partial \eta^i} \Big|_{q=0}, \\ \theta_i(\eta) = \frac{1}{i!} \frac{\partial^i \theta(\eta; q)}{\partial \eta^i} \Big|_{q=0}, \\ \phi_i(\eta) = \frac{1}{i!} \frac{\partial^i \phi(\eta; q)}{\partial \eta^i} \Big|_{q=0}, \end{array} \right\} \quad (8.4.12)$$

The convergence in the series is largely reliant on  $\hbar_f$ ,  $\hbar_h$ ,  $\hbar_\theta$  and  $\hbar_\phi$ . If non-zero auxiliary parameters are chosen to estimate Equation (8.4.11) at  $q = 1$ .

Thus, the following can be obtained

$$\left. \begin{array}{l} f(\eta) = f_0(\eta) + \sum_{i=1}^{\infty} f_i(\eta), \\ h(\eta) = h_0(\eta) + \sum_{i=1}^{\infty} h_i(\eta), \\ \theta(\eta) = \theta_0(\eta) + \sum_{i=1}^{\infty} \theta_i(\eta), \\ \phi(\eta) = \phi_0(\eta) + \sum_{i=1}^{\infty} \phi_i(\eta), \end{array} \right\} \quad (8.4.13)$$

### 8.4.2 i-th order deformation

The deformation equations in  $i^{th}$  order can be presented in the form

$$\left. \begin{array}{l} \mathcal{L}_f [f_i(\eta) - \chi_i f_{i-1}(\eta)] = \hbar_f \mathcal{R}_{f,i}(\eta), \\ \mathcal{L}_h [h_i(\eta) - \chi_i h_{i-1}(\eta)] = \hbar_h \mathcal{R}_{h,i}(\eta), \\ \mathcal{L}_\theta [\theta_i(\eta) - \chi_i \theta_{i-1}(\eta)] = \hbar_\theta \mathcal{R}_{\theta,i}(\eta), \\ \mathcal{L}_\phi [\phi_i(\eta) - \chi_i \phi_{i-1}(\eta)] = \hbar_\phi \mathcal{R}_{\phi,i}(\eta), \end{array} \right\} \quad (8.4.14)$$

under the conditions of the boundary

$$\begin{aligned} f_i(0) &= 0, f'_i(0) = \gamma(1 + K(1 - n))f''_i(0), f'_i(\infty) = 0, h_i(+\infty; q) = 0, \\ h_i(0; q) &= -nf''_i(0; q), h_i(+\infty; q) = 0, \theta'_i(0; q) = \lambda_1 \theta_i(0; q), \\ \theta_i(+\infty; q) &= 0, \phi'_i(0; q) = \lambda_2 \phi_i(0; q), \phi_i(+\infty; q) = 0. \end{aligned} \quad (8.4.15)$$

where

$$\mathcal{R}_{f,i}(\eta) = (1 + K)f'''_{i-1} + \sum_{j=0}^{i-1} f_j f''_{i-1-j} - \sum_{j=0}^{i-1} f'_j f'_{i-1-j} + K h'_{i-1} - M^2 f'_{i-1} + M^2 E, \quad (8.4.16)$$

$$\mathcal{R}_{h,i}(\eta) = \left(1 + \frac{K}{2}\right) h''_{i-1} + \sum_{j=0}^{i-1} f_j h''_{i-1-j} - \sum_{j=0}^{i-1} h_j f'_{i-1-j} - K(2h_{i-1} + f''_{i-1}). \quad (8.4.17)$$

$$\begin{aligned} \mathcal{R}_{\theta,i}(\eta) &= \frac{4}{3}Rd(\theta_w - 1)^3 \left\{ \sum_{j=0}^{i-1} \theta''_{i-1-j} \sum_{l=0}^j \theta_{j-l} \sum_{p=0}^l \theta_{l-p} \theta_p \right\} + \left(1 + \frac{4}{3}Rd\right) \theta''_{i-1} \\ &+ 4Rd(\theta_w - 1)^3 \left\{ \sum_{j=0}^{i-1} \theta'_{i-1-j} \sum_{l=0}^j \theta'_{j-l} \sum_{p=0}^l \theta_{l-p} \theta_p \right\} + 4Rd(\theta_w - 1) \sum_{j=0}^{i-1} \theta''_{i-1-j} \theta_j \\ &+ 4Rd(\theta_w - 1)^2 \sum_{j=0}^{i-1} \theta''_{i-1-j} \sum_{l=0}^j \theta_{j-l} \theta_l + 8Rd(\theta_w - 1)^2 \sum_{j=0}^{i-1} \theta'_{i-1-j} \sum_{l=0}^j \theta'_{j-l} \theta_l \\ &+ 4Rd(\theta_w - 1) \sum_{j=0}^{i-1} \theta'_{i-1-j} \theta'_j + (1 + K) PrEc \sum_{j=0}^{i-1} f''_j f''_{i-1-j} \\ &+ M^2 EcPr \left( \sum_{j=0}^{i-1} f'_j f'_{i-1-j} + E^2 - 2Ef'_{i-1} \right) + Pr \sum_{j=0}^{i-1} f_j \theta'_{i-1-j} \end{aligned} \quad (8.4.18)$$

$$\mathcal{R}_{\phi,i}(\eta) = \phi''_{i-1} + Sc \sum_{j=0}^{i-1} f_j \phi'_{i-1-j}. \quad (8.4.19)$$

with

$$\chi_i = \begin{cases} 0, & i < 1 \\ 1, & i \geq 1 \end{cases} \quad (8.4.20)$$

The general solutions  $f_i$ ,  $h_i$ ,  $\theta_i$  and  $\phi_i$  comprising the special solution  $f_i^*$ ,  $h_i^*$ ,  $\theta_i^*$  and  $\phi_i^*$  are given by

$$\left. \begin{aligned} f_i(\eta) &= f_i^*(\eta) + k_1 + k_2 e^\eta + k_3 e^{-\eta}, \\ h_i(\eta) &= h_i^*(\eta) + k_4 e^\eta + k_5 e^{-\eta}, \\ \theta_i(\eta) &= \theta_i^*(\eta) + k_6 e^\eta + k_7 e^{-\eta}, \\ \phi_i(\eta) &= \phi_i^*(\eta) + k_8 e^\eta + k_9 e^{-\eta} \end{aligned} \right\} \quad (8.4.21)$$

here  $f_i^*$ ,  $h_i^*$ ,  $\theta_i^*$  and  $\phi_i^*$  are given by the corresponding specific solutions  $i^{th}$ -order and constants equations  $k_j$  ( $j = 1, 2, \dots, 9$ ) the boundary conditions shall be determined.

## 8.5 Convergence Analysis

Auxiliary parameters  $\hbar_f$ ,  $\hbar_h$ ,  $\hbar_\theta$  and  $\hbar_\phi$  significantly affects the convergence and approximation rate of HAM solutions.  $\hbar$ -curve is plotted taking appropriate order in Figure 8.2, for this purpose. For the auxiliary parameter  $\hbar_f$ , Figure 8.2 clearly suggests an allowable range of  $-1.6 < \hbar_f < -0.1$ . We use  $\hbar_f = -0.50$  in this case. Similarly,  $\hbar_h = -1.17$ ,  $\hbar_\theta = -0.22$  and  $\hbar_\phi = -1.84$ . Convergence of series solution with varying order of approximation is calculated in Table 8.1.

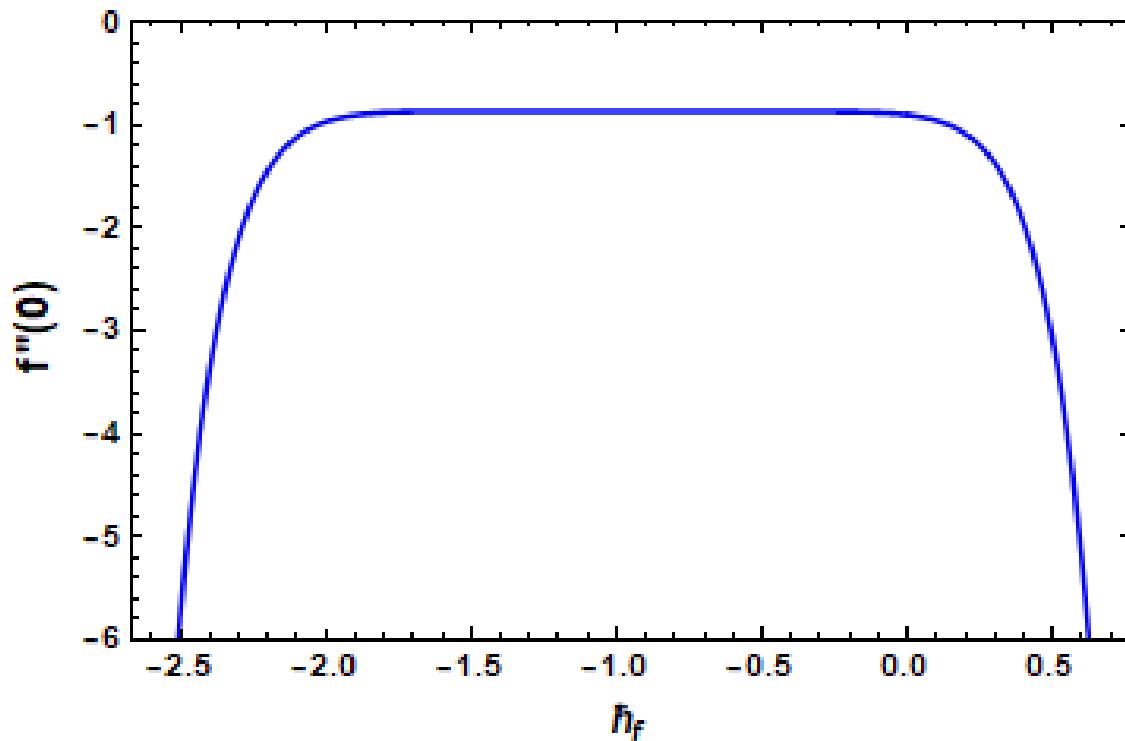


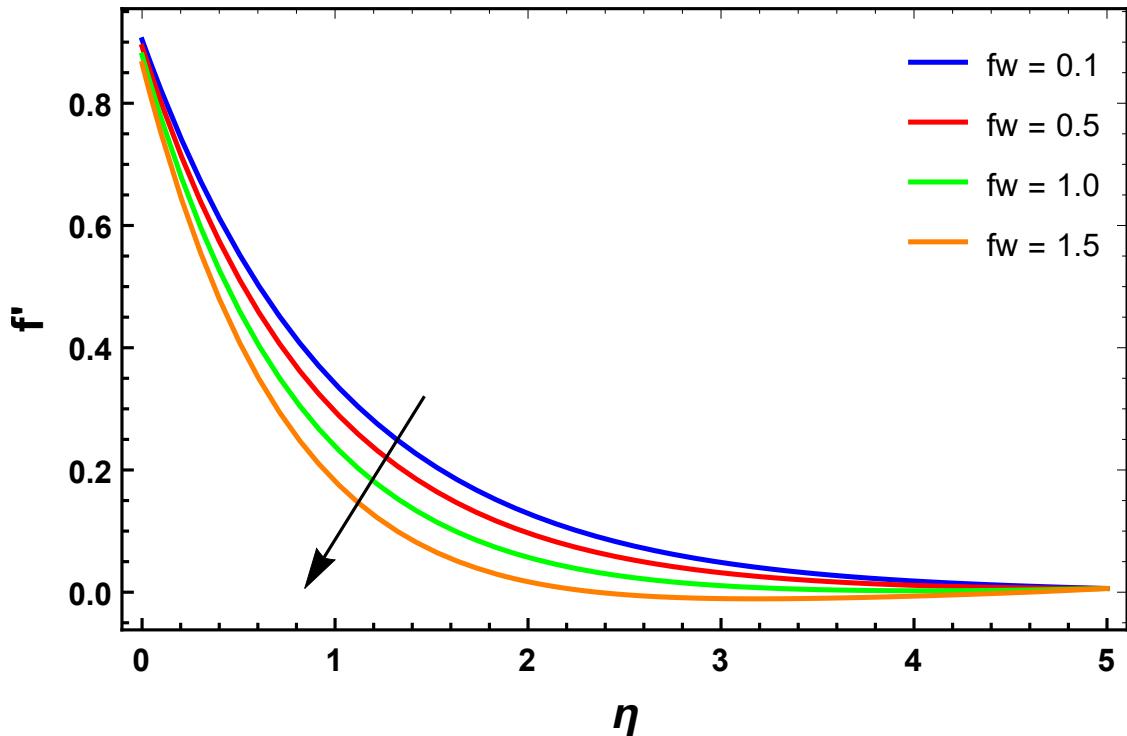
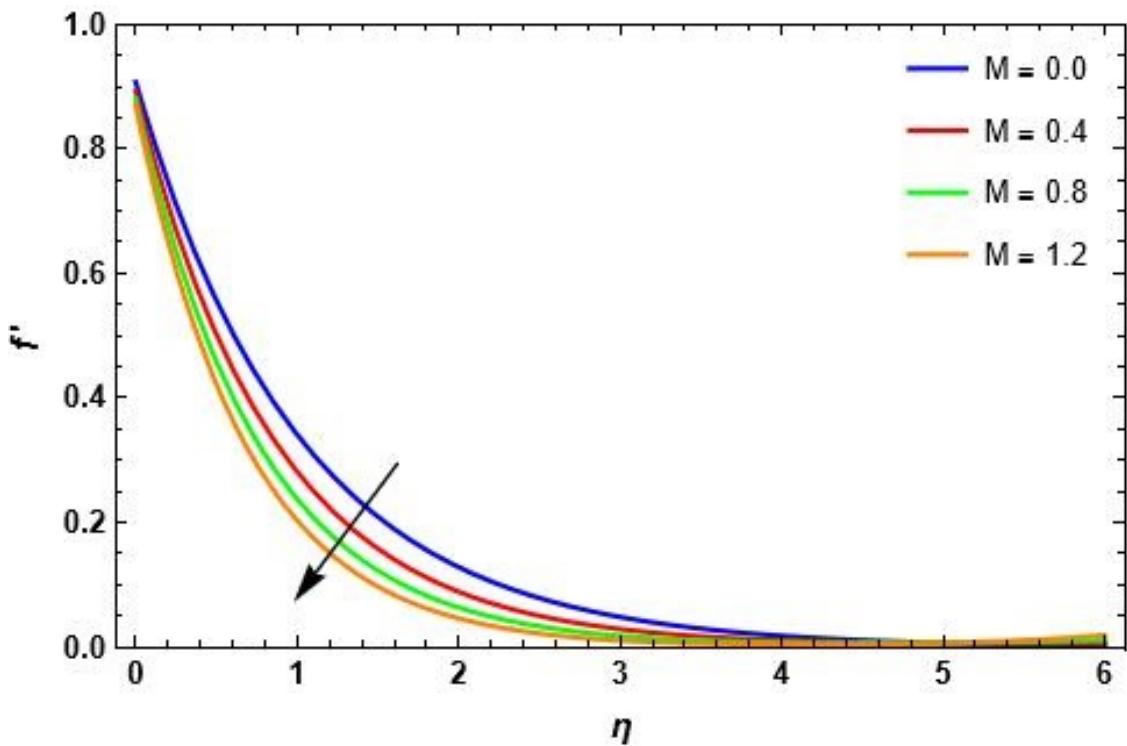
Figure 8.2:  $f''(0)$  via  $\hbar_f$ .

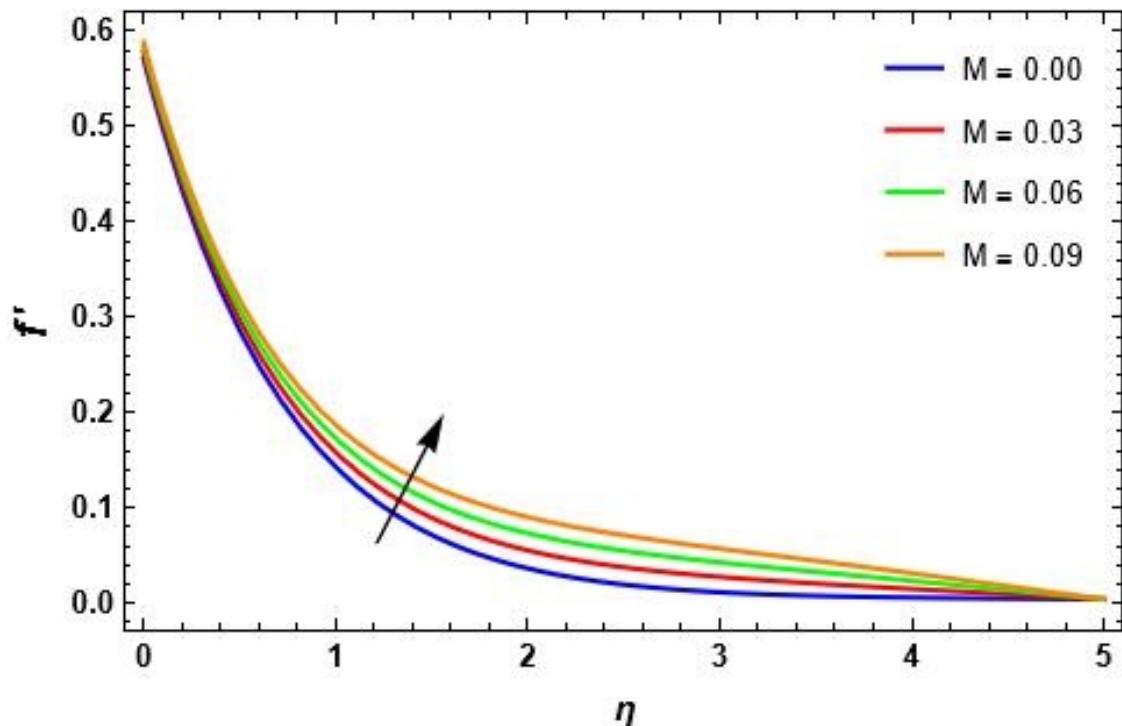
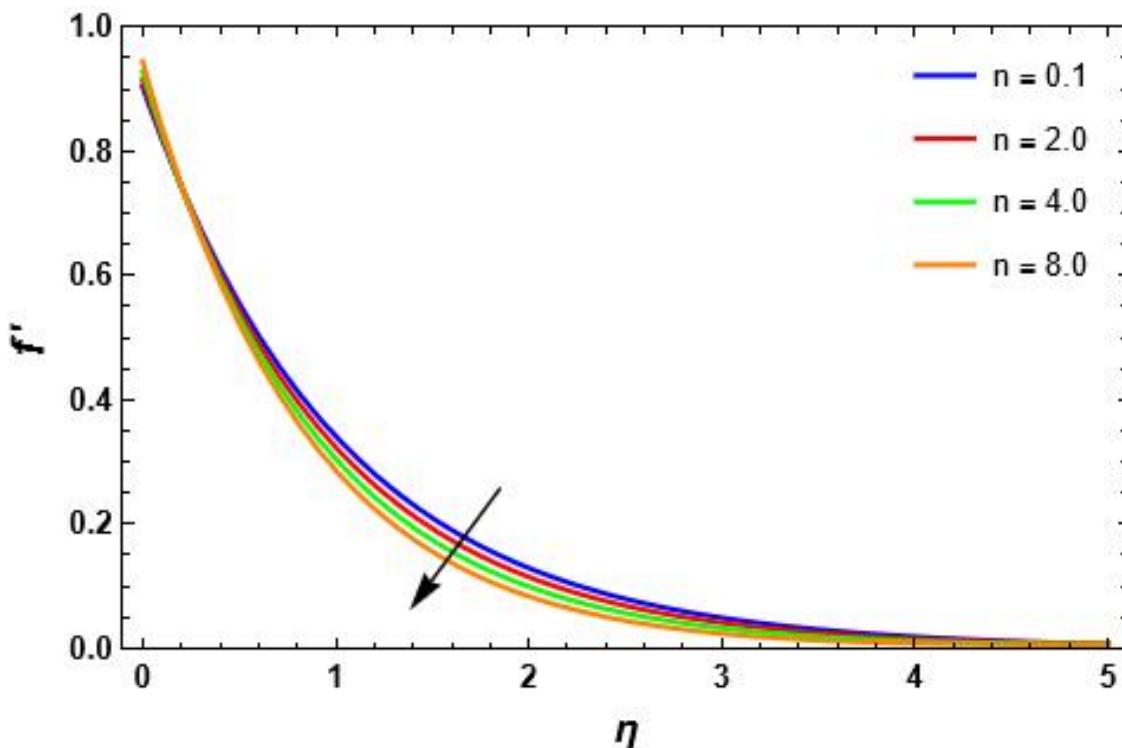
## 8.6 Result and Discussion

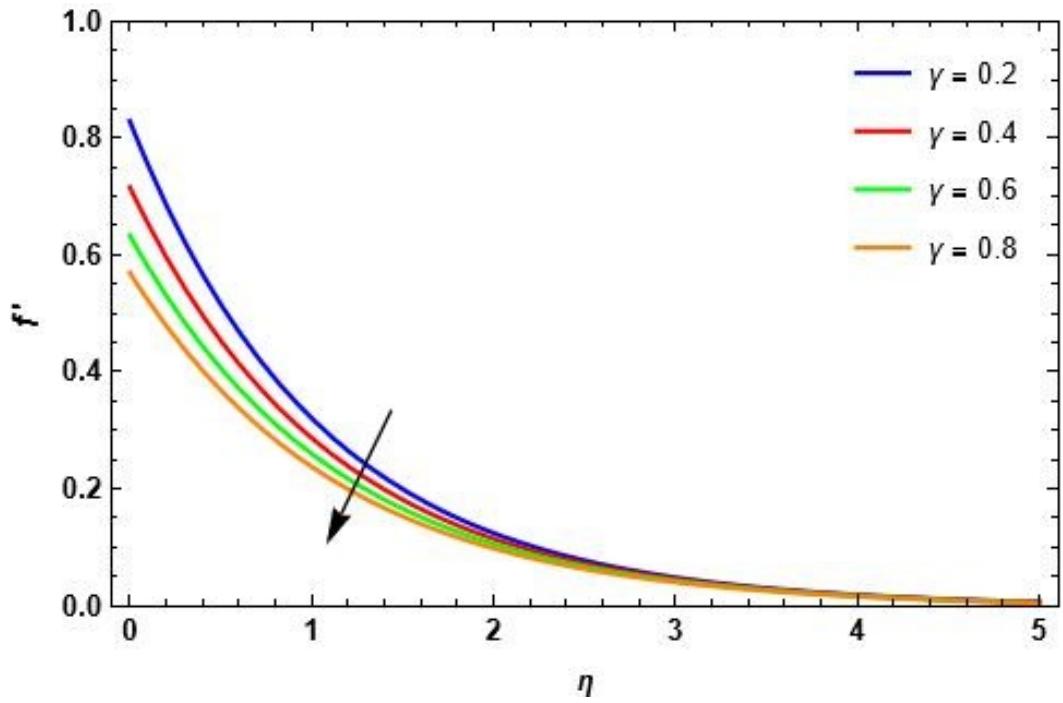
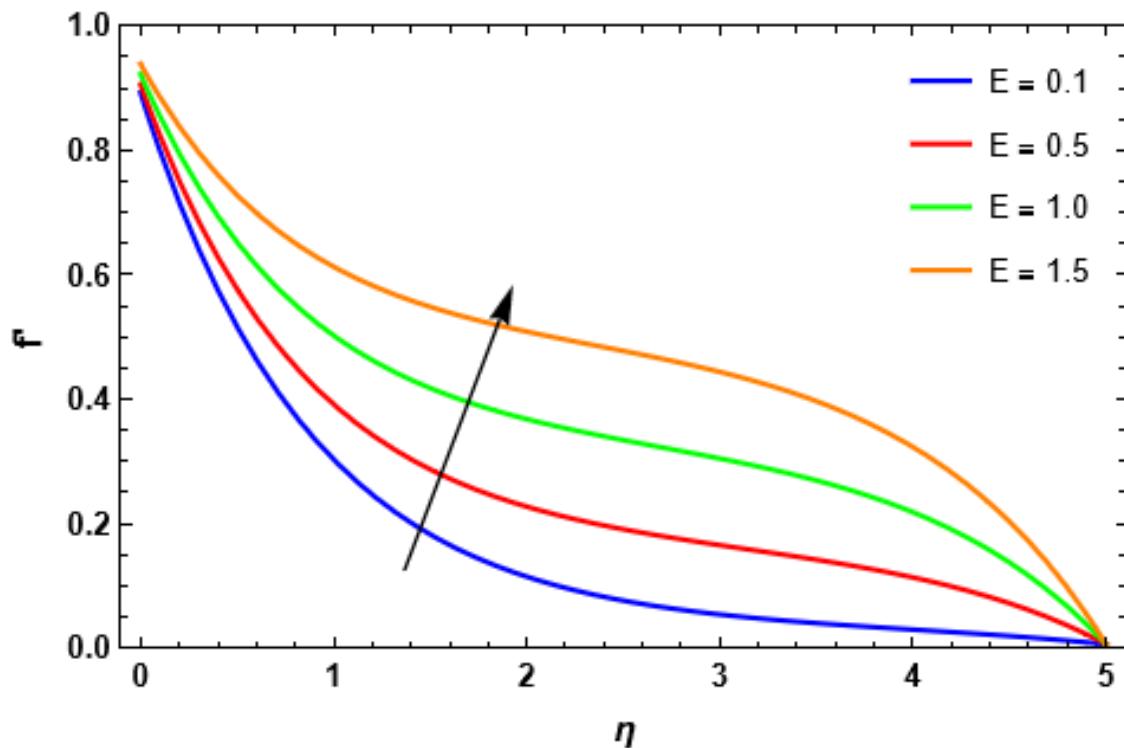
In this section the solutions are found using suitable Mathematica code. Figures 8.3-8.23 show effects on velocity, angular velocity, temperature, and concentration profile of various variables. Graphs are used to describe the findings achieved. This section covers the results of various parameters including Slip parameter  $\gamma$ , Suction parameter  $f_w$ , Electric parameter  $E$ , Material parameter  $K$ , Micro element concentration  $n$ , Prandtl number  $Pr$ , Radiation parameter  $Rd$ , Magnetic parameter  $M$ , and Schmidt number  $Sc$  on Profiles of velocity, angular velocity, temperature, and concentration. Because of the porous medium, velocity decreases for the higher values of Suction parameter  $f_w$  in Figure 8.3. In Figure 8.4 shows velocity decreases for the increasing values of  $M$  in the absence of electric fields. Figure 8.6 shows velocity diminishes for the increasing value of concentration of micro elements  $n$ . Figure 8.7 shows velocity decreases for the higher values of slip parameter  $\gamma$ . In Figure 8.8, velocity increases for the higher values of Electric parameter  $E$ . Figure 8.9 shows velocity enhances for the larger values of Material parameter  $K$ .

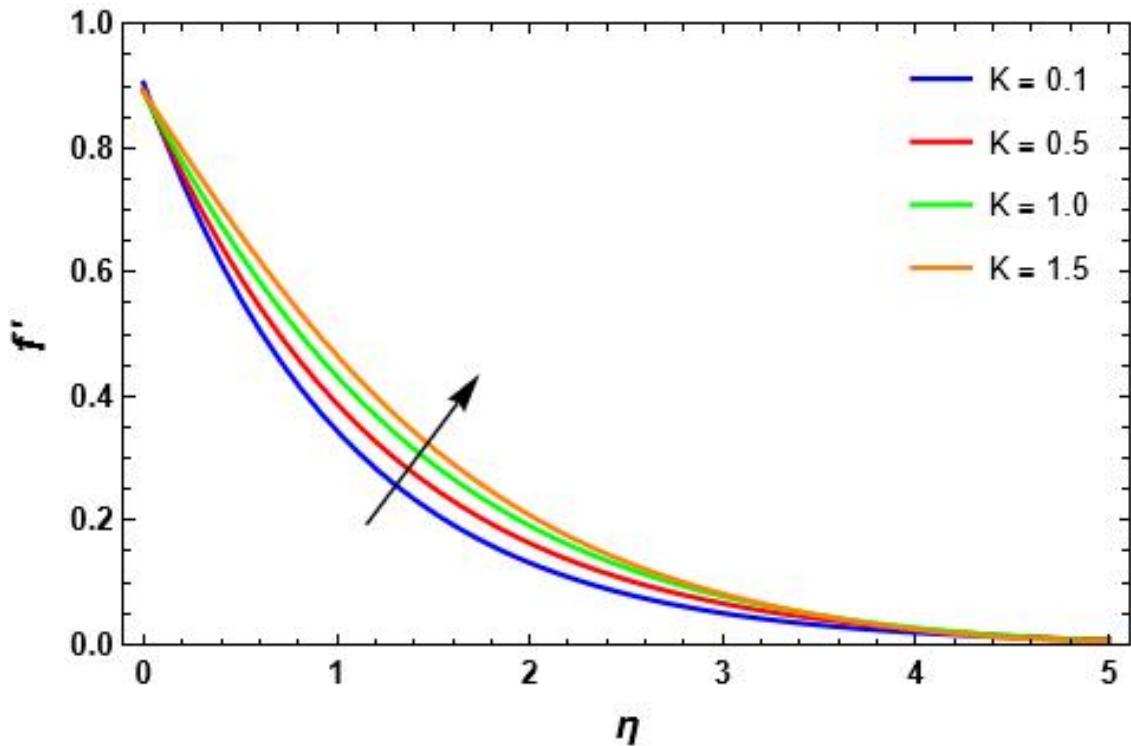
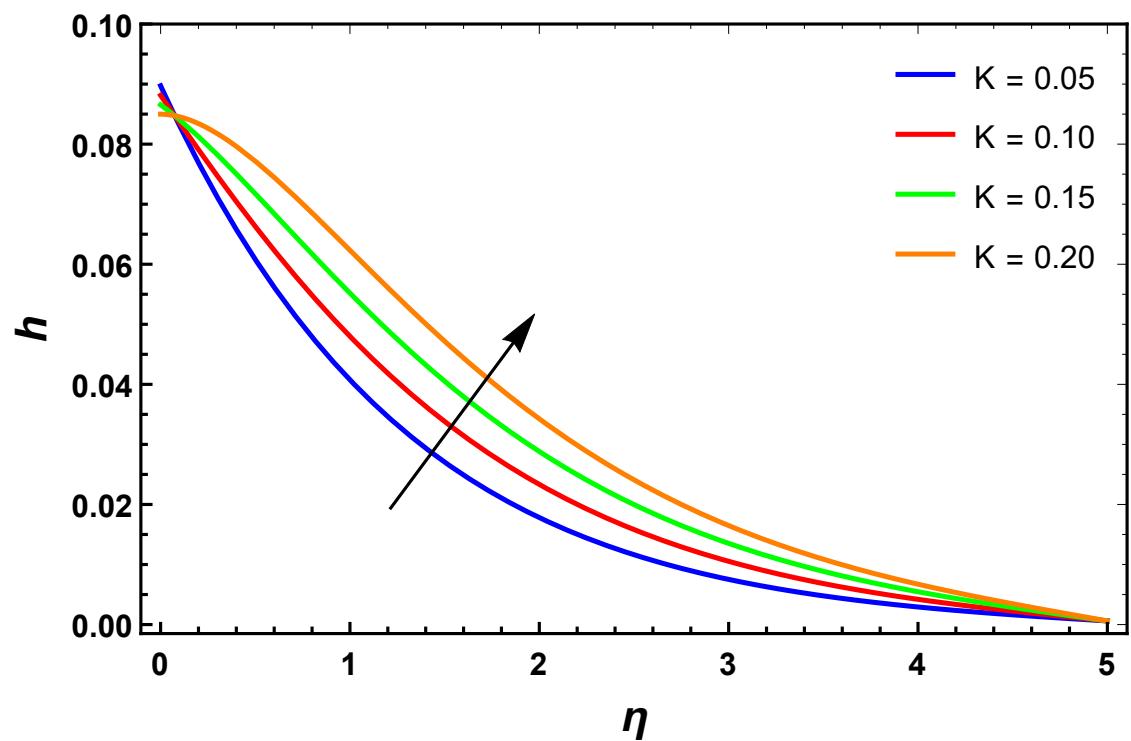
Figure 8.10 illustrates angular velocity increases for the large amount of Material parameter  $K$ . Figure 8.11 shows angular velocity increases for the higher value of concentration of microelements  $n$ . For higher value of Magnetic parameter  $M$  temperature increases, it can be seen in Figure 8.12. Figure 8.13 shows that temperature enhances when Temperature ratio parameter  $\theta_w$  has been increased. Figure 8.14 shows an increment in temperature for higher values of radiation parameter  $Rd$ . Figure 8.15 shows an increment in temperature for higher values of Electric parameter  $E$ . For higher values of thermal Biot number  $\lambda_1$ , temperature enhances, it can be seen in Figure 8.16.

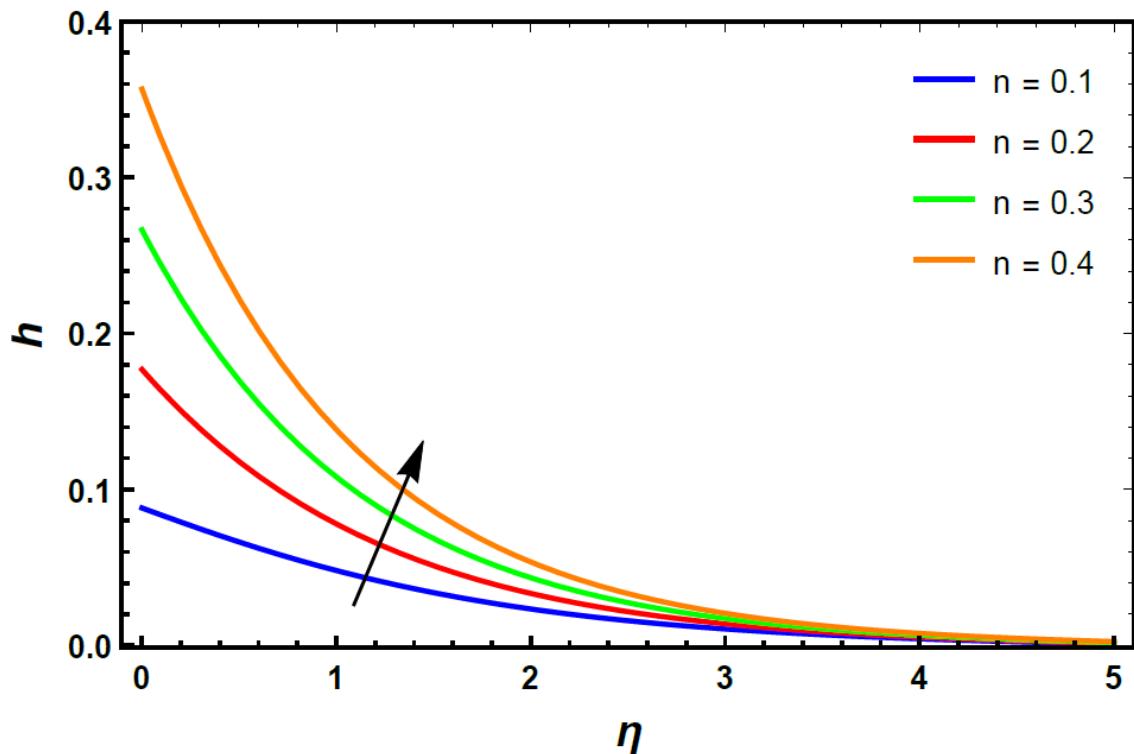
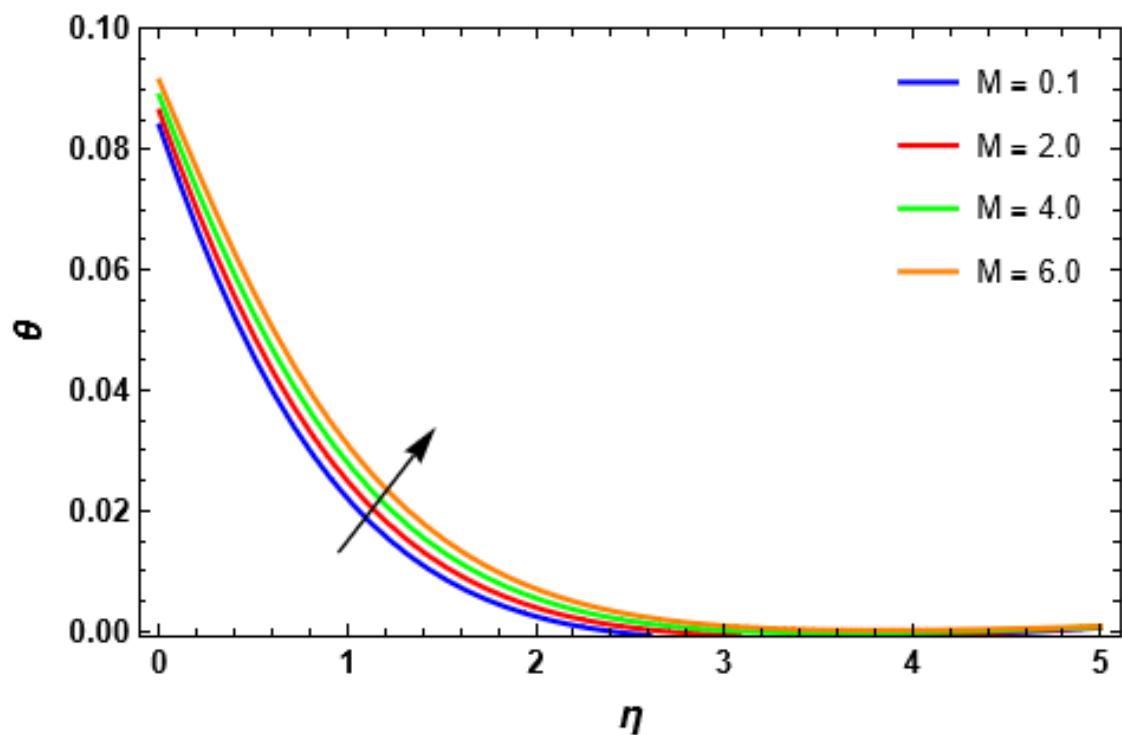
Figure 8.17 shows concentration diminishes for higher values of Schmidt numbers  $Sc$ . Figure 8.18 shows concentration is increased for the higher Solutal Biot numbers  $\lambda_2$ . Skin friction coefficient increased for higher values of  $M$ , it can be found from Figure 8.19. Figure 8.20 illustrates Local wall couple stress increases for rising values of Material parameter  $K$ . Figure 8.22 and Figure 8.21 shows Nusselt number increases with increasing values of  $\theta_w$ ,  $Rd$  and  $\gamma_1$ . Figure 8.23 illustrates Sherwood number enhanced by the higher values of  $Sc$ . Numerical calculation for Nusselt number can be found in Table 8.2.

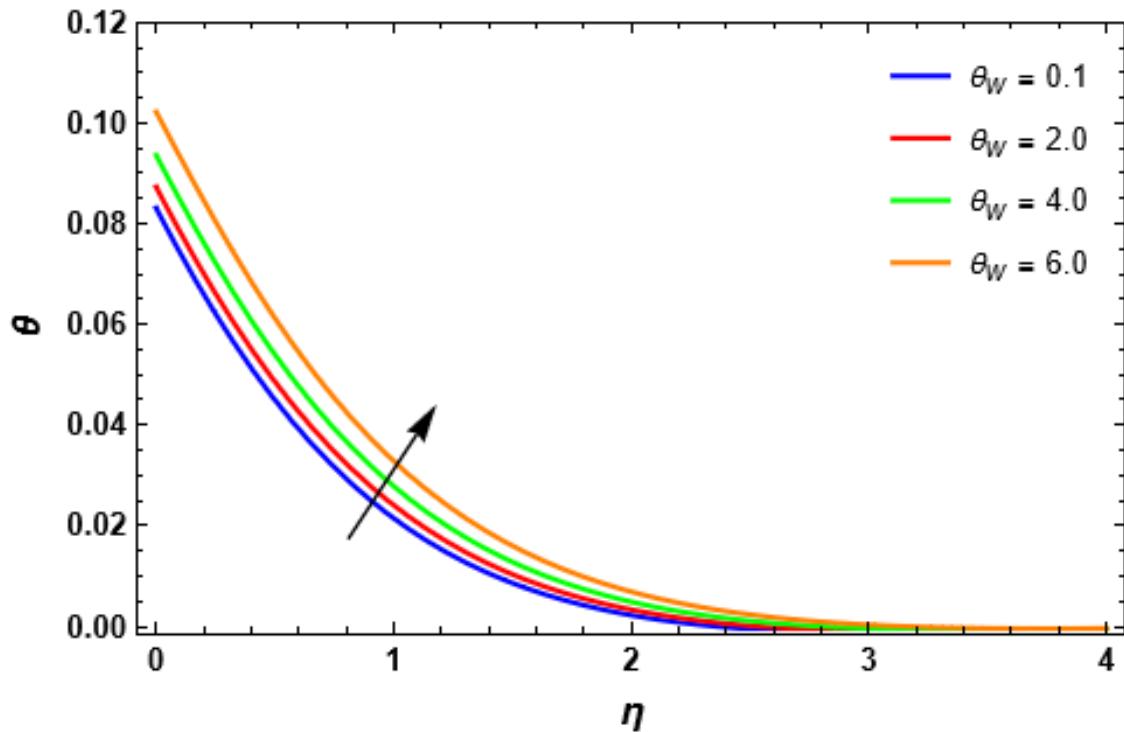
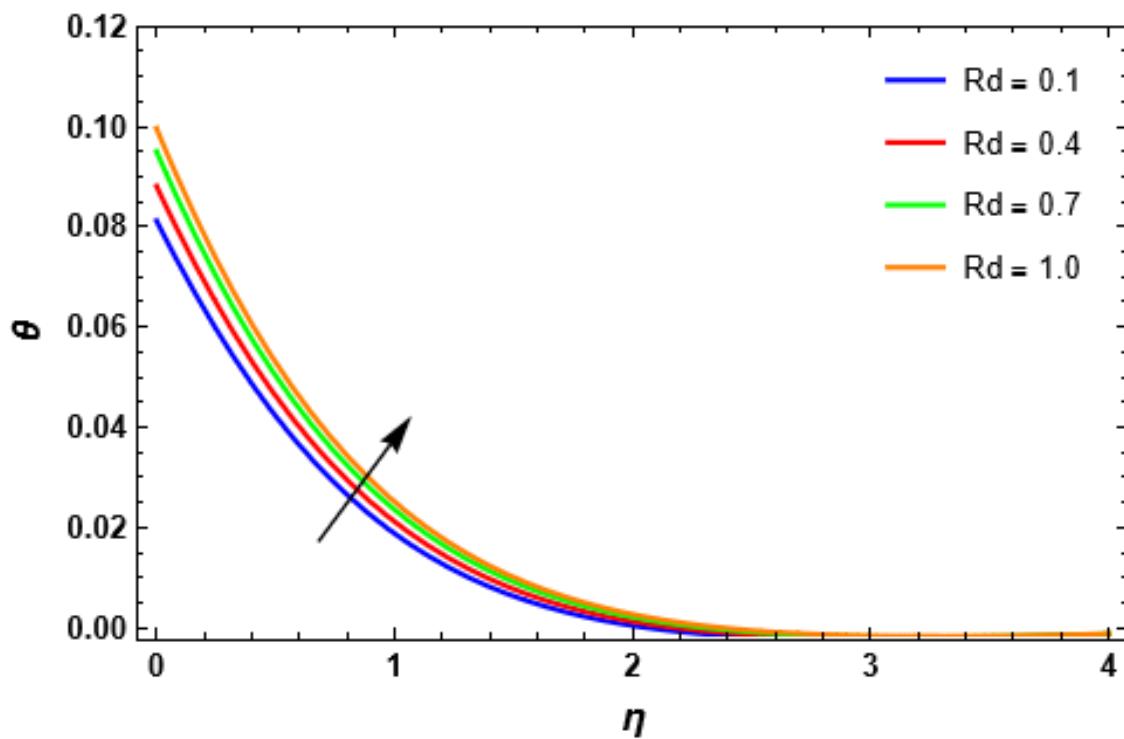
Figure 8.3:  $f'(\eta)$  via  $f_w$ .Figure 8.4:  $f'(\eta)$  when  $E = 0$  via  $M$ .

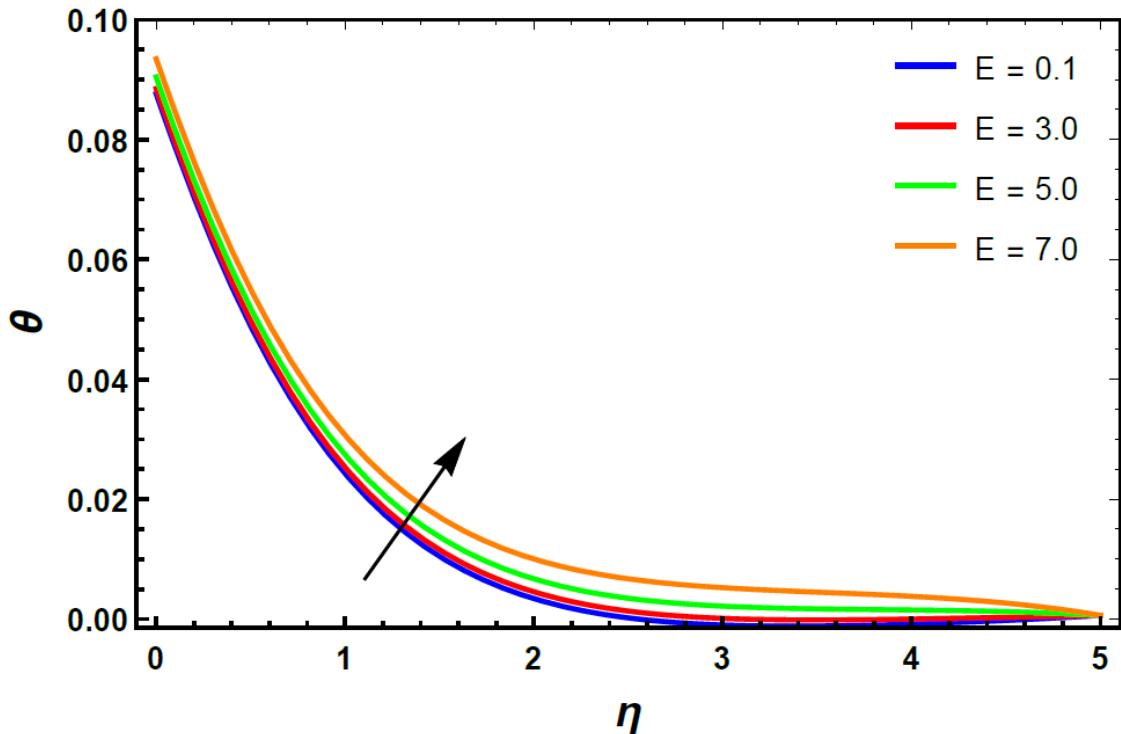
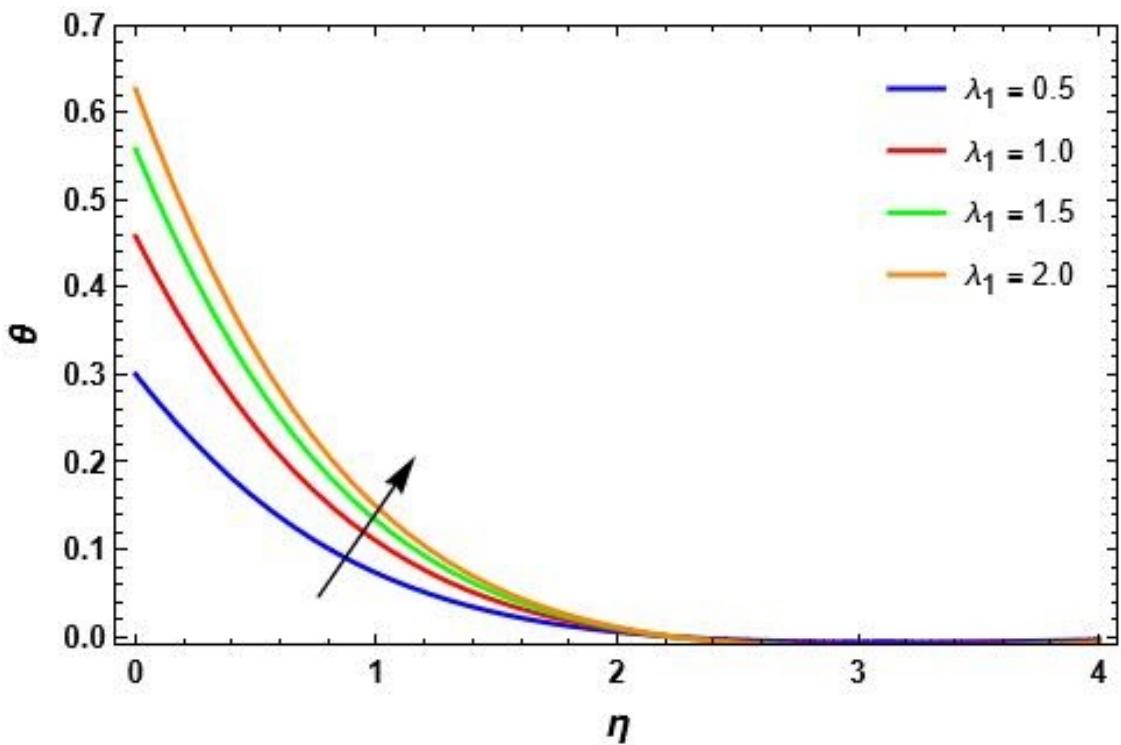
Figure 8.5:  $f'(\eta)$  when  $E = 0.5$  via  $M$ .Figure 8.6:  $f'(\eta)$  via  $n$ .

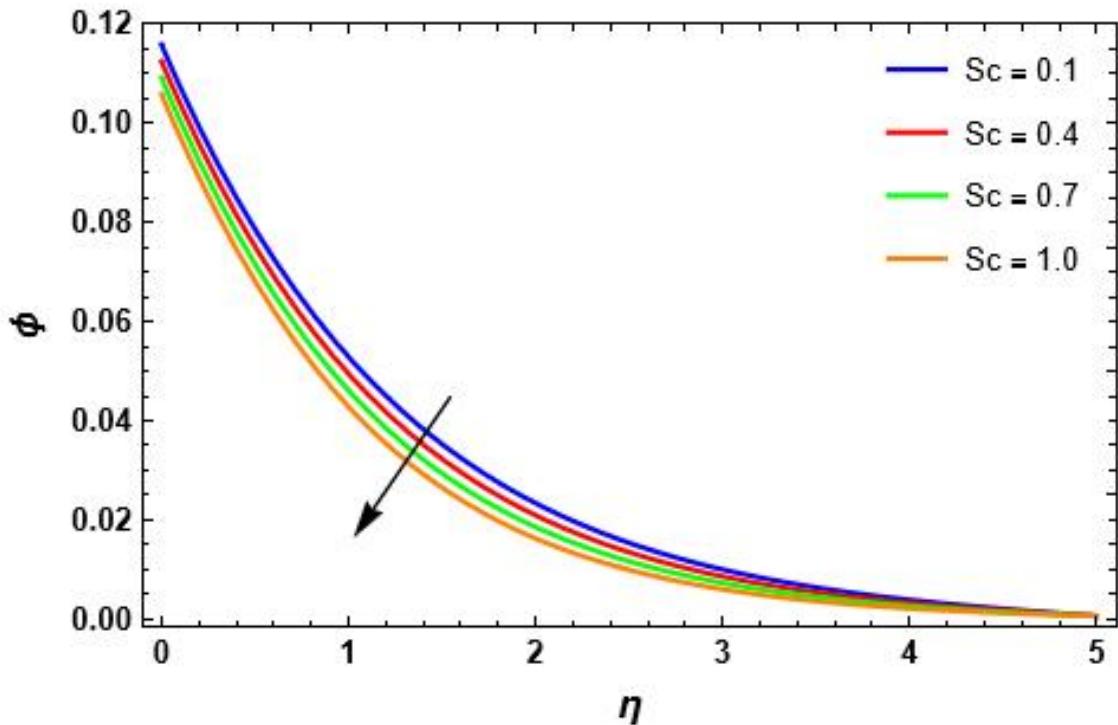
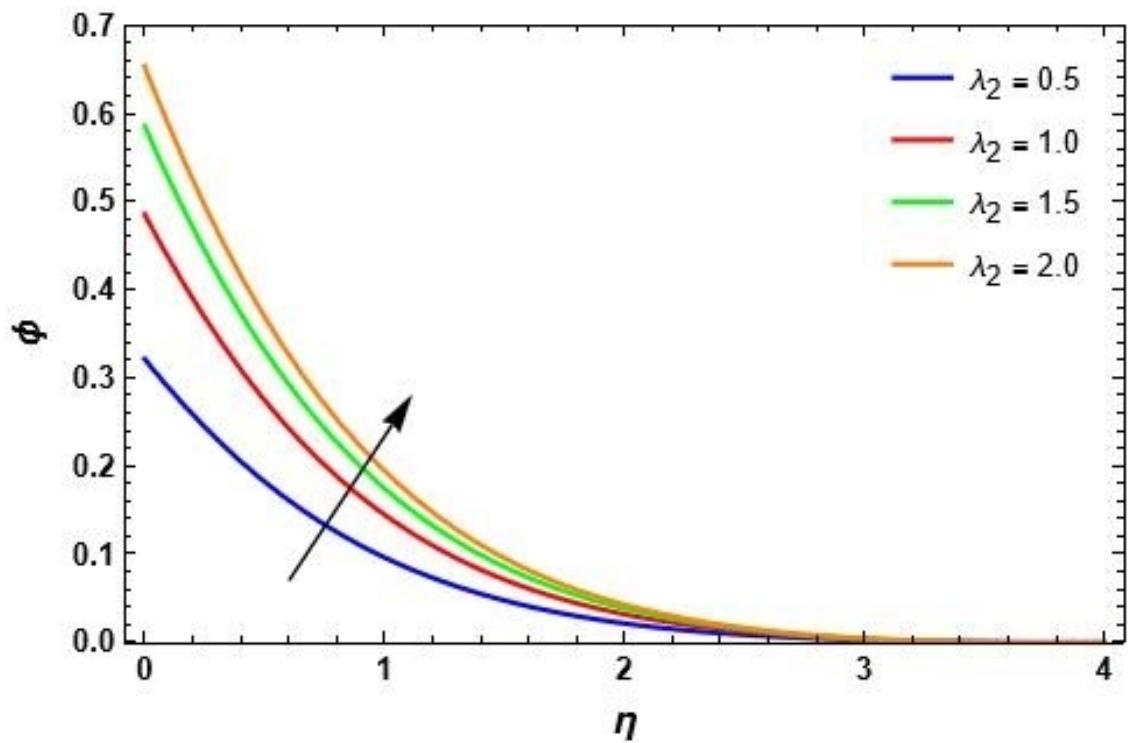
Figure 8.7:  $f'(\eta)$  via  $\gamma$ .Figure 8.8:  $f'(\eta)$  via  $E$ .

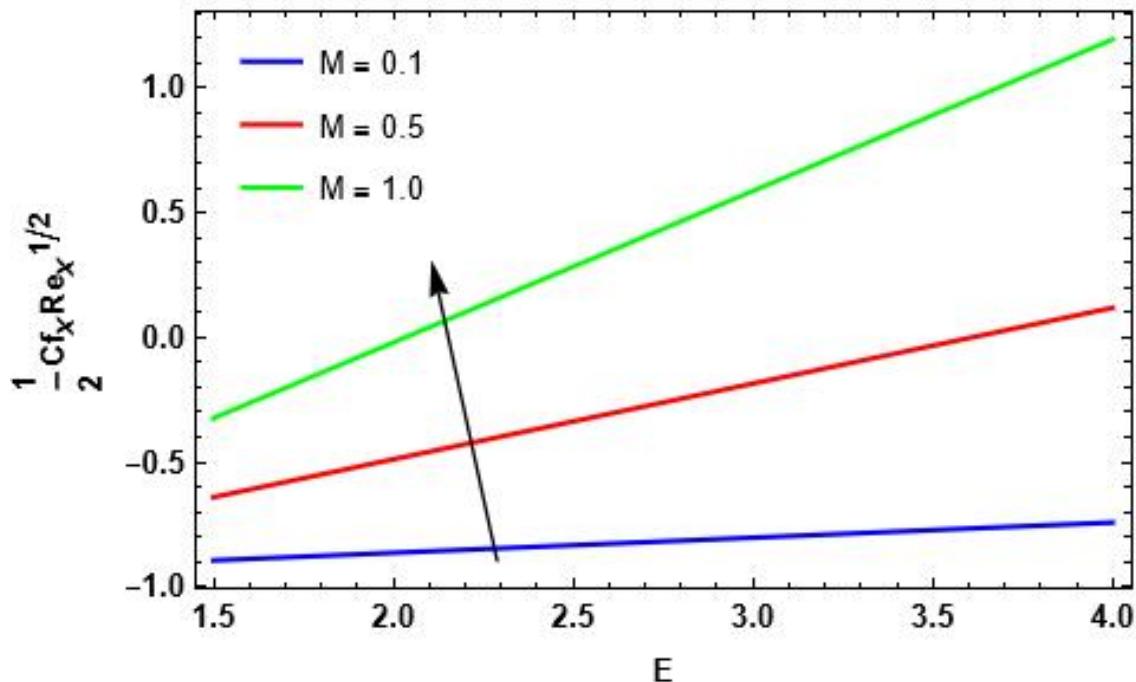
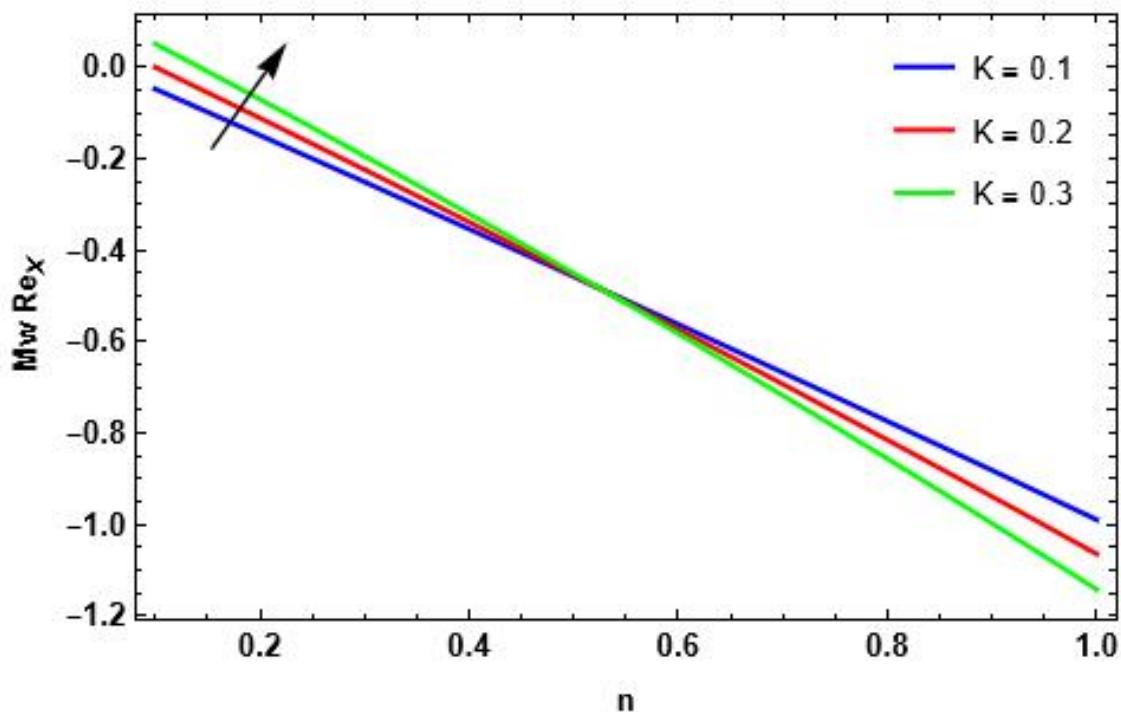
Figure 8.9:  $f'(\eta)$  via  $K$ .Figure 8.10:  $h(\eta)$  via  $K$ .

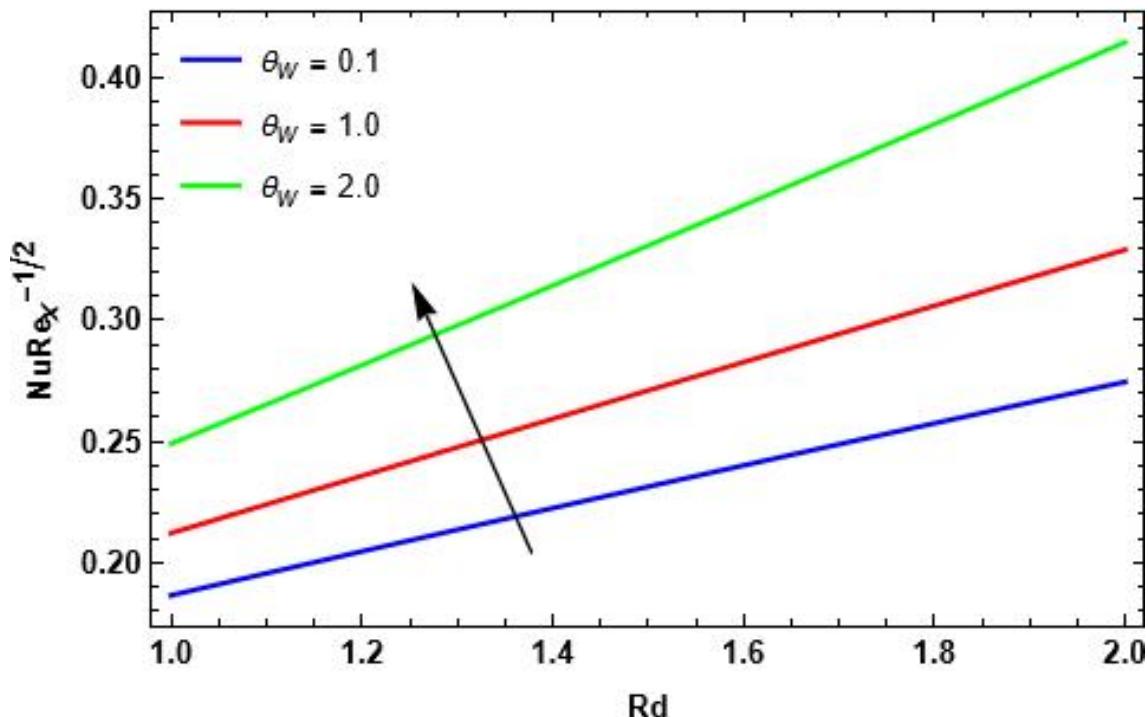
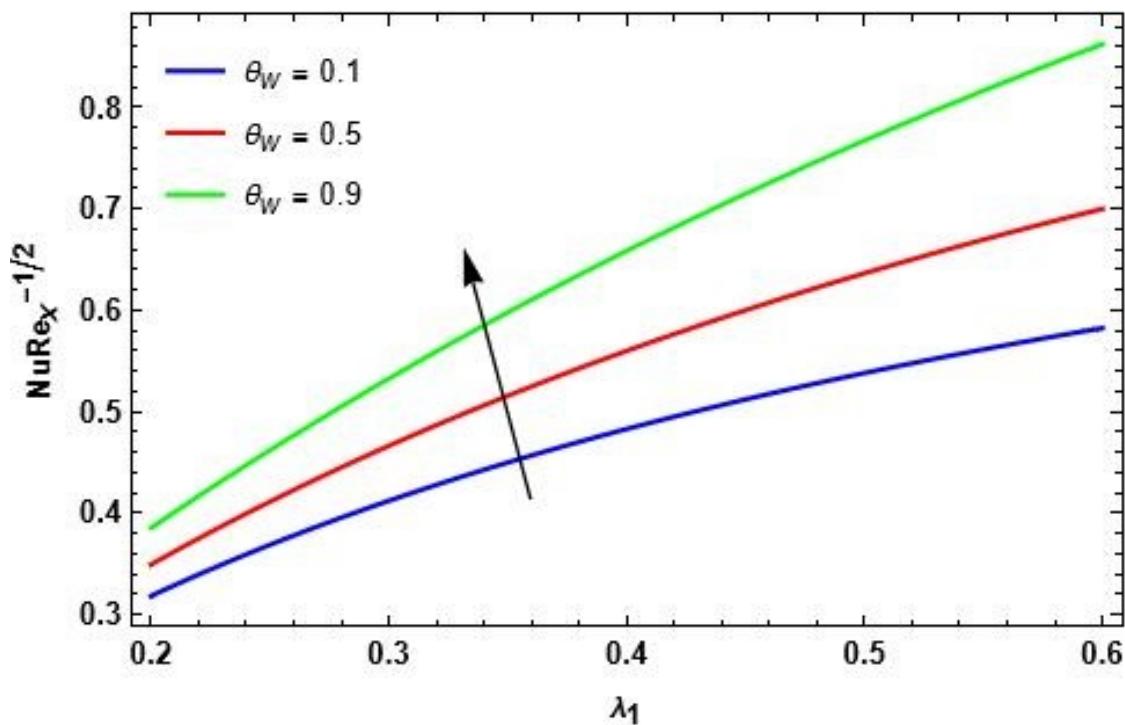
Figure 8.11:  $h(\eta)$  via  $n$ .Figure 8.12:  $\theta(\eta)$  via  $M$ .

Figure 8.13:  $\theta(\eta)$  via  $\theta_w$ .Figure 8.14:  $\theta(\eta)$  via  $Rd$ .

Figure 8.15:  $\theta(\eta)$  via  $E$ .Figure 8.16:  $\theta(\eta)$  via  $\lambda_1$ .

Figure 8.17:  $\phi(\eta)$  via  $Sc$ .Figure 8.18:  $\phi(\eta)$  via  $\lambda_2$ .

Figure 8.19:  $C_{f_x} R e_x^{1/2}$  via  $M$ .Figure 8.20:  $M_wx R e_x$  via  $K$ .

Figure 8.21:  $Nu_x Re_x^{1/2}$  via  $\theta_w$ .Figure 8.22:  $Nu_x Re_x^{1/2}$  via  $\theta_w$ .

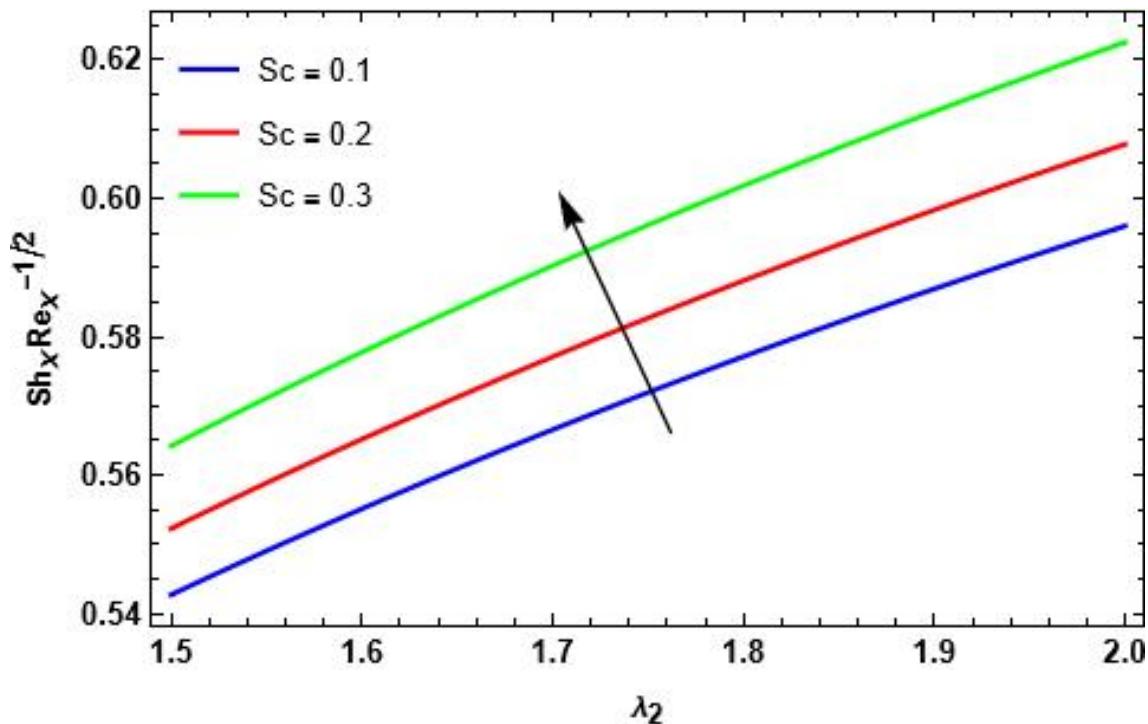
Figure 8.23:  $Sh_x Re_x^{1/2}$  via  $Sc$ .

Table 8.1: Convergence of series.

Order of Approximations	$-f''(0)$	$-h'(0)$	$-\theta'(0)$	$-\phi'(0)$
1	1.003244	0.066081	0.09101	0.084782
5	0.996594	0.055059	0.09228	0.080484
10	0.996694	0.055061	0.09203	0.079963
15	0.996693	0.055062	0.09201	0.079919
20	0.996693	0.055062	0.09201	0.079919

Table 8.2: Numerical calculation of Nusselt number.

$K$	$M$	$Rd$	$\theta_w$	$E$	$\gamma$	$\lambda_1$	$Ec$	$Pr$	$Nu_x Re_x^{-1/2}$
0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.2	8.0	0.0886728112312340
0.2									0.0877926621310645
0.3									0.0869460900339288
	0.3								0.0878993228765361
	0.4								0.0871292499113140
		0.2							0.0950280026875549
		0.3							0.1013352240321531
			0.2						0.0890899054572508
			0.3						0.0895248727257786
				0.2					0.0936954842292853
				0.3					0.0943091801034734
				1.1					0.0949447573637243
				1.2					0.0891655414211307
				1.3					0.0894984433810905
					0.2				0.0908204466328325
					0.3				0.092456165624022
						0.2			0.1637989432763814
						0.3			0.2283895299246889
							0.3		0.0819690494354928
							0.4		0.0755037261999629
								9.0	0.0874310327573451
								10.0	0.0861980091535024

## 8.7 Conclusion

The following are the research's findings:

- For higher values of Electric parameter and Material parameter, both increase velocity.
- With increasing values of Slip parameter, Magnetic parameter and Suction parameter, velocity decreases.
- For increasing values of Material parameter and concentration of microelements, both increase angular velocity.
- With increasing values of Prandtl number, temperature diminishes.
- For increased values of Magnetic parameter, Electric parameter, Thermal Biot number, Temperature ratio parameter, and Non-linear thermal radiation all enhance temperature.
- With higher values of Solutal Biot number raises concentration, while Schmidt number decreases it.
- For a large amount of Magnetic parameter, Skin friction coefficient increases.
- Local wall couple stress rises for a wide number of Material parameter.
- Nusselt number rises during large amount of thermal Biot number.
- Sherwood number increases for large amount of Schmidt number.

- 
- Skin friction factor enhances with increase in Weissenberg number  $We$ , velocity slip parameter  $\gamma$ , Thermal Grashof number  $Gr_T$  and Solutal Gashof number  $Gr_C$  whereas declined for Magnetic parameter  $M$ , Variable viscosity parameter  $\zeta$  and Suction parameter  $f_w$ .
  - Nusselt number is enhanced against rising values of Prandtl number  $Pr$ , temperature ratio parameter  $\theta_w$ , Thermal Biot number  $\lambda_1$ , Thermal Grashof number  $Gr_T$  and Radiation parameter  $Rd$ , while decreases for Dufour number  $Du$ , Heat generation/absorption coefficient  $\beta$  and Brinkman number  $Br$ .
  - Skin friction and number of Sherwood decreases for the broad unsteadiness parameter  $A$  values.
  - Sherwood number is raised for Schmidt number  $Sc$ .
  - $N_G$  augmented when increment occurs in Magnetic parameter  $M$ , Diffusion parameter  $L^*$ , Thermal Grashof number  $Gr_T$ , temperature difference parameter  $\alpha_1$  and Brinkman number  $Br$ .
  - Bejan number has increment for large amount of  $\alpha_1$ ,  $L^*$  and  $M$  while decreases for higher values of  $Br$ .

## **Future Works**

From studying previous research papers, it is observed that this work can be expanded in three dimensional steady and unsteady flow of Entropy optimized fluid. Also, effects of other physical parameters for different types of non-Newtonian fluids can be carried out. It may also be tried to develop mathematical models of two and three dimensional blood flow with magnetic field which can be used for human life. Study of effects of induced magnetic field for higher dimensional problems is rarely seen previous published works. It can also be tried to find the solutions of magnetic field effects on blood flow with heat transfer in different types of stenosed arteries and also discussed comparative study of different types of blood flow of different type's stenosed arteries.

# Published/Accepted Research Articles

1. H. R. Kataria, M. H. Mistry, Effect of Non-linear Radiation on MHD Mixed Convection Flow of a Micropolar fluid Over an Unsteady Stretching Sheet. In **Journal of Physics**, (2021), 1964, p. 022005, ISSN : 1742-6596.  
doi:10.1088/1742-6596/1964/2/022005, (**Scopus**).
2. H. R. Kataria, M. H. Mistry, A. S. Mittal, Influence of nonlinear radiation on MHD Micropolar fluid flow with viscous dissipation, **Heat Transfer (Wiley)**, (2022), 51(2), 1449-1467, ISSN : 2688-4542. <https://doi.org/10.1002/htj.22359>, (**Scopus**).
3. H. R. Kataria, M. H. Mistry, Entropy optimized MHD fluid flow over a vertical stretching sheet in presence of radiation, **Heat Transfer (Wiley)**, (2022), 51(3), 2546-2564, ISSN : 2688-4542. <https://doi.org/10.1002/htj.22412>, (**Scopus**).
4. H. R. Kataria, A. S. Mittal, M. H. Mistry, Effect of nonlinear radiation on entropy optimised MHD fluid flow, **International Journal of Ambient Energy (Taylor and Francis)**, (2022), 1-10, ISSN : 2162-8246.  
<https://doi.org/10.1080/01430750.2022.2059000>, (**Scopus**).
5. H. R. Kataria, A. S. Mittal, M. H. Mistry, Consequences of nonlinear radiation, variable viscosity, Soret and Dufour effects on MHD Carreau fluid flow, **International Journal of Applied and Computational Mathematics (Springer)**, (2022), ISSN : 2199-5796, (**Scopus**).

# **Communicated Research Work**

1. H. R. Kataria, M. H. Mistry, A. S. Mittal, Soret and Dufour impact on MHD Williamson fluid flow with non linear radiation and varying viscosity.
2. H. R. Kataria, M. H. Mistry, Entropy optimized unsteady MHD natural convective flow of Williamson fluid including onlinear radiation and viscous dissipation effects.

# **Presented Research Work in Conferences**

1. M. H. Mistry, H. R. Kataria, Mathematical Modelling of two dimensional electrically conducting Williamson fluid considering variable viscosity and radiation, in YOUNG SCIENTIST CONFERENCE as a part of India International Science Festival-2019 held at Biswa Bangla Convention Centre, Kolkata during November 5-8, 2019.
2. M. H. Mistry, H. R. Kataria, Mathematical modelling of two dimensional magnetohydrodynamic fluid flow considering variable viscosity and radiation, at the Science Conclave 2020 organized by Faculty of Science, The Maharaja Sayajirao University of Baroda, Vadodara on 28th February 2020.
3. M. H. Mistry, H. R. Kataria, Magneto-Micropolar electrically conducting fluid flow over a linearly stretching sheet for heat and mass transfer in International conference on Advances in Differential equations and Numerical analysis organized by Department of Mathematics, Indian Institute of Technology, Guwahati, India during October 12-15, 2020.
4. M. H. Mistry, H. R. Kataria, Effect of radiation on two dimensional electrically conducting Williamson fluid considering variable viscosity in the 21st International conference on Science, Engineering and Technology (ISCET-2020), organized by Vellore Institute of Technology, Vellore during November 30, 2020 to December 1, 2020.
5. M. H. Mistry, H. R. Kataria, Influence of Variable Thermal Conductivity and Viscosity on MHD Boundary Layer Flow of Carreau Fluid Over Stretching/Shrinking Sheet in the International Science Symposium on Recent trends in Science and Technology, organized by Christ college, Rajkot on 8th and 9th, April 2021. I stood second in the competition.
6. M. H. Mistry, H. R. Kataria, Entropy optimized MHD natural convective flow of Williamson fluid with nonlinear radiation and viscous dissipation effects,

---

International Conference on Recent Advances in Fluid Mechanics (ICRAFM-2022) organized Manipal Institute of Technology, MAHE, Manipal on 4th-6th, October 2022.

# Bibliography

- [1] A. Ajibade, A. Umar, Effects of viscous dissipation and boundary wall thickness on steady natural convection Couette flow with variable viscosity and thermal conductivity, International Journal of Thermofluids, 7(2020) 100052. <https://doi.org/10.1016/j.ijft.2020.100052>.
- [2] A. A. Afify, MHD free convective flow and mass transfer over a stretching sheet with chemical reaction, Heat and Mass Transfer, 40(2004) 495-500. <https://doi.org/10.1007/s00231-003-0486-0>.
- [3] A. T. Akinshilo, Mixed convective heat transfer analysis of MHD fluid flowing through an electrically conducting and non-conducting walls of a vertical micro-channel considering radiation effect, Applied Thermal Engineering, 156(2019) 506-513. <https://doi.org/10.1016/j.applthermaleng.2019.04.100>.
- [4] A. Bejan, A study of entropy generation in fundamental convective heat transfer, Journal of heat transfer, 101(1979) 718-725. <https://doi.org/10.1115/1.3451063>.
- [5] A. Bejan, Second law analysis in heat transfer, Energy, 5(1980) 720-732. [https://doi.org/10.1016/0360-5442\(80\)90091-2](https://doi.org/10.1016/0360-5442(80)90091-2).
- [6] A. Borrelli, G. Giantesio, M. C. Patria, N. C. Roșca, A. V. Roșca, I. Pop, Buoyancy effects on the 3D MHD stagnation-point flow of a Newtonian fluid, Communications in Nonlinear Science and Numerical Simulation, 43(2017) 1-13. <https://doi.org/10.1016/j.cnsns.2016.06.022>.
- [7] A. Chamkha, Mass transfer with chemical reaction in MHD mixed convective flow along a vertical stretching sheet, International Journal of Energy & Technology, 4(2012) 1-12.
- [8] A. J. Chamkha, A. M. Rashad, Unsteady heat and mass transfer by MHD mixed convection flow from a rotating vertical cone with chemical reaction

- and Soret and Dufour effects, *The Canadian Journal of Chemical Engineering*, 92(2014) 758-767. <https://doi.org/10.1002/cjce.21894>.
- [9] A. C. Eringen, Theory of micropolar fluids, *Journal of mathematics and Mechanics*, 16(1966) 1-18. <https://www.jstor.org/stable/24901466>.
  - [10] A. C. Eringen, Theory of thermomicrofluids, *Journal of Mathematical analysis and Applications*, 38(1972) 480-496. [https://doi.org/10.1016/0022-247X\(72\)90106-0](https://doi.org/10.1016/0022-247X(72)90106-0).
  - [11] A. Idowu, M. Akolade, J. Abubakar, B. Falodun, MHD free convective heat and mass transfer flow of dissipative Casson fluid with variable viscosity and thermal conductivity effects, *Journal of Taibah University for Science*, 14(2020) 851-862. <https://doi.org/10.1080/16583655.2020.1781431>.
  - [12] A. Ishak, Unsteady MHD flow and heat transfer over a stretching plate, *Journal of Applied Sciences*, 10(2010) 2127-2131. <https://dx.doi.org/10.3923/jas.2010.2127.2131>.
  - [13] A. V. Lemoff, A. P. Lee, An AC magnetohydrodynamic micropump, *Sensors and Actuators B: Chemical*, 63(2000) 178–185. [https://doi.org/10.1016/S0925-4005\(00\)00355-5](https://doi.org/10.1016/S0925-4005(00)00355-5).
  - [14] A. S. Mittal, H. R. Patel, R. R. Darji, Mixed Convection Micropolar Ferrofluid Flow with Viscous Dissipation, Joule Heating and Convective Boundary Conditions, *International Communications in Heat and Mass Transfer*, 108(2019) 104320. <https://doi.org/10.1016/j.icheatmasstransfer.2019.104320>.
  - [15] A. S. Mittal, H. R. Kataria, Three dimensional CuO-Water nanofluid flow considering Brownian motion in presence of radiation, *Karbala International Journal of Modern Science*, 4(2018) 275-286. <https://doi.org/10.1016/j.kijoms.2018.05.002>.
  - [16] A. S. Mittal, H. R. Patel, Influence of thermophoresis and Brownian motion on mixed convection two dimensional MHD Casson fluid flow with non-linear radiation and heat generation, *Physica A: Statical Mechanicsa and its Applications*, 537(2020) 122710. <https://doi.org/10.1016/j.physa.2019.122710>.
  - [17] A. M. Rashad, S. Abbasbandy, A. J. Chamkha, Mixed convection flow of a micropolar fluid over a continuously moving vertical surface immersed in

- a thermally and solutally stratified medium with chemical reaction, Journal of the Taiwan Institute of Chemical Engineers, 45(2014) 2163-2169. <https://doi.org/10.1016/j.jtice.2014.07.002>.
- [18] A. Raptis, A. K. Singh, MHD free convection flow past an accelerated vertical plate, International Communications in Heat and Mass Transfer, 10(1983) 313-321. [https://doi.org/10.1016/0735-1933\(83\)90016-7](https://doi.org/10.1016/0735-1933(83)90016-7).
  - [19] A. Ullah, A. Hafeez, W. K. Mashwani, W. Kumam, P. Kumam, M. Ayaz, Non-linear thermal radiations and mass transfer analysis on the processes of magnetite carreau fluid flowing past a permeable stretching/shrinking surface under cross diffusion and Hall effect, Coatings, 10(2020) 523. <https://doi.org/10.3390/coatings10060523>.
  - [20] B. J. Gireesha, A. Roja, Second law analysis of MHD natural convection slip flow of Casson fluid through an inclined microchannel, Multidiscipline Modeling in Materials and Structures, 16(2020) 1435-1455. <https://doi.org/10.1108/MMMS-11-2019-0189>.
  - [21] B. Gireesha, M. Umeshia, B. Prasannakumara, N. Shashikumar, M. Archana, Impact of nonlinear thermal radiation on magnetohydrodynamic three dimensional boundary layer flow of Jeffrey nanofluid over a nonlinearily permeable stretching sheet, Physica A: Statistical Mechanics and its Applications, 549(2020) 124051. <https://doi.org/10.1016/j.physa.2019.124051>.
  - [22] B. Gireesha, M. Archana, P. Kumar, R. Gorla, Significance of temperature dependent viscosity, nonlinear thermal radiation and viscous dissipation on the dynamics of water conveying cylindrical and brick shaped molybdenum disulphide nanoparticles, International Journal of Applied and Computational Mathematics, 5(2019) 1-15. <https://doi.org/10.1007/s40819-019-0649-4>.
  - [23] B. Raftari, K. Vajravelu, Homotopy analysis method for MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall, Communications in nonlinear science and numerical simulation, 17(2012) 4149-4162. <https://doi.org/10.1016/j.cnsns.2012.01.032>.
  - [24] B. K. Swain, B. C. Parida, S. Kar, N. Senapati, Viscous dissipation and joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium, Heliyon, 6(2020) e05338. <https://doi.org/10.1016/j.heliyon.2020.e05338>.

- [25] C. C. Cho, Mixed convection heat transfer and entropy generation of Cu-water nanofluid in wavy-wall lid-driven cavity in presence of inclined magnetic field, International Journal of Mechanical Sciences, 151(2019) 703-714. <https://doi.org/10.1016/j.ijmecsci.2018.12.017>.
- [26] C. J. Huang, Influence of non-Darcy and MHD on free convection of non-Newtonian fluids over a vertical permeable plate in a porous medium with soret/dufour effects and thermal radiation, International Journal of Thermal Sciences, 130(2018) 256-263. <https://doi.org/10.1016/j.ijthermalsci.2018.04.019>.
- [27] D. D. Gray, A. Giorgini, The validity of the Boussinesq approximation for liquids and gases, International Journal of Heat and Mass Transfer, 19(1976) 545-551. [https://doi.org/10.1016/0017-9310\(76\)90168-X](https://doi.org/10.1016/0017-9310(76)90168-X).
- [28] D. Pal, G. Mandal, K. Vajravalu, Soret and Dufour effects on MHD convective-radiative heat and mass transfer of nanofluids over a vertical non-linear stretching/shrinking sheet, Applied Mathematics and Computation, 287(2016) 184-200. <https://doi.org/10.1016/j.amc.2016.04.037>.
- [29] F. Mabood, W. A. Khan, A. I. M. Ismail, Approximate analytical modeling of heat and mass transfer in hydromagnetic flow over a non-isothermal stretched surface with heat generation/absorption and transpiration, Journal of the Taiwan Institute of Chemical Engineers, 54(2015) 11-19. <https://doi.org/10.1016/j.jtice.2015.03.022>.
- [30] F. Mabood, T. Yusuf, G. Bognár, Features of entropy optimization on MHD couple stress nanofluid slip flow with melting heat transfer and nonlinear thermal radiation, Scientific Reports, 10(2020) 1-13. <https://doi.org/10.1038/s41598-020-76133-y>.
- [31] F. Sultan, S. Mustafa, W. Khan, M. Shahzad, M. Ali, W. Adnan, S. Rehman, A numerical treatment on rheology of mixed convective Carreau nanofluid with variable viscosity and thermal conductivity, Applied Nanoscience, 10(2020) 3295-3303. <https://doi.org/10.1007/s13204-020-01294-1>.
- [32] F. A. Soomro, M. Usman, R. U. Haq, W. Wang, Thermal and velocity slip effects on MHD mixed convection flow of Williamson nanofluid along a vertical surface: Modified Legendre wavelets approach, Physica E: Low-dimensional Systems and Nanostructures, 104(2018) 130-137. <https://doi.org/10.1016/j.physe.2018.07.002>.

- [33] F. Selimefendigil, H. F. Öztop, MHD mixed convection and entropy generation of power law fluids in a cavity with a partial heater under the effect of a rotating cylinder, International Journal of Heat and Mass Transfer, 98(2016) 40-51. <https://doi.org/10.1016/j.ijheatmasstransfer.2016.02.092>.
- [34] F. Wang, M. I. Asjad, M. Zahid, A. Iqbal, H. Ahmad, M. D. Alsulami, Unsteady Thermal Transport Flow of Casson Nanofluids with Generalized Mittag-Leffler Kernel of Prabhakar's Type, Journal of Materials Research and Technology, 14(2021) 1292–1300. <https://doi.org/10.1016/j.jmrt.2021.07.029>.
- [35] G. Lukaszewicz, Micropolar fluids: theory and applications, Springer Science & Business Media, (1999).
- [36] G. Nagaraju, J. Srinivas, J. R. Murthy, A. M. Rashad, Entropy generation analysis of the MHD flow of couple stress fluid between two concentric rotating cylinders with porous lining, Heat Transfer-Asian Research, 46(2017) 316-330. <https://doi.org/10.1002/htj.21214>.
- [37] G. J. Reddy, R. S. Raju, J. A. Rao, Influence of viscous dissipation on unsteady MHD natural convective flow of Casson fluid over an oscillating vertical plate via FEM, Ain Shams Engineering Journal, 9(2018) 1907-1915. <https://doi.org/10.1016/j.asej.2016.10.012>.
- [38] H. R. Patel, A. S. Mittal, R. R. Darji, MHD Flow of Micropolar Nanofluid over a Stretching/Shrinking Sheet Considering Radiation, International Communications in Heat and Mass Transfer, 108(2019) 104322. <https://doi.org/10.1016/j.icheatmasstransfer.2019.104322>.
- [39] H. R. Kataria, A. S. Mittal, Mathematical model for velocity and temperature of gravity-driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation, Journal of Nigerian Mathematical Society, 34(2015) 303-317. <https://doi.org/10.1016/j.jnnms.2015.08.005>.
- [40] H. R. Kataria, A. S. Mittal, Velocity, Mass and Temperature Analysis of Gravity-Driven Convection Nanofluid Flow Past an Oscillating Vertical Plate in the Presence of Magnetic Field in a Porous Medium, Applied Thermal Engineering, 110(2017a) 864-874. <https://doi.org/10.1016/j.applthermaleng.2016.08.129>.

- [41] H. R. Kataria, H. R. Patel, Radiation and chemical reaction effects on MHD Casson fluid flow past an oscillating vertical plate embedded in porous medium, *Alexandria Engineering Journal*, 55(2016) 583-595. <https://doi.org/10.1016/j.aej.2016.01.019>.
- [42] H. R. Kataria, H. R. Patel, Soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium, *Alexandria Engineering Journal*, 55(2016) 2125-2137. <https://doi.org/10.1016/j.aej.2016.06.024>.
- [43] H. R. Kataria, H. R. Patel, Effects of chemical reaction and heat generation/absorption on magnetohydrodynamic (MHD) Casson fluid flow over an exponentially accelerated vertical plate embedded in porous medium with ramped wall temperature and ramped surface concentration, *Propulsion and Power Research* 8(2019) 35-46. <https://doi.org/10.1016/j.jppr.2018.12.001>.
- [44] H. R. Kataria, H. R. Patel, R. Singh, Effect of magnetic field on unsteady natural convective flow of a micropolar fluid between two vertical walls, *Ain Shams Engineering Journal*, 8(2017) 87-102. <https://doi.org/10.1016/j.asej.2015.08.013>.
- [45] H. Kataria, H. Patel, Effect of thermo-diffusion and parabolic motion on MHD Second grade fluid flow with ramped wall temperature and ramped surface concentration, *Alexandria engineering journal*, 57(2018) 73-85. <https://doi.org/10.1016/j.aej.2016.11.014>.
- [46] H. F. Öztop, N. S. Bondareva, M. A. Sheremet, N. Abu-Hamdeh, Unsteady natural convection with entropy generation in partially open triangular cavities with a local heat source, *International Journal of Numerical Methods for Heat & Fluid Flow*, 27(2017) 2696-2716. <https://doi.org/10.1108/HFF-12-2016-0510>.
- [47] H. S. Takhar, A. A. Raptis, C. P. Perdikis, MHD asymmetric flow past a semi-infinite moving plate, *Acta Mechanica*, 65(1987) 287-290. <https://doi.org/10.1007/BF01176888>.
- [48] H. Vaidya, C. Rajashekhar, F. Mebarek-Oudina, K. V. Prasad, K. Vajravelu, B. Ramesh Bhat, Examination of Chemical Reaction on Three Dimensional Mixed Convective Magnetohydrodynamic Jeffrey Nanofluid Over a Stretching Sheet, *Journal of Nanofluids*, 11(2022) 113-124. <https://doi.org/10.1166/jon.2022.1817>.

- [49] J. Hartmann, Hg-dynamics I theory of the laminar flow of an electrically conductive liquid in a homogenous magnetic field, Det Kal. Danske Videnskabernes selskab, Mathematisk-fysiske Meddeleser, 15(1937) 1-27.
- [50] J. Wang, R. Muhammad, M. I. Khan, W. A. Khan, S. Z. Abbas, Entropy optimized MHD nanomaterial flow subject to variable thicked surface, Computer Methods and Programs in Biomedicine, 189(2020) 105311. <https://doi.org/10.1016/j.cmpb.2019.105311>.
- [51] J. Zhong, M. Yi, H. H. Bau, Magnetohydrodynamic (MHD) pump fabricated with ceramic tapes, Sensors and Actuators A: Physical, 96(2002) 59–66. [https://doi.org/10.1016/S0924-4247\(01\)00764-6](https://doi.org/10.1016/S0924-4247(01)00764-6).
- [52] K. Anantha Kumar, V. Sugunamma, N. Sandeep, Physical aspects on unsteady MHD-free convective stagnation point flow of micropolar fluid over a stretching surface, Heat Transfer—Asian Research, 48(2019) 3968-3985. <https://doi.org/10.1002/htj.21577>.
- [53] K. Aslani, U. S. Mahabaleshwar, J. Singh, I. E. Sarris, Combined effect of radiation and inclined MHD flow of a micropolar fluid over a porous stretching/shrinking sheet with mass transpiration, International Journal of Applied and Computational Mathematics, 7(2021) 1-21. <https://doi.org/10.1007/s40819-021-00987-7>.
- [54] K. Batchlor, An introduction to fluid dynamics, London: Cambridge University Press, 158(1987).
- [55] K. G. Kumar, G. K. Ramesh, B. J. Gireesha, R. S. R. Gorla, Characteristics of Joule heating and viscous dissipation on three-dimensional flow of Oldroyd B nanofluid with thermal radiation, Alexandria Engineering Journal, 57(2018) 2139-2149. <https://doi.org/10.1016/j.aej.2017.06.006>.
- [56] K. Jabeen, M. Mushtaq, R. M. A. Muntazir, Analysis of MHD Fluids around a Linearly Stretching Sheet in Porous Media with Thermophoresis, Radiation and Chemical Reaction, Mathematical Problems in Engineering, 2020(2020) 9685482. <https://doi.org/10.1155/2020/9685482>.
- [57] K. L. Hsiao, Combined electrical MHD heat transfer thermal extrusion system using Maxwell fluid with radiative and viscous dissipation effects, Applied Thermal Engineering, 112(2017) 1281-1288. <https://doi.org/10.1016/j.applthermaleng.2016.08.208>.

- [58] K. R. Rajagopal, M. Ruzicka, A. R. Srinivasa, On the Oberbeck-Boussinesq approximation, Mathematical Models and Methods in Applied Sciences, 6(1996). 1157-1167. <https://doi.org/10.1142/S0218202596000481>.
- [59] K. U. Rehman, A. S. Alshomrani, M. Y. Malik, Carreau fluid flow in a thermally stratified medium with heat generation/absorption effects, Case studies in thermal engineering, 12(2018) 16-25. <https://doi.org/10.1016/j.csite.2018.03.001>.
- [60] K. Sarada, R. J. P. Gowda, I. E. Sarris, R. N. Kumar, B. C. Prasannakumara, Effect of Magnetohydrodynamics on Heat Transfer Behaviour of a Non-Newtonian Fluid Flow over a Stretching Sheet under Local Thermal Non-Equilibrium Condition, Fluids, 6(2021) 264. <https://doi.org/10.3390/fluids6080264>.
- [61] K. Sharma, K. Bhaskar, Influence of Soret and Dufour on Three-Dimensional MHD Flow Considering Thermal Radiation and Chemical Reaction, International Journal of Applied and Computational Mathematics, 6(2020) 1-17. <https://doi.org/10.1007/s40819-019-0753-5>.
- [62] K. A. Yih, Free convection effect on MHD coupled heat and mass transfer of a moving permeable vertical surface, International Communications in Heat and Mass Transfer, 26(1999) 95-104. [https://doi.org/10.1016/S0735-1933\(98\)00125-0](https://doi.org/10.1016/S0735-1933(98)00125-0).
- [63] L. A. Lund, Z. Omar, S. Dero, I. Khan, Linear stability analysis of MHD flow of micropolar fluid with thermal radiation and convective boundary condition: Exact solution, Heat Transfer—Asian Research, 49(2020) 461-476. <https://doi.org/10.1002/htj.21621>.
- [64] M. S. Astanina, M. A. Sheremet, H. F. Öztürk, N. Abu-Hamdeh, MHD natural convection and entropy generation of ferrofluid in an open trapezoidal cavity partially filled with a porous medium, International Journal of Mechanical Sciences, 136(2018) 493-502. <https://doi.org/10.1016/j.ijmecsci.2018.01.001>.
- [65] M. Bibi, A. Zeeshan, M. Malik, Numerical analysis of unsteady flow of three-dimensional Williamson fluid-particle suspension with MHD and nonlinear thermal radiations, The European Physical Journal Plus, 135(2020) 1-26. <https://doi.org/10.1140/epjp/s13360-020-00857-z>.]

- [66] M. Bhatti, L. Phali, C. M. Khalique, Heat transfer effects on electro-magnetohydrodynamic Carreau fluid flow between two micro-parallel plates with Darcy-Brinkman-Forchheimer medium, *Archive of Applied Mechanics*, 91(2021) 1683-1695. <https://doi.org/10.1007/s00419-020-01847-4>.
- [67] M. Dada, C. Onwubuoya, Variable viscosity and thermal conductivity effects on Williamson fluid flow over a slendering stretching sheet, *World Journal of Engineering*, 17(2020) 357-371. <https://doi.org/10.1108/WJE-08-2019-0222>.
- [68] M. F. El-Amin, Magnetohydrodynamic free convection and mass transfer flow in micropolar fluid with constant suction, *Journal of magnetism and magnetic materials*, 234(2001) 567-574. [https://doi.org/10.1016/S0304-8853\(01\)00374-2](https://doi.org/10.1016/S0304-8853(01)00374-2).
- [69] M. Farooq, M. I. Khan, M. Waqas, T. Hayat, A. Alsaedi, M. I. Khan, MHD Stagnation Point Flow of Viscoelastic Nanofluid with Non-Linear Radiation Effects, *Journal of Molecular Liquids*, 221(2016) 1097–1103. <https://doi.org/10.1016/j.molliq.2016.06.077>.
- [70] M. A. Hossain, Viscous and Joule heating effects on MHD free convection flow with variable plate temperature (No. IC-90/265). International Centre for Theoretical Physics, (1990).
- [71] M. Hussain, A. Ghaffar, A. Ali, A. Shahzad, K. Nisar, M. Alharthi, W. Jamshed, MHD thermal boundary layer flow of a Casson fluid over a penetrable stretching wedge in the existence of nonlinear radiation and convective boundary condition, *Alexandria Engineering Journal*, 60(2021) 5473-5483. <https://doi.org/10.1016/j.aej.2021.03.042>.
- [72] M. Imtiaz, H. Nazar, T. Hayat, A. Alsaedi, Soret and Dufour effects in the flow of viscous fluid by a curved stretching surface, *Pramana*, 94(2020) 1-11. <https://doi.org/10.1007/s12043-020-1922-0>.
- [73] M. Kayalvizhi, R. Kalaivanan, N. V. Ganesh, B. Ganga, A. A. Hakeem, Velocity slip effects on heat and mass fluxes of MHD viscous–Ohmic dissipative flow over a stretching sheet with thermal radiation, *Ain Shams Engineering Journal*, 7(2016) 791-797. <https://doi.org/10.1016/j.asej.2015.05.010>.
- [74] M. I. Khan, F. Alzahrani, Activation Energy and Binary Chemical Reaction Effect in Nonlinear Thermal Radiative Stagnation Point Flow of Walter-B Nanofluid: Numerical Computations, *International Journal of Modern Physics B*, 34(2020) 2050132. <https://doi.org/10.1142/S0217979220501325>.

- [75] M. I. Khan, A. Kumar, T. Hayat, M. Waqas, R. Singh, Entropy generation in flow of Carreau nanofluid, *Journal of Molecular Liquids*, 278(2019) 677-687. <https://doi.org/10.1016/j.molliq.2018.12.109>.
- [76] M. V. Krishna, K. Vajravelu, Hall effects on the unsteady MHD flow of the Rivlin-Ericksen fluid past an infinite vertical porous plate, *Waves in Random and Complex Media*, (2022)1-24. <https://doi.org/10.1080/17455030.2022.2084178>.
- [77] M. Massoudi, I. Christie, Effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe, *International Journal of Non-Linear Mechanics*, 30(1995) 687-699. [https://doi.org/10.1016/0020-7462\(95\)00031-I](https://doi.org/10.1016/0020-7462(95)00031-I).
- [78] M. Madhu, B. Mahanthesh, N. S. Shashikumar, S. A. Shehzad, S. U. Khan, B. J. Gireesha, Performance of second law in Carreau fluid flow by an inclined microchannel with radiative heated convective condition, *International Communications in Heat and Mass Transfer*. 117(2020) 104761. <https://doi.org/10.1016/j.icheatmasstransfer.2020.104761>.
- [79] M. K. Nayak, A. A. Hakeem, B. Ganga, M. I. Khan, M. Waqas, O. D. Makinde, Entropy optimized MHD 3D nanomaterial of non-Newtonian fluid: a combined approach to good absorber of solar energy and intensification of heat transport, *Computer methods and programs in biomedicine*, 186(2020) 105131. <https://doi.org/10.1016/j.cmpb.2019.105131>.
- [80] M. K. Nayak, S. Shaw, A.J. Chamkha, 3D MHD Free Convective Stretched Flow of a Radiative Nanofluid Inspired by Variable Magnetic Field, *Arabian Journal of science and engineering*, 44( 2019) 1269-1282. <https://doi.org/10.1007/s13369-018-3473-y>.
- [81] M. K. Nayak, N. S. Akbar, D. Tripathi, V. S. Pandey, Three dimensional MHD flow of nanofluid over an exponential porous stretching sheet with convective boundary conditions, *Thermal Science and Engineering Progress*, 3(2017) 133-140. <https://doi.org/10.1016/j.tsep.2017.07.006>.
- [82] M. K. Nayak, G. C. Dash, L. P. Singh, Heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction, *Propulsion and Power Research*, 5(2016) 70-80. <https://doi.org/10.1016/j.jppr.2016.01.006>.

- [83] M. Ramzan, M. Farooq, T. Hayat, J. D. Chung, Radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary condition, *Journal of Molecular Liquids*, 221(2016) 394-400. <https://doi.org/10.1016/j.molliq.2016.05.091>.
- [84] M. M. Rashidi, S. Abelman, N. F. Mehr, Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid, *International journal of Heat and Mass transfer*, 62(2013) 515-525. <https://doi.org/10.1016/j.ijheatmasstransfer.2013.03.004>.
- [85] M. M. Rashidi, B. Rostami, N. Freidoonimehr, S. Abbasbandy, Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects, *Ain Shams Engineering Journal*, 5(2014) 901-912. <https://doi.org/10.1016/j.asej.2014.02.007>.
- [86] M. M. Rashidi, S. A. Mohimanian Pour, A novel analytical solution of heat transfer of a micropolar fluid through a porous medium with radiation by DTM-Padé, *Heat Transfer-Asian Research*, 39(2010) 575-589. <https://doi.org/10.1002/htj.20317>.
- [87] M. G. Reddy, P. Padma, B. Shankar, Effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet, *Ain Shams Engineering Journal*, 6(2015) 1195-1201. <https://doi.org/10.1016/j.asej.2015.04.006>.
- [88] M. Sheikholeslami, H. R. Kataria, A. S. Mittal, Radiation Effects on Heat Transfer of Three Dimensional Nanofluid Flow Considering Thermal Interfacial Resistance and Micro Mixing in Suspensions, *Chinese Journal of Physics*, 55(2017) 2254–2272. <https://doi.org/10.1016/j.cjph.2017.09.010>.
- [89] M. Sheikholeslami, H. R. Kataria, A. S. Mittal, Effect of thermal diffusion and heat-generation on MHD nanofluid flow past an oscillating vertical plate through porous medium, *Journal of Molecular Liquids*, 257(2018) 12-25. <https://doi.org/10.1016/j.molliq.2018.02.079>.
- [90] M. Sheikholeslami, D. D. Ganji, M. Y. Javed, R. Ellahi, Effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model, *Journal of magnetism and Magnetic materials*, 374(2015) 36-43. <https://doi.org/10.1016/j.jmmm.2014.08.021>.
- [91] M. Sulemana, I. Y. Seini, O. D. Makinde, Hydrodynamic Boundary Layer Flow of Chemically Reactive Fluid over Exponentially Stretching Vertical Surface with Transverse Magnetic Field in Unsteady

- Porous Medium, Journal of Applied Mathematics, 2022(2022) 7568695.  
<https://doi.org/10.1155/2022/7568695>.
- [92] M. Turkyilmazoglu, MHD fluid flow and heat transfer due to a stretching rotating disk, International journal of thermal sciences, 51(2012) 195-201.  
<https://doi.org/10.1016/j.ijthermalsci.2011.08.016>.
- [93] M. Turkyilmazoglu, I. Pop, Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid, International Journal of Heat and Mass Transfer, 57(2013) 82-88.  
<https://doi.org/10.1016/j.ijheatmasstransfer.2012.10.006>.
- [94] M. Turkyilmazoglu, MHD fluid flow and heat transfer due to a shrinking rotating disk, Computers & Fluids, 90(2014) 51-56.  
<https://doi.org/10.1016/j.compfluid.2013.11.005>.
- [95] M. Usman, Z. H. Khan, M. B. Liu, MHD natural convection and thermal control inside a cavity with obstacles under the radiation effects, Physica A: Statical Mechanics and its applications, 535(2019) 122443.  
<https://doi.org/10.1016/j.physa.2019.122443>.
- [96] M. Waqas, M. Farooq, M. I. Khan, A. Alsaedi, T. Hayat, T. Yasmeen, Magnetohydrodynamic (MHD) mixed convection flow of micropolar liquid due to nonlinear stretched sheet with convective condition, International Journal of Heat and Mass Transfer, 102(2016) 766-772.  
<https://doi.org/10.1016/j.ijheatmasstransfer.2016.05.142>.
- [97] N. Kumar, S. Gupta, MHD free-convective flow of micropolar and Newtonian fluids through porous medium in a vertical channel, Meccanica, 47(2012) 277-291. <https://doi.org/10.1007/s11012-011-9435-z>.
- [98] N. K. Ranjit, G. C. Shit, Entropy generation on electromagnetohydrodynamic flow through a porous asymmetric micro-channel, European Journal of Mechanics-B/Fluids, 77(2019) 135-147.  
<https://doi.org/10.1016/j.euromechflu.2019.05.002>.
- [99] N. R. Devi, M. Shivananda, H. F. Öztürk, N. Abu-Hamdeh, P. Padmanathan, A. Satheesh, A review on ferrofluids with the effect of MHD and entropy generation due to convective heat transfer, The European Physical Journal Plus, 137(2022) 482. <https://doi.org/10.1140/epjp/s13360-022-02616-8>.

- [100] N. Z. Basha, K. Vajravelu, F. Mebarek-Oudina, I. Sarris, K. Vaidya, K. V. Prasad, C. Rajashekhar, MHD Carreau nanoliquid flow over a nonlinear stretching surface, *Heat Transfer*, 51(2022) 5262-5287. <https://doi.org/10.1002/htj.22546>.
- [101] O. A. Bég, A. Y. Bakier, V. R. Prasad, Numerical study of free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects, *Computational Materials Science*, 46 (2009) 57-65. <https://doi.org/10.1016/j.commatsci.2009.02.004>.
- [102] P. J. Carreau, D. Kee, M. Daroux, An analysis of the viscous behaviour of polymeric solutions, *The Canadian Journal of Chemical Engineering*, 57(1979) 135-140. <https://doi.org/10.1002/cjce.5450570202>.
- [103] P. S. Reddy, A. J. Chamkha, A. Al-Mudhaf, MHD heat and mass transfer flow of a nanofluid over an inclined vertical porous plate with radiation and heat generation/absorption, *Advanced Powder Technology*, 28(2017) 1008-1017. <https://doi.org/10.1016/j.apt.2017.01.005>.
- [104] P. G. Siddheshwar, U. S. Mahabaleswar, H. I. Andersson, A new analytical procedure for solving the non-linear differential equation arising in the stretching sheet problem, *International Journal of Applied Mechanics and Engineering*, 18(2013) 955-964. <https://doi.org/10.2478/ijame-2013-0059>.
- [105] P. G. Siddheshwar, A. Chan, U. S. Mahabaleswar, Suction-induced magnetohydrodynamics of a viscoelastic fluid over a stretching surface within a porous medium, *The IMA Journal of Applied Mathematics*, 79(2014) 445-458. <https://doi.org/10.1093/imamat/hxs074>.
- [106] R. C. Chaudhary, A. K. Jha, Effects of chemical reactions on MHD Micropolar fluid flow past a vertical plate in slip-flow regime, *Applied mathematics and Mechanics*, 29(2008) 1179-1194. <https://doi.org/10.1007/s10483-008-0907-x>.
- [107] R. J. P. Gowda, R. N. Kumar, A. M. Jyothi, B. C. Prasannakumara, I. E. Sarris, Impact of Binary Chemical Reaction and Activation Energy on Heat and Mass Transfer of Marangoni Driven Boundary Layer Flow of a Non-Newtonian Nanofluid, *Processes*, 9(2021) 702. <https://doi.org/10.3390/pr9040702>.
- [108] R. Mehta, H. R. Kataria, Brownian motion and thermophoresis effects on MHD flow of viscoelastic fluid over stretching/shrinking sheet in the presence of thermal radiation and chemical reaction, *Heat Transfer*, 51(2022) 274-295. <https://doi.org/10.1002/htj.22307>.

- [109] R. Saravana, R. Hemadri Reddy, K. V. Narasimha Murthy, O. D. Makinde, Thermal radiation and diffusion effects in MHD Williamson and Casson fluid flows past a slendering stretching surface, *Heat Transfer*, 51(2022) 3187-3200. <https://doi.org/10.1002/htj.22443>.
- [110] R. V. Williamson, The flow of pseudoplastic materials, *Industrial & Engineering Chemistry*, 21(1929), 1108-1111. <https://doi.org/10.1021/ie50239a035>.
- [111] S. Abbasbandy, Homotopy analysis method for heat radiation equations. *International Communications in Heat and Mass Transfer*, 34(2007) 380–387. <https://doi.org/10.1016/j.icheatmasstransfer.2006.12.001>.
- [112] S. K. Adegbie, O. K. Koriko, I. L. Animasaun, Melting heat transfer effects on stagnetion point flow of micropolar fluid with variable dynamic viscosity and thermal conductivity at constant vortex viscosity, *Journal of the Nigerian Mathematical society*, 35(2016) 34-47. <http://dx.doi.org/10.1016/j.jnnms.2015.06.004>.
- [113] S. Afsana, M. M. Molla, P. Nag, L. K. Saha, S. Siddiqua, MHD Natural Convection and Entropy Generation of non-Newtonian Ferrofluid in a Wavy Enclosure, *International Journal of Mechanical Sciences*, 198(2021) 106350. <https://doi.org/10.1016/j.ijmecsci.2021.106350>.
- [114] S. M. Arifuzzaman, M. S. Khan, M. F. U. Mehedi, B. M. J. Rana, S.F. Ahmed, Chemically reactive and naturally convective high speed MHD fluid flow through an oscillatory vertical porous plate with heat and radiation absorption effect, *Engineering Science and Technology, an International Journal*, 21(2018) 215-228. <https://doi.org/10.1016/j.jestch.2018.03.004>.
- [115] S. Das, R. N. Jana, O. D. Makinde, MHD boundary layer slip flow and heat transfer of nanofluid past a vertical stretching sheet with non-uniform heat generation/absorption, *International Journal of Nanoscience*, 13(2014), 1450019. <https://doi.org/10.1142/S0219581X14500197>.
- [116] S. Das, R. N. Jana, O. D. Makinde, Magnetohydrodynamic mixed convective slip flow over an inclined porous plate with viscous dissipation and Joule heating. *Alexandria Engineering Journal*, 54(2015) 251-261. <https://doi.org/10.1016/j.aej.2015.03.003>.
- [117] S. R. Elkoumy, E. I. Barakat, S. I. Abdelsalam, Hall and Transverse Magnetic Field Effects on Peristaltic Flow of a Maxwell Fluid through a Porous Medium,

Global Journal of Pure and Applied Mathematics, 9(2013) 187–203. [https://buescholar.bue.edu.eg/basic\\_sci\\_eng/30](https://buescholar.bue.edu.eg/basic_sci_eng/30).

- [118] S. Gupta, D. Kumar, J. Singh, Analytical study for MHD flow of Williamson nanofluid with the effects of variable thickness, nonlinear thermal radiation and improved Fourier's and Fick's Laws, SN Applied Sciences, 2(2020) 1-12. <https://doi.org/10.1007/s42452-020-1995-x>.
- [119] S. Jena, G. C. Dash, S. R. Mishra, Chemical reaction effect on MHD viscoelastic fluid flow over a vertical stretching sheet with heat source/sink, Ain Shams Engineering Journal, 9(2018) 1205-1213. <https://doi.org/10.1016/j.asej.2016.06.014>
- [120] S. Liao, Beyond Perturbation: Introduction to the Homotopy Analysis Method, Chapman and Hall/CRC Press, (2003). <http://doi.org/10.1201/9780203491164>.
- [121] S. Liao, Homotopy analysis method in nonlinear differential equations, Beijing: Higher education press, 2012.
- [122] S. R. Mishra, I. Khan, Q. M. Al-Mdallal, T. Asifa, Free convective micropolar fluid flow and heat transfer over a shrinking sheet with heat source, Case studies in thermal engineering, 11(2018) 113-119. <https://doi.org/10.1016/j.csite.2018.01.005>.
- [123] S. Nadeem, S. T. Hussain, C. Lee, Flow of a Williamson fluid over a stretching sheet, Brazilian journal of chemical engineering, 30(2013) 619-625. <https://doi.org/10.1590/S0104-66322013000300019>.
- [124] S. Nadeem, R. U. Haq, C. Lee, MHD boundary layer flow over an unsteady shrinking sheet: analytical and numerical approach, Journal of the Brazilian society of Mechanical sciences and Engineering, 37(2015) 1339-1346. <https://doi.org/10.1007/s40430-014-0261-9>.
- [125] S. R. Pradhan, S. Baag, S. R. Mishra, M. R. Acharya, Free convective MHD micropolar fluid flow with thermal radiation and radiation absorption: A numerical study, Heat Transfer—Asian Research, 48(2019) 2613-2628. <https://doi.org/10.1002/htj.21517>.
- [126] S. Rosseland, Astrophysik und atom-theoretische Grundlagen, Springer-Verlag; Berlin, 1931.

- [127] T. M. Ajayi, A. J. Omowaye, I. L. Animasaun, Effects of viscous dissipation and double stratification on MHD Casson fluid flow over a surface with variable thickness: boundary layer analysis, In International journal of engineering research in Africa, 28(2017) 73-89. <https://doi.org/10.4028/www.scientific.net/JERA.28.73>.
- [128] T. Abbas, S. Rehman, R. Shah, M. Idrees, M. Qayyum, Analysis of MHD Carreau fluid flow over a stretching permeable sheet with variable viscosity and thermal conductivity, Physica A-statistical Mechanics and Its Applications, 551(2020) 124225. <https://doi.org/10.1016/j.physa.2020.124225>.
- [129] T. Tayebi, A. S. Dogonchi, A. J. Chamkha, M. B. B. Hamida, S. El-Sapa, A. M. Galal, Micropolar nanofluid thermal free convection and entropy generation through an inclined I-shaped enclosure with two hot cylinders, Case Studies in Thermal Engineering, 31(2022) 101813. <https://doi.org/10.1016/j.csite.2022.101813>.
- [130] T. A. Yusuf, F. Mabood, B. C. Prasannakumara, I. E. Sarris, Magneto-bioconvection flow of Williamson nanofluid over an inclined plate with gyrotactic microorganisms and entropy generation, Fluids, 6(2021), 109. <https://doi.org/10.3390/fluids6030109>.
- [131] U. Ali, M. Y. Malik, A. A. Alderremy, S. Aly, K. U. Rehman, A generalized findings on thermal radiation and heat generation/absorption in nanofluid flow regime, Physica A: Statistical Mechanics and its Applications, 553(2020) 124026. <https://doi.org/10.1016/j.physa.2019.124026>.
- [132] U. Ghosh, Electro-magneto-hydrodynamics of non-linear viscoelastic fluids, Journal of Non-Newtonian Fluid Mechanics, 277(2020) 104234. <https://doi.org/10.1016/j.jnnfm.2020.104234>.
- [133] U. S. Mahabaleshwar, K. R. Nagaraju, M. A. Sheremet, P. N. Vinay Kumar, G. Lorenzini, Effect of mass transfer and MHD induced Navier's slip flow due to a non linear stretching sheet, Journal of Engineering Thermophysics, 28(2019) 578-590. <https://doi.org/10.1134/S1810232819040131>.
- [134] U. Nazir, S. Saleem, M. Nawaz, M. A. Sadiq, A. Alderremy, Study of transport phenomenon in Carreau fluid using Cattaneo-Christov heat flux model with temperature dependent diffusion coefficients, Physica A: Statistical Mechanics and its Applications, 554(2020) 123921. <https://doi.org/10.1016/j.physa.2019.123921>.

- [135] W. A. Khan, I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, International journal of heat and mass transfer, 53(2010) 2477-2483. <https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032>.
- [136] W. Wang, B. W. Li, P. L. Varghese, X. Y. Leng, X. Y. Tian, Numerical analysis of three-dimensional MHD natural convection flow in a short horizontal cylindrical annulus, International Communications in Heat and Mass Transfer, 98(2018) 273-285. <https://doi.org/10.1016/j.icheatmasstransfer.2018.09.009>.
- [137] W. Ibrahim, Magnetohydrodynamics (MHD) flow of a tangent hyperbolic fluid with nanoparticles past a stretching sheet with second order slip and convective boundary condition. Results in physics, 7(2017) 3723-3731. <https://doi.org/10.1016/j.rinp.2017.09.041>.
- [138] Y. S. Daniel, Z. A. Aziz, Z. Ismail, A. Bahar, Unsteady EMHD dual stratified flow of nanofluid with slips impacts, Alexandria Engineering Journal, 59(2020) 177-189. <https://doi.org/10.1016/j.aej.2019.12.020>.
- [139] Y. S. Daniel, Z. A. Aziz, Z. Ismail, F. Salah, Impact of thermal radiation on electrical MHD flow of nanofluid over nonlinear stretching sheet with variable thickness, Alexandria Engineering Journal, 57(2018) 2187-2197. <https://doi.org/10.1016/j.aej.2017.07.007>.
- [140] Y. S. Daniel, Z. A. Aziz, Z. Ismail, F. Salah, Double stratification effects on unsteady electrical MHD mixed convection flow of nanofluid with viscous dissipation and Joule heating, Journal of applied research and technology, 15(2017) 464-476. <https://doi.org/10.1016/j.jart.2017.05.007>.
- [141] Y. Lin, L. Zheng, B. Li, L. Ma, A new diffusion for laminar boundary layer flow of power law fluids past a flat surface with magnetic effect and suction or injection, International Journal of Heat and Mass Transfer, 90(2015) 1090-1097. <https://doi.org/10.1016/j.ijheatmasstransfer.2015.07.067>.
- [142] Y. D. Reddy, B. S. Goud, A. J. Chamkha, M. A. Kumar, Influence of radiation and viscous dissipation on MHD heat transfer Casson nanofluid flow along a nonlinear stretching surface with chemical reaction, Heat Transfer, 51(2022) 3495-3511. <https://doi.org/10.1002/htj.22460>.
- [143] Z. Li, M. Sheikholeslami, A. S. Mittal, A. Shafee, R. U. Haq, Nanofluid heat transfer in a porous duct in the presence of Lorentz forces using the lattice Boltzmann method, The European Physical Journal Plus, 134(2019) 1-10. <https://doi.org/10.1140/epjp/i2019-12406-8>.