Chapter 1

Introduction

General relativity, formulated by Albert Einstein more than a century ago, revolutionized our understanding of gravity. The General Theory of Relativity (GTR) is based on two postulates. **The principle of covariance** suggests that the laws must be expressible in a form which is independent of the particular spacetime coordinates chosen, that is, laws of nature remain invariant with respect to any spacetime coordinate system. **The principle of equivalance** says that in the neighbourhood of any given point, we distinguish between the gravitational field produced by the attraction of masses and the field produced by the accelerating frame of reference.

1.1 Tensor Calculus

The tensor calculus is highly used in general relativity to describe Einstein's field equation. These field equations relate the geometry of spacetime to the distribution of matter and energy.

According to the general theory of relativity, the presence of matter curves up the geometry of associated spacetime, where the metric is described by the Riemannian metric. According to John A. Wheeler, "Spacetime tells matter how to move, matter tells spacetime how to curve".

$$ds^2 = g_{ij}dx^i dx^j, \quad i = 0,1,2,3; \quad j = 0,1,2,3. \tag{1.1}$$

where g_{ij} is fundamental tensor and $g = |g_{ij}| = \begin{vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{vmatrix}$.

We define $g^{ij} = \frac{1}{|g|} \times$ Co-factor of g_{ij}, in this determination, $|g| \neq 0$.

The tensor g^{ij} is called reciprocal tensor of g_{ij}. The Riemann-Christoffel Curvature tensor is defined as

$$R^a_{ijk} = -\frac{\partial \Gamma^a_{ij}}{\partial x^k} + \frac{\partial \Gamma^a_{ik}}{\partial x^j} - \Gamma^b_{ij}\Gamma^a_{bk} + \Gamma^b_{ik}\Gamma^a_{bj}, \tag{1.2}$$

where, Christoffel symbols of first and second kind are respectively defined by

$$\Gamma_{ij,a} = \frac{1}{2}\left(\frac{\partial g_{aj}}{\partial x^i} + \frac{\partial g_{ai}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^a}\right), \tag{1.3}$$

$$\Gamma^a_{ij} = g^{al}\Gamma_{ij,l}. \tag{1.4}$$

Christoffel symbols are not tensor quantities but are used to describe the Riemann-Christoffel Curvature tensor. By contracting R^a_{ijk} w.r.t., a and k, gives Ricci tensor denoted by R_{ij} and defined as

$$R_{ij} = R^a_{ija} = -\frac{\partial \Gamma^a_{ij}}{\partial x^a} + \frac{\partial \Gamma^a_{ia}}{\partial x^j} - \Gamma^b_{ij}\Gamma^a_{ba} + \Gamma^b_{ia}\Gamma^a_{bj}, \tag{1.5}$$

and by contracting the Ricci tensor R_{ij}, we get the Ricci scalar R in the following manner

$$R = g^{ij}R_{ij}. \tag{1.6}$$

The behavior of spacetime in the presence of matter is governed by the Einstein's field equations. Throughout the thesis, we use stable spherically symmetric spacetime metric, which can be written in two ways. One with signature (+,-,-,-) in this case Einstein's field equations are described as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^2}T_{ij}, \tag{1.7}$$

and other with signature (-,+,+,+) in this case Einstein's field equations are described as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi G}{c^2}T_{ij}, \tag{1.8}$$

where, $i,\ j = 0,\ 1,\ 2,\ 3$. R_{ij} denotes Ricci tensor, R denotes Ricci scalar, and T_{ij} is the energy-momentum tensor that contains information on physical properties of matter distribution. G is the Newtonian gravitational constant and c is the speed of light.

Einstein's field equations are a collection of 10 second-order, non-linear partial differential equations. The Bianchi identity $\nabla^i G_{ij} = 0$, reduces the number of independent equations to six.

1.2 Energy Momentum Tensor

Einstein's field equations (1.7) connect the geometry of the spacetime with the matter content producing curvatures in the spacetime.

1.2.1 Dust Fluid

The stress-energy tensor of a relativistic pressureless fluid is

$$T^{ij} = \rho u^i u^j \tag{1.9}$$

where ρ is the mass density in the dust's rest frame and u^i is the dust's four-velocity.

1.2.2 Perfect Fluid

The energy-momentum tensor for a perfect fluid is a mathematical representation of the distribution of energy, momentum and stress within a fluid that behaves as an idealized fluid. For a perfect fluid, the energy-momentum tensor takes the following form

$$T^{ij} = (\rho + p)u^i u^j - p g_{ij}, \tag{1.10}$$

where ρ is the energy density of the fluid in the comoving frame (rest frame of the fluid), p is the pressure of the fluid in the comoving frame, u^i is the four-velocity vector of the fluid, and g_{ij} is the metric tensor of spacetime.

1.2.3 Electromagnetic Field

The energy-momentum tensor associated with a distribution of charge is given by

$$E_i^j = \frac{1}{4\pi}\left(-F_{ik}F^{kj} + \frac{1}{4}F_{mn}F^{mn}\delta_i^j\right), \tag{1.11}$$

where $F'_{ij}s$ are components of electromagnetic field tensor satisfying Maxwell's equations

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0, \tag{1.12}$$

and

$$\frac{\partial}{\partial x^k}(F^{ik}\sqrt{-g}) = 4\pi\sqrt{-g}J^i. \tag{1.13}$$

The four current J^i is defined as

$$J^i = \sigma u^i, \tag{1.14}$$

where σ denotes the charge density of the distribution. For a static distribution

$$u^i = (e^{-\frac{\nu}{2}}, 0, 0, 0). \tag{1.15}$$

The spherical symmetry implies that electromagnetic field tensor F_{ij} has F_{10} = -F_{01} as its only non-vanishing component. The Maxwell's equations (1.12) and (1.13) admit

$$F_{01} = -\frac{e^{\frac{(\lambda+\nu)}{2}}}{r^2}\int_0^r 4\pi r^2 \sigma e^{\frac{\nu}{2}} dr \tag{1.16}$$

as their solution.

1.2.4 Anisotropic Fluid Distribution

A fluid distribution with radial pressure different from tangential pressure is termed as an anisotropic fluid distribution. According to Maharaj and Maarten [122], we write the energy-momentum tensor as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} + \pi_{ij}, \tag{1.17}$$

where ρ is the proper density, p is the isotropic pressure, u^i the four-velocity field of the fluid. The anisotropic stress tensor is given by

$$\pi_{ij} = \sqrt{3}S[C_iC_j - \frac{1}{3}(u_iu_j - g_{ij})], \tag{1.18}$$

where $C^i = (0, -e^{\frac{\lambda}{2}}, 0, 0)$ is a radially directed vector and $S = S(r)$ denotes the magnitude of anisotropy. The non-vanishing components of the energy-momentum tensor are

$$T_0^0 = \rho, \qquad T_1^1 = -\left(p + \frac{2S}{\sqrt{3}}\right), \qquad T_2^2 = T_3^3 = -\left(p - \frac{S}{\sqrt{3}}\right), \tag{1.19}$$

and define the radial and tangential pressures as

$$p_r = p + \frac{2S}{\sqrt{3}}, \qquad p_\perp = p - \frac{S}{\sqrt{3}}. \tag{1.20}$$

The magnitude of anisotropy obtained as

$$S = \frac{p_r - p_\perp}{\sqrt{3}}, \tag{1.21}$$

as the magnitude of anisotropy. For a perfect fluid distribution $p_r = p_\perp$ and hence S = 0.

1.3 Field Equations

For static spherically symmetric spacetime metric,

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.22}$$

Einstein's field equations lead to four-second-order, non-linear ordinary differential equations, of which three equations are independent. Due to the non-linear nature of Einstein's field equations, it is difficult to obtain a closed-form (exact) solution of Einstein's field equations in general. Exact solution plays a significant role in understanding the properties of compact stars.

The energy-momentum tensor for anisotropic fluid distribution can be defined as

$$T_{ij} = (\rho + p_\perp)u_i u_j + p_\perp g_{ij} + (p_r - p_\perp)\chi_i \chi_j, \tag{1.23}$$

where ρ is the matter density, p_r is the radial pressure, $p_\perp$ is the tangential pressure, u^i is the four-velocity of the fluid and χ^i is a unit spacelike four-vector along the radial direction so that $u^i u_i = -1, \chi^i \chi_j = 1$ and $u^i \chi_j = 0$. For spacetime metric (1.22) and energy-momentum tensor (1.23), with $G = c^2 = 1$ the Einstein's field equations takes the form

$$8\pi\rho = \frac{1 - e^{-\lambda}}{r^2} + \frac{e^{-\lambda}\lambda'}{r}, \tag{1.24}$$

$$8\pi p_r = \frac{e^{-\lambda}\nu'}{r} + \frac{e^{-\lambda} - 1}{r^2}, \tag{1.25}$$

$$8\pi p_\perp = e^{-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu' - \lambda'}{2r}\right), \tag{1.26}$$

$$8\pi\sqrt{3}S = 8\pi p_r - 8\pi p_\perp, \tag{1.27}$$

where prime ($'$) denotes differentiation with respect to r. The system of equations (1.24-1.26) governs the behavior of the gravitational field for anisotropic fluid distribution.

Karl Schwarzschild [170] first described the exact solution of Einstein's field equation for empty spacetime then the interior of stellar objects with constant density was studied by Schwarzschild [171]. This model is reasonably good at describing the interior of stellar structures with low pressure.

Chandrasekhar [34] has articulated '...The life history of a small-mass star must be fundamentally different from that of a large-mass star... A tiny mass star enters the white-dwarf stage... A massive star cannot enter this stage, leaving us to speculate on other possibilities.' When a star's gravitational pull is balanced by its internal pressure, it remains in an equilibrium state. When the star's nuclear fuel is exhausted, it loses its equilibrium and begins to collapse. A star with an initial mass less than the Chandrasekhar limit of 1.4 $M_{\odot}$, where $M_{\odot}$ is the solar mass, falls into an equilibrium state known as a white dwarf star, whose gravitational pull is balanced by pressure electron degeneracy.

Theoretical investigation of Ruderman [168] and Canuto [31] suggests that pressure may not be isotropic at ultra-high density. Bowers and Liang [28] demonstrated that anisotropy in pressure has a non-negligible impact on the maximum equilibrium mass and surface redshift of astrophysical objects. Herrera and Santos [76] provided a comprehensive overview of anisotropic fluid distributions. Tolman [208] developed a method to find exact solutions to Einstein's field equations for static fluid spheres. Pant and Sah [145] generalized Tolman's VI solution (with parameter B = 0) by considering charge into account. Bayin [7] discovered the solution for the anisotropic fluid sphere and also examined the radiating anisotropic fluid sphere. Pant and Sah [146] extended Tolman's I, IV, and V solutions, as well as the de Sitter solution, to derive a new class of static solutions by assuming an equation of state. Using the ansatz $e^{\nu} \propto (1+x)^n$, Durgapal [52] generated a class of new exact solutions for spherically symmetric static fluid spheres. With a modification in the Tolman III, IV, V, and VI Solutions, Krori *et. al.* [94] obtained exact solutions of Einstein's field equations for the anisotropic matter. A new ansatz has been designed by Maartens and Maharaj [129] to determine the exact solution to Einstein's field equations.

Patel and Koppar [153] obtained the charged analog of Vaidya and Tikekar [211] solution on spheroidal spacetime. Two exact analytical solutions to Einstein's field equations for anisotropic matter distribution that describe the maximum mass, causality condition, and surface and central redshifts have been studied by de Leȯn [46]. Maharaj and Maarten [122] obtained an exact solution for Einstein's field equations. Delgaty and Lake [45] examined 127 solutions of Einstein's field equations out of which only 9 satisfy physical plausibility conditions including causality conditions.

Many authors worked on geometrically significant spacetime metric, whose space part has specific geometry.

1.4 Geometrically Significance Spacetime Metric

Considering static spherically symmetric fluid distribution as

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.28}$$

with an ansatz

$$e^{\lambda(r)} = \frac{1 + ar^2}{1 + br^2} \tag{1.29}$$

where a and b are geometrical parameters.

The $t =$ constant section satisfies the cartesian equation

$$\frac{x^2 + y^2 + z^2}{L^2} - \frac{w^2}{A^2} + \frac{w}{A} = 0, \tag{1.30}$$

immersed in a four-dimensional Euclidean space with metric

$$d\sigma^2 = dx^2 + dy^2 + dz^2 + dw^2, \tag{1.31}$$

where the parameter a and b are related by

$$a = \frac{4(1+\frac{A^2}{L^2})}{L^2}, \quad b = \frac{4}{L^2}, \tag{1.32}$$

The space part of metric (1.28) is obtained by introducing,

$$\begin{aligned} x &= rsin\theta cos\phi, \\ y &= rsin\theta sin\phi, \\ z &= rcos\theta, \\ w &= \frac{A + A\sqrt{1+4\frac{r^2}{L^2}}}{2}. \end{aligned} \tag{1.33}$$

Combining equation(1.28) and (1.29) gives,

$$ds^2 = e^{\nu(r)}dt^2 - \frac{1+ar^2}{1+br^2}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.34}$$

This metric is also a consequence of Matese and Whitman's function [97].

(i) Case-I : $a = -\frac{k}{R^2}, b = -\frac{1}{R^2}$, where k and R are geometric parameter.

In this case, the metric (1.34) takes the form

$$ds^2 = e^{\nu(r)}dt^2 - \frac{1-k\frac{r^2}{R^2}}{1-\frac{r^2}{R^2}}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.35}$$

which is known as spheroidal spacetime metric developed by Vaidya and Tikekar [211], for $k < 1$ the space part of the metric (1.35) is regular, for $k = 1$ the space part of the metric (1.35) represents flatspace and for $k = 0$ the space part of the metric (1.35) represents a 3-sphere.

In the case of perfect fluid,

(i) when k = 0 and $e^{\nu(r)} = [A + B\sqrt{1-\frac{r^2}{R^2}}]^2$, metric (1.35) gives Schwarzschild interior solution.

(ii)when k = 0 and $\nu = 0$, metric (1.35) gives the Einstein's universe.

(iii)when k = 0 and $e^{\nu(r)} = 1 - \frac{r^2}{R^2}$, metric (1.35) gives the de Sitter's universe.

A number of researchers have used spacetime metric (1.35) to describe physically feasible models of superdense stars with matter as charged fluid distribution, anisotropic fluid distribution, and perfect fluid distribution. Few of these are Tikekar [202], Finch and Skea [57], Singh and Kotambkar [176], Paul *et. al.* [156], Thirukkanesh [191].

(ii) Case-II : $a = \frac{k}{R^2}, b = \frac{1}{R^2}$, where k and R are geometric parameters.

In this case, the metric (1.34) takes the form

$$ds^2 = e^{\nu(r)}dt^2 - \frac{1 + k\frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.36}$$

which is known as the pseudo-spheroidal spacetime metric. The metric (1.36) is regular for $k > 1$. This metric has been developed and studied by Tikekar and Thomas [203]. Later, several authors considered pseudo-spheroidal spacetime metric to model physically viable stars. Few of these are Tikekar and Thomas [204], Tikekar and Thomas [205], Thomas *et. al.* [196], Thomas and Ratanpal [197], Paul *et. al.* [156], Chattopadhyay and Paul [36], Chattopadhyay *et. al.* [37], Thomas and Pandya ([198], [199]), and Ratanpal *et. al.*([159], [158]).

(iii) Case-III : $a = \frac{1}{R^2}, b = 0$, where R is geometric parameter.

In this case, the metric (1.34) takes the form

$$ds^2 = e^{\nu(r)}dt^2 - \left(1 + \frac{r^2}{R^2}\right)dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.37}$$

which is called paraboloidal (also known as Finch Skea[57]) spacetime metric. However, paraboloidal can not be obtained from cartesian equation (1.30) due to which it is not possible to obtain a solution on paraboloidal spacetime metric by setting b=0 in equation (1.29). The paraboloidal case has to be tackled separately. This form of Finch Skea metric was considered by Jotania and Tikekar [85]. Various authors considered this spacetime metric to develop models of compact stars. Few of them include Ratanpal *et. al.* [160], Banerjee *et. al.* [8], Hansraj and Maharaj [68], Maharaj *et. al.* [118].

The spacetime metric (1.34) was considered by several authors. Sharma *et. al.* [177] studied the general solution for a class of static charged spheres by assuming $E^2 = \frac{\alpha^2(1-x^2)}{R^2(1+\lambda-\lambda x^2)^2}$ with metric potential $g_{rr} = e^{2\mu} = \frac{1+\lambda\frac{r^2}{R^2}}{1-\frac{r^2}{R^2}}$. Thirukkanesh and Maharaj [188] studied the exact models for isotropic matter with metric potential $g_{rr} = \frac{1+ar^2}{1+r^2}$. Maharaj and Komathiraj [125] studied the exact solutions to the Einstein-Maxwell system of equations in spherically symmetric gravitational fields with an electric field intensity. Komathiraj and Maharaj [90] presented exact solutions to the Einstein-Maxwell system of equations with a specific form of $E^2 = \frac{\alpha k(x^2-1)}{R^2(1-k+kx^2)^2}$. Sharma and Maharaj [178] presented an exact solution to the Einstein field equations with anisotropic matter distribution by assuming mass function $m(r) = \frac{br^3}{2(1+ar^2)}$ using the linear equation of state. Komathiraj and Maharaj [91] obtained a new class of exact solution to the Einstein-Maxwell system of equations which can be used to model the interior of charged relativistic objects. Feroze and siddiqui [56] studied a charged anisotropic matter with the quadratic equation of state. Takisa and Maharaj [184] studied the Einstein-Maxwell system of equations with anisotropic pressures and electromagnetic field using the polytropic equation of state. Takisa and Maharaj [186] studied a charged compact object with anisotropic pressure in a core envelope star using the quadratic equation of state in the core part and a linear equation of state in the envelope part. Thirukkanesh *et. al.* [194] studied the impact of anisotropy on the superdense relativistic stars.

Utilising a spherically symmetric spacetime metric, Mak and Harko [121] obtained exact anisotropic solutions of Einstein's field equations. Sharma and Ratanpal [179] generated the exact solution of Einstein's field equations on a static spherically symmetric spacetime metric. Gupta and Maurya [65] obtained a class of charged analog of Durgapal and Fuloria [53] solution for the superdense star. Maurya and Gupta [99] studied the charged analog of a neutral solution with ansatz $g_{rr} = (1 + cr^2)^6$ on a static spherically symmetric spacetime metric. Maurya and Gupta [98] obtained extremum mass of charged superdense star models using metric potential $g_{rr} = B(1 + cr^2)^n$. Murad [114] studied a class of interior solutions of the Einstein-Maxwell system of equations for a static spherically symmetric distribution of a charged perfect fluid. Murad and Fatema [115] and Fatema and Murad [55] obtained an exact solution of static spherically symmetric perfect fluid spheres of the Einstein-Maxwell field equations. Murad and Fatema [117] obtained charged and anisotropic models in generalized Tolman IV spacetime. Pandya *et. al.* [149] studied the static spherically symmetric anisotropic system using modified Finch and Skea ansatz. Dayanandan *et. al.* [43] investigated the stability of an anisotropic compact star model using Matese and Whitman solutions in general relativity. Bhar *et. al.* [17] obtained a relativistic anisotropic compact star model having a metric potential $g_{rr} = 1 + \frac{a^2r^2}{(1+br^2)^4}$ using embedding class one. Sharma *et. al.* [180] studied superdense relativistic stars with anisotropic matter distribution. Khunt *et. al.* [96] studied the core envelope model of highly compact stars using the quadratic equation of state. By assuming pressure anisotropy Bhar and Rej [20] obtained a new model of anisotropic compact star with modified Finch Skea spacetime. Bhar *et. al.* [21] obtained a singularity-free spherically symmetric stellar model with anisotropic pressure using the Tolman ansatz (Tolman [208]). Das *et. al.* [41] studied the anisotropic extension of the well-known Tolman IV solution to model realistic compact stellar objects.

1.5 Compact Stars

A compact star is the final stage of stellar evolution, arising from a star's gravitational collapse after its nuclear fuel runs out. White dwarfs, neutron stars, and black holes are examples of these objects. All three states described for a star during its evolution are known as compact stars (Shapiro *et. al.* [172]). White dwarfs are dense objects of low to medium mass; neutron stars emerge when more massive stars collapse; and black holes form when very massive stars crash. Extreme densities and gravitational forces characterize these objects.

In comparison to normal stars, compact stars have two distinguishing characteristics. The first characteristic of compact stars is their ultra-high density. Despite having masses comparable to or lower than the Sun, these stars have been reduced into significantly smaller volumes, resulting in extraordinarily dense cores. The second characteristic of compact stars is gravitational collapse. This collapse occurs because the star's internal pressure is no longer sufficient to overcome gravity, resulting in an enormous decrease in size and a rise in density.

1.5.1 White Dwarfs

White dwarf stars have a radius of about 5000 km and a density of about 1 ton/cm^3 (Misner *et. al.* [134]). White dwarfs are low to medium-mass stars (up to $8M_\odot$) that have used all their nuclear fuel. Electron degeneracy pressure, a quantum mechanical process, protects these objects from gravitational collapse. Over time, white dwarfs cool and darken, eventually becoming cold and dark "black dwarfs."

1.5.2 Neutron Star

A neutron star is an astronomical object known for its exceptional compactness and density, primarily composed of neutrons. These unique celestial bodies form as a consequence of the supernova explosion of a massive star. Large stars undergo a catastrophic collapse when their nuclear fuel is exhausted. The collapse is driven by gravitational forces, leading to the formation of neutron stars. The small size is a result of gravitational collapse during their formation. Neutron stars exhibit extraordinarily strong gravitational fields. Surface gravity on a neutron star is approximately 100 billion times greater than that of Earth. Neutron stars often have fast rotation rates. Some neutron stars, known as pulsars, emit beams of electromagnetic radiation as they rotate. When these pulsars move across the earth's line of sight, the radiation is observed as regular pulses. Neutron stars maintain stability as long as their mass does not exceed a limit close to 2 $M_\odot$.

1.5.3 Black Holes

A black hole is a region of spacetime characterized by an incredibly powerful gravitational field, preventing the escape of anything, including particles and electromagnetic radiation like light. The boundary surrounding this region is known as the event horizon. Black holes are formed when massive stars, having exhausted their fuel, succumb to gravity and collapse. The gravitational pull of a black hole is so powerful that it warps the fabric of spacetime around it. It earned the name "black" because it absorbs all light that strikes it, making the region invisible. The gravitational pull of a black hole is exceptionally strong, warping spacetime in its vicinity. Supermassive black holes can have masses ranging from hundreds of thousands to billions of times that of the sun. The exact procedure for the formation of black holes is currently under study. The first strong case for a black hole in a binary system was that of Cygnus X-1, an X-ray source investigated by Bowyer *et. al.* [25]. This structure provides a clear overview of black holes, covering their formation, key characteristics, event horizon, mass range, the ongoing study of their creation, and the historical context of the first identified black hole.

The study of compact stars remains an area of interest in relativistic astrophysics as they provide a platform for the observational work of these objects and vice-versa. Hewish *et. al.* [4] identified the first pulsar, CP 1919, subsequently known as PSR B1919+21, in the year 1968. The finding of the first pulsar was a significant breakthrough in relativistic astrophysics. Antony Hewish and Sir Martin Ryle shared the Nobel Prize in Physics in 1974 for discovering pulsars. Oppenheimer and Volkov [143] established a relativistic theory of neutron stars. Observational data show that the masses of compact pulsating objects may range from 1 to 2 $M_\odot$, even though the majority of neutron star masses are grouped around 1.4 $M_\odot$. The diameters of such tiny objects might range between 5 - 15 km given by Shapiro and Teukolsky [173], Prakash *et. al.* [157], Akmal *et. al.* [2]. In Table (1.1) masses and radii of some of the observed compact stars are given.

Table 1.1: Masses and radii of some observed compact stars.

Object	**Mass** $M_\odot$	**Radius (Km)**	**References**
PSR J1614-2230	1.97 ± 0.04	9.69 ± 0.2	Demorest *et. al.* [49]
Cen X-3	1.49 ± 0.08	9.178 ± 0.13	Rawls *et. al.* [163]
Vela X-1	1.77 ± 0.08	9.56 ± 0.08	Rawls *et. al.* [163]
PSR J1903 + 327	1.667 ± 0.021	9.438 ± 0.03	Freire *et. al.* [60]
SMC X-4	1.29 ± 0.05	8.831 ± 0.09	Rawls *et. al.* [163]
LMC X-4	1.04 ± 0.09	8.301 ± 0.2	Rawls *et. al.* [163]
Her X-1	0.85 ± 0.15	8.1 ± 0.41	Rawls *et. al.* [163]
4U1608-52	1.74 ± 0.14	9.528 ± 0.15	Güver *et. al.* [66]
4U1820-30	1.58 ± 0.06	9.316 ± 0.086	Güver *et. al.* [67]
EXO 1785-248	1.3 ± 0.2	8.849 ± 0.4	Özel *et. al.* [144]

1.6 Layout of the thesis

The thesis is organized as follows:

Chapter 1 contains an introduction to the general theory of relativity. It also contains the summary of each chapter of the thesis.

Chapter 2 describes a class of new solutions for Einstein's field equations under Karmarkar [86] conditions, by choosing the ansatz $e^{\lambda(r)} = \frac{1+k\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}$ in static spherically symmetric spacetime metric (1.22). The Karmarkar [86] conditions provides a relation between Riemann curvature tensor R_{ijkl} in the form

$$R_{1414}R_{2323} = R_{1212}R_{3434} + R_{1224}R_{1334}. \tag{1.38}$$

This can be written in the form

$$\frac{\nu''}{\nu'} + \frac{\nu'}{2} = \frac{\lambda' e^{\lambda}}{2\left(e^{\lambda}-1\right)}, \tag{1.39}$$

for the metric (1.22). The general solution of equation (1.39) is given by

$$e^{\nu} = \left[A + B\int\sqrt{(e^{\lambda(r)}-1)}dr\right]^2, \tag{1.40}$$

where A and B are constants of integration.

The pressure anisotropy takes the form

$$8\pi\sqrt{3}S = 8\pi p_r - 8\pi p_\perp = -\frac{\nu' e^{-\lambda}}{4}\left[\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1}\right]\left[\frac{\nu' e^{\nu}}{2rB^2} - 1\right]. \tag{1.41}$$

In the case of isotropic distribution of matter, we have $S = 0$ which leads to either

$\frac{2}{r} - \frac{\lambda'}{e^{\lambda}-1} = 0$ or $\frac{\nu' e^{\nu}}{2rB^2} - 1 = 0$. The former case leads to Schwarzschild [169] exterior solution and the latter gives the solution given by Kohler and Chao [92]. It is found that some pulsars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3 can be accommodated in this model. We have displayed the nature of physical parameters and energy conditions throughout the distribution using numerical and graphical methods for a particular pulsar 4U 1820-30 and found that the solution satisfies all physical requirements.

Chapter 3 deals with a new class of singularity-free interior solutions describing anisotropic matter distribution on static spherically symmetric spacetime metric. Das *et. al.* ([39], [40]) considered metric potential g_{rr} as $B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^4}$, and $B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^6}$, respectively and developed the models of relativistic stars. We have generalized the work of Das *et. al.* ([39], [40]) by considering metric potential g_{rr} as

$$B_0^2(r) = \frac{1}{(1-\frac{r^2}{R^2})^n} \, , \tag{1.42}$$

where $n > 2$ is a positive integer. Also, $B_0^2(r) = 1$ ensures that it is finite at the centre. It is regular at the centre since $(B_0^2(r))' = 0$, we obtained the models of relativistic stars and it is observed that all the physical quantities are well behaved up to n= 70. The various physical characteristics of the model are examined for the pulsar PSRJ1903+327. Analysis shows that all the physical acceptability conditions are satisfied.

Chapter 4, In this chapter the new exact solutions of Einstein-Maxwell system of equations for charged anisotropic models have been obtained by choosing ansatz $e^{\lambda} = 1+\frac{r^2}{R^2}$, here we consider linear equation of state for radial pressure $p_r = A\rho - B$, where A and B are constants. The expression of charge is considered as

$$E^2 = \frac{\alpha \frac{r^2}{R^2}}{R^2(1+\frac{r^2}{R^2})^2} , \tag{1.43}$$

The physical acceptability conditions of the model have been investigated, and it is shown that the model is compatible with several compact star candidates like 4U 1820-30, PSR J1903+327, EXO 1785-248, LMC X-4, SMC X-4, Cen X-3. A noteworthy feature of the model is that it satisfies all the conditions needed for a physically acceptable model. It is observed that when $\alpha = 0$. i.e. in the case of uncharged matter distribution the model reduces to the Thomas and Pandya [200].

Chapter 5, contains a new exact solution of Einsteins's field equations on Finch Skea spacetime. In the literature, one assumes a linear equation of state of the form $p_r = \alpha\rho - \beta$, where ρ is the density and p_r is the radial pressure and α and β are constants. Note that the linearity is in terms of density and not in terms of the radial variable r. This implies that α and β might not be constants and could be the functions of the radial variable r as well. Keeping this in mind in our work, to develop an anisotropic stellar model, we assume a linear equation of state of the form $p_r = \alpha\left(1 - \frac{r^2}{R^2}\right)\rho$, where $0 < \alpha < 1$. This assumption allows us to generate a new class of exact solution to the Einstein field equations which is physical plausible. The solution of field equations has been obtained and the expression of density, radial pressure, and tangential pressure have been calculated. The interior spacetime metric is matched with the Schwarzschild exterior spacetime metric

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.44}$$

and the values of constants of integration and mass have been obtained. It is observed that the total mass of stellar configuration is one-fourth of the radius. The bound on parameter α has been calculated and it is observed that all the physical plausibility conditions are satisfied for $0.06 < \alpha < 0.17$. In particular the pulsar 4U 18020 30 has been considered to demonstrate that all physically viable conditions are satisfied.

Chapter 6, In this chapter, we have reported that Nasheeha *et. al.* [137] studied that models of steller configuration by considering metric potential $g_{rr} = \frac{1+ar^2}{1+(a-b)r^2}$ and equation of state

$$p_r = \tau\rho^{(1+\frac{1}{p})} + \eta\rho - \omega, \tag{1.45}$$

where τ, η, ω and p are real constants. It is noted that the metric potential g_{tt} and many physical entities are not well-behaved in the case of a = b. We consider metric potential $g_{rr} = 1 + ar^2$ which is particular case of $g_{rr} = \frac{1+ar^2}{1+(a-b)r^2}$ when a = b. If p = 1 in equation (1.45), then it becomes a quadratic equation of state. If $\tau = 0$ in equation (1.45), then it becomes a linear equation of state. If $\eta = 0$, in equation (1.45), then it becomes a polytrope with polytropic index p. If $p = \frac{-1}{2}$, $\omega = 0$ and $\tau = -\alpha$, in equation (1.45), then it becomes a Chaplygin equation of state. If $p = -2$, then it becomes a color-flavor-locked (CFL) equation of state. The physical viability of models is tested for strange star candidate 4U 1820 - 30 having mass $M = 1.58M_{\odot}$ and radius R = 9.1 km. All the models are found to be physically plausible. The stability of our model with various equations of state has been compared with the work of Nasheeha *et. al.* [137].

Appendix contains units conversion then the list of publications is given. At last bibliography is provided.